Calculus 3, Exam 1 Formulas

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Vector Operations/Identities

$v \cdot w = ||v||\ ||w||\ cos\theta$

$v \cdot v = ||v||^2$

$v \perp w \Leftrightarrow v \cdot w = 0$

<u>Unit vector:</u>

$e_u = \frac{u}{||u||}$

<u>Projection of u along v:</u>

$u_{||v} = \left(\frac{u \cdot v}{v \cdot v}\right) v = \left(\frac{u \cdot v}{||v||^2}\right) v = \left(\frac{u \cdot v}{||v||}\right) e_v$

<u>Component of u along v:</u>

$\frac{u \cdot v}{||v||} = ||u||\ cos\theta$

<u>u perpendicular to v:</u>

$u_{\perp v} = u - u_{||v}$

<u>Decomposition of vector u with respect to v (derived from above):</u>

$u = u_{\perp v} + u_{||v}$

Cross Product

Cross product of v and w:

$$v \times w = \begin{pmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix} = \begin{pmatrix} v_2 & v_3 \\ w_2 & w_3 \end{pmatrix} i - \begin{pmatrix} v_1 & v_3 \\ w_1 & w_3 \end{pmatrix} j + \begin{pmatrix} v_1 & v_2 \\ w_1 & w_2 \end{pmatrix} k$$

(Example for row i): $\begin{pmatrix} v_2 & v_3 \\ w_2 & w_3 \end{pmatrix} i \ ... \ = (v_2 w_3 - v_3 w_2) i \ ...$

About cross product:

$(v \times w) \perp v$ and $(v \times w) \perp w$

$||v \times w|| = ||v|| \ ||w|| \ sin\theta$

Area of parallelogram spanned by v and w = $||v \times w||$

Lines and Planes in Space

Equation of a line:

A line through $p_0 = (x_0, y_0, z_0)$ in the direction of $v =< a, b, c >$ is described as:

$$r(t) = r_0 + t \ v \ = < x_0, y_0, z_0 > + \ t < a, b, c >$$

where $r_0 = \overline{Op_0}$ (a vector from O the origin to p_0), and t is independent.

Parametric equations for line (derived from above):
$x(t) = x_0 + at$
$y(t) = y_0 + bt$
$z(t) = z_0 + ct$

Equation of a plane:

Vector form:
$n \cdot < x, y, z > = \ d$

Scalar form:
$a(x - x_0) + b(y - y_0) + c(z - z_0) \ = \ 0$
or
$ax + by + cz = d$
where
$d = ax_0 + by_0 + cz_0$

Other

Arc length:

$s(t) = \int_0^T ||r'(t)||\ dt$

Speed:

$\frac{d}{dt}s(t) = ||r'(t)||$

Tangent plane:

For $z = f(x, y)$:
tangent plane at (x_0, y_0): $f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$