

Graph Theory Fundamentals

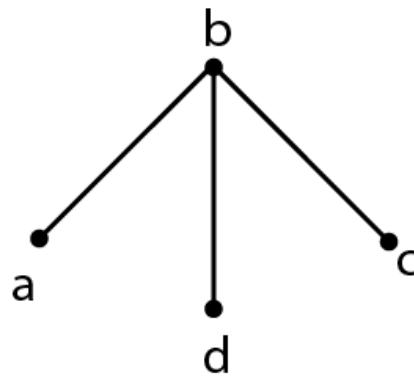
Graph

Notation: $G = (V, E)$

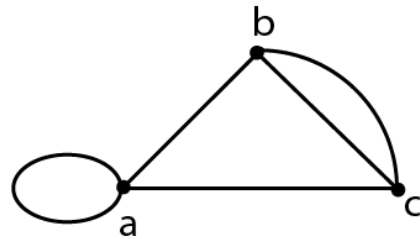
Where G is the graph, V represents the set of vertices and E represents the set of edges.

Simple Graph

A simple graph has no loops or multiple edges.



Simple



Not simple

Neighborhood

A set of all vertices adjacent to a vertex.

Using the simple graph above:

$$N(b) = \{a, d, c\}$$

$$N(a) = \{b\}$$

Degree

Cardinality (count) of a neighborhood.

Using the simple graph shown earlier:

$$\deg(b) = |N(b)| = 3$$

$$\deg(a) = 1$$

Handshaking Theorem

The sum of the degrees of a graph is equal to twice the the number of edges.

Using the simple graph shown earlier:

$$2|E| = \sum_{v \in V} \deg(v)$$

$$2 \times 3 = \deg(a) + \deg(b) + \deg(c) + \deg(d)$$

$$2 \times 3 = 1 + 3 + 1 + 1$$

Complete Graph

Contains exactly one edge between every pair of different vertices.

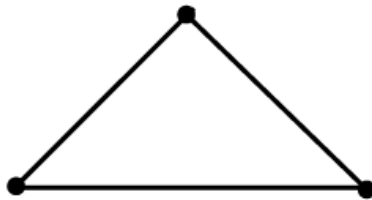
Notation: K_n where n is the number of vertices.



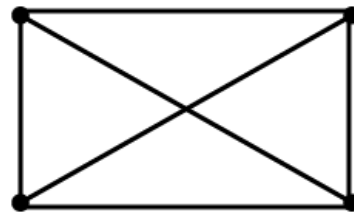
K_1



K_2



K_3



K_4