Graph theory is the study of graphs which are composed of mathematical structures used to model relations between objects. A graph is a combination of nodes (points or vertices) that are connected to one another creating edges. Graph theory was originated in the Eighteenth century by Leonhard Euler in the German city of Königsberg, when he posed the question of whether or not it was possible to traverse 4 bodies of land connected by seven bridges, where each bridge would be crossed exactly once. From the question he posed in image A, the first representation of a modern graph was generated, consisting of four vertices, and seven edges. As a result, a famous Euler path theorem was born, which states that “A connected multigraph has a Euler path if and only if it has exactly two vertices of odd degrees”.

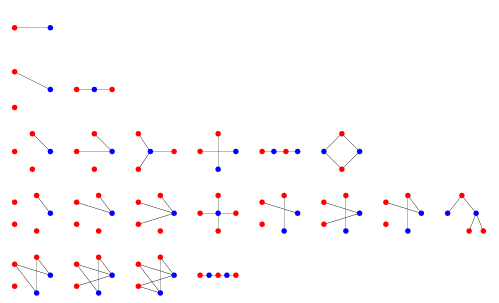
A close up of a map

Description automatically generated

In the modern day, graph theory is utilized as a tool to quantify and simplify the moving parts of dynamic systems (city layouts, computer systems, etc.). Although graph theory is a powerful tool that we use, there are some problems that have been encountered throughout its history, one of the most famous being the graph coloring problem. One of the most interesting problems in graph theory is the graph coloring theory which works closely with whether or not a certain graph is **bipartite**. A graph is considered to be bipartite when the vertices can be partitioned into two separate sets V1 and V2 such that all the edges connect a vertex within one set to a vertex in the other set. There are no edges comprised of vertices in a single set within a bipartite graph. We can consider bipartite graphs to be equivalent to two-colorable graphs, in which the vertices are colored in a manner where the color does repeat along an edge. Cycles are considered to be bipartite, while wheels, and complete graphs are not. An application of bipartite graphs is to find matchings within a graph. Matching of a graph is a set of edges in the graph, but no two edges share the same vertex.

Graph coloring is used as a method of labeling graph components such as vertices and edges with some certain constraints. The purpose of graph coloring is to assign labels (colors) to vertices of a graph in a manner such that no two adjacent vertices, edges or regions have the same color. Within a graph there exists a chromatic number, which is the representation of minimal number of colors that will need to be used in order for graph coloring to be possible. Certain types of graphs, such as complete (Kn) or bipartite (Km,n) only have a few choices possible for how they could be colored.

A graph can also have a chromatic number, which would be least number of colors needed to color the vertices of a graph so that no two adjacent vertices share the same color. Bipartite graphs would then have the chromatic number 2, and could also be referred to as a 2-chromatic graph.



Depicted above are some 2-chromatic (bipartite) graphs.

When a graph’s chromatic number is less than or equal to some *k,* it can be considered to be *k*-colorable. Disconnected graphs have the special distinction of being 1-colorable.

There is currently no way to calculate the chromatic number of an arbitrary graph, however, and this problem is classified as NP-Complete, meaning it is NP and NP-Hard. There is however an algorithm by Mehrotra and Trick used for vertex coloring and finding chromatic numbers that works for reasonably-sized graphs.