

1. For the system described, y has a degree of 3 compared to the output u , because the first derivative of y involved only a partial derivative of $h(x_3)$, and also $f_3(x_1, x_2, x_3)$. The second rule contains partial derivatives of $f_3(x_1, x_2, x_3)$, which may include u , depending on f_3 , because of the first derivative of x_1 (which contains $g_1(x_1)u$). In this second derivative of y , u only appears inside other functions, not explicitly. So, the third derivative of y , which contains the second derivative of x_1 , where both u and the first derivative of u appear, determines the degree above.
2. To get our input/output feedback linearizing controller, we need to use the output y differentiated 3 times, as explained above:

$$y^{(3)} = L_f^3 h(x) + L_g L_f^2 h(x) \cdot u$$

So, our control law for u can be defined as:

$$u = \frac{1}{L_g L_f^2 h(x)} (-L_f^3 h(x) + v)$$

(assuming $L_g L_f^2 h(x) \neq 0$ around the origin)

To control the output dynamics, we choose a v so that:

$$y^{(3)} = v$$

Which would take the form of something like (I'm just expressing in terms of y instead of introducing a z coordinate system):

$$v = -\delta_1 y - \delta_2 y' - \delta_3 y''$$

For the zero dynamics, since $y = h(x_3)$, we know that as $x_3 \rightarrow 0$, $y \rightarrow 0$.

So, the remaining states x_1, x_2 must stabilize on their own ($y' = x_3' = f_3(x_1, x_2, 0) = 0$)

The controller stabilizes the origin locally asymptotically only if the dynamics of x_1, x_2 are locally asymptotically stable.