ENGY 604 Project - MILP Formulation (Part A)

Team 3

1. Sets

- R: Set of Primary Resources, $r \in$ {Natural Gas, Biomass, Grid Electricity}
- G: Set of Generators, $g \in$ {Biomass ST, NatGas CHP, Solar PV, Wind Farm}
- D: Set of Conversion Devices, $d \in$ {Refrigerator, LED, Heater}

2. Parameters (Base)

General Parameters

- H: Operating horizon (20 years)
- CF_1: Conversion factor from kW-year to GJ ($\approx$ 31.54 GJ/kW-year)
- CF_2: Conversion factor from GJ to PJ (10^{-6} PJ/GJ)

Resource Parameters

- Price_r: Price of resource r (\$/GJ)
- CO_2Emis_r: CO_2 Emissions for resource r (kg/PJ)

Generator Parameters

- capex_g: Capital cost for generator g (\$/kW)
- opex_g: Operational cost for generator g (\$/kW-year)
- η_g^{elec}: Electrical efficiency of generator g (as fraction)
- η_g^{heat}: Heating efficiency of generator g (as fraction)
- $C_g^{\min}$: Minimum capacity of generator g (kW)
- $C_g^{\max}$: Maximum capacity of generator g (kW)

Conversion Devices Parameters

- capex_d: Capital cost for conversion device d (\$/kW)
- opex_d: Operational cost for conversion device d (\$/kW-year)
- η_d: Efficiency of conversion device d (as fraction, or COP)
- Demand_d: Demand for conversion device d (kW)

3. Parameters (Intermediate)

Total Electrical Demand (kW)

This is the total power required by the electricity-driven conversion devices.

$$\text{ElecDemand}_{\text{total}} = \frac{\text{Demand}_{\text{refrig}}}{\eta_{\text{refrig}}} + \frac{\text{Demand}_{\text{led}}}{\eta_{\text{led}}}$$

Total Heat Demand (kW)

This is the total heat required by the heat-driven conversion devices.

$$\text{HeatDemand}_{\text{total}} = \frac{\text{Demand}_{\text{heater}}}{\eta_{\text{heater}}}$$

Resource Prices ($/kW-year)

This converts the price from $/GJ to $/kW-year using CF_1.

$$\begin{aligned}
\text{Price}^{\text{kwyr}}_{\text{natgas}} &= \text{Price}_{\text{natgas}} \cdot CF_1 \\
\text{Price}^{\text{kwyr}}_{\text{biomass}} &= \text{Price}_{\text{biomass}} \cdot CF_1 \\
\text{Price}^{\text{kwyr}}_{\text{grid}} &= \text{Price}_{\text{grid}} \cdot CF_1
\end{aligned}$$

Resource Emissions (kg/kW-year)

This converts emissions from kg/PJ to kg/kW-year using CF_1 and CF_2.

$$\begin{aligned}
\text{CO}_2\text{Emis}^{\text{kwyr}}_{\text{natgas}} &= \text{CO}_2\text{Emis}_{\text{natgas}} \cdot CF_1 \cdot CF_2 \\
\text{CO}_2\text{Emis}^{\text{kwyr}}_{\text{biomass}} &= \text{CO}_2\text{Emis}_{\text{biomass}} \cdot CF_1 \cdot CF_2 \\
\text{CO}_2\text{Emis}^{\text{kwyr}}_{\text{grid}} &= \text{CO}_2\text{Emis}_{\text{grid}} \cdot CF_1 \cdot CF_2
\end{aligned}$$

Fixed Converter Cost

Capacity for each converter

From Table 1, the required input capacities are:

$$\begin{aligned}
C^{\text{input}}_{\text{refrigerator}} &= \frac{\text{Demand}_{\text{refrigeration}}}{\eta_{\text{refrigerator}}} \\
C^{\text{input}}_{\text{LED}} &= \frac{\text{Demand}_{\text{lighting}}}{\eta_{\text{LED}}} \\
C^{\text{input}}_{\text{heater}} &= \frac{\text{Demand}_{\text{heating}}}{\eta_{\text{heater}}}
\end{aligned}$$

Total fixed converter cost

For capex we use the nameplate demand, and for opex we use the calculated capacity.

$$Cost_{\text{conv}} = \sum_{d \in D} \left(\text{Demand}_d \cdot \text{capex}_d + H \cdot C^{\text{input}}_d \cdot \text{opex}_d \right)$$

4. Mathematical Problem Formulation

4.1 Decision Variables

C_g : Continuous variable for the capacity (size) built for each generator g (kW).

P_g^{elec} : Continuous variable for the actual electric power produced by generator g (kW).

P_r^{cons} : Continuous variable for the amount of primary resource r consumed, in equivalent units of power (kW).

y_g : Binary variable; 1 if generator g is built, 0 otherwise.

4.2 Objective Functions

Objective 1: Minimize Total Cost (Z_{COST})

$$\min \underbrace{\sum_{g \in G} (C_g \cdot \text{capex}_g)}_{\text{Gen CAPEX}} + \underbrace{H \cdot \sum_{g \in G} (C_g \cdot \text{opex}_g)}_{\text{Gen OPEX (20 yr)}} + \underbrace{H \cdot \sum_{r \in R} (P_r^{\text{cons}} \cdot \text{Price}^{\text{kwyr}}{}_r)}_{\text{Resource Cost (20 yr)}} + \underbrace{Cost_{\text{conv}}}_{\text{Fixed Converter Cost}}$$

Objective 2: Minimize Total Emissions (Z_{EMIS})

$$\min H \cdot \sum_{r \in R} (P_r^{\text{cons}} \cdot \text{CO}_2\text{Emis}_{\text{r}}^{\text{kwyr}})$$

Objective 3: Maximize Efficiency (Minimize Primary Input, Z_{INPUT})

$$\min \sum_{r \in R} P_r^{\text{cons}}$$

4.3 Constraints

Note: Wind and solar have no resource costs, so they do not require a fuel balance equation.

C1: Electricity Balance

$$\sum_{g \in G} P_g^{\text{elec}} + P_{\text{Grid}}^{\text{cons}} = \text{ElecDemand}_{\text{total}}$$

C2: Heat Balance

$$P_{\text{CHP}}^{\text{elec}} \cdot \frac{\eta_{\text{CHP}}^{\text{heat}}}{\eta_{\text{CHP}}^{\text{elec}}} = \text{HeatDemand}_{\text{total}} \quad (\text{since } P_{\text{CHP}}^{\text{heat}} = P_{\text{CHP}}^{\text{elec}} \cdot \frac{\eta_{\text{CHP}}^{\text{heat}}}{\eta_{\text{CHP}}^{\text{elec}}})$$

C3: Fuel Balance (Biomass)

$$P_{\text{BioST}}^{\text{elec}} = P_{\text{Biomass}}^{\text{cons}} \cdot \eta_{\text{BioST}}^{\text{elec}}$$

C4: Fuel Balance (Natural Gas)

$$P_{\text{CHP}}^{\text{elec}} = P_{\text{Natural Gas}}^{\text{cons}} \cdot \eta_{\text{CHP}}^{\text{elec}}$$

C5: Generation Output Limit

$$P_g^{\text{elec}} \leq C_g \quad \forall g \in G$$

C6: Capacity Limits (Min/Max)

$$y_g \cdot C_g^{\text{min}} \leq C_g \leq y_g \cdot C_g^{\text{max}} \quad \forall g \in G$$

C7: Integer Cut

$$\sum_{g \in G} y_g \leq 1$$

C8: Non-Negativity & Binary

$$C_g, P_g^{\text{elec}}, P_r^{\text{cons}} \geq 0$$
$$y_g \in \{0, 1\}$$

Appendix A: Parameter Value Tables

Table 1: Technical and economic parameters of on-site energy conversion technologies

Sources	Refrigerator	LED	Heater
Input Resource	Electricity	Electricity	Heat
Output Resource	Refrigeration	Lighting	Space Heating
η_d [%]	300 (COP)	80	85
capex_d [$/kW]	70	10	30
opex_d [$/kW-year]	4	1	3

Table 2: Technical and economic parameters of energy generation technologies

Processes	BioST	CHP	PV	Wind
Input Resource	Biomass	Natural Gas	Solar	Wind
Output Resource	Electricity	Electricity & Heat	Electricity	Electricity
η_g^{elec} [%]	68	44	9	22
η_g^{heat} [%]	0	28	0	0
$C_g^{\min}$ [kW]	100	800	10	10
$C_g^{\max}$ [kW]	1×10^6	1×10^6	300	500
capex_g [$/kW]	250	500	2000	2000
opex_g [$/kW year]	15	15	500	1200

Table 3: Prices and CO2 emissions of the primary energy resources and grid electricity

Sources	Natural Gas	Biomass	Grid Electricity
Price_r [$/GJ]	8.89	9.72	36.11
CO_2Emis_r [kg/PJ]	56	100	90

Table 4: Energy demands for various utilities

Uses	Lighting	Refrigeration	Space Heating
Demand_d [kW]	200	1000	100

Appendix B: Intermediate Parameter Calculations

These are the full worked out conversions for the intermediate parameters.

Total Electrical Demand (kW)

This is the total power required by the electricity-driven conversion devices.

$$\begin{aligned}
\text{ElecDemand}_{\text{total}} &= \frac{\text{Demand}_{\text{refrig}}}{\eta_{\text{refrig}}} + \frac{\text{Demand}_{\text{led}}}{\eta_{\text{led}}} \\
&= \frac{1000\text{ kW}}{3.0} + \frac{200\text{ kW}}{0.80} \\
&= 333.33\text{ kW} + 250\text{ kW} \\
&= 583.33\text{ kW}
\end{aligned}$$

Total Heat Demand (kW)

This is the total heat required by the heat-driven conversion devices.

$$\begin{aligned}
\text{HeatDemand}_{\text{total}} &= \frac{\text{Demand}_{\text{heater}}}{\eta_{\text{heater}}} \\
&= \frac{100\text{ kW}}{0.85} \\
&= 117.65\text{ kW}
\end{aligned}$$

Resource Prices (\$/kW-year)

This converts the price from \$/GJ to \$/kW-year using CF_1.

$$\begin{aligned}
\text{Price}^{\text{kwyr}}_{\text{natgas}} &= \text{Price}_{\text{natgas}} \cdot CF_1 = 8.89 \cdot 31.54 \approx 280.39\ \$/\text{kW-year} \\
\text{Price}^{\text{kwyr}}_{\text{biomass}} &= \text{Price}_{\text{biomass}} \cdot CF_1 = 9.72 \cdot 31.54 \approx 306.57\ \$/\text{kW-year} \\
\text{Price}^{\text{kwyr}}_{\text{grid}} &= \text{Price}_{\text{grid}} \cdot CF_1 = 36.11 \cdot 31.54 \approx 1138.80\ \$/\text{kW-year}
\end{aligned}$$

Resource Emissions (kg/kW-year)

This converts emissions from kg/PJ to kg/kW-year using CF_1 and CF_2.

$$\begin{aligned}
\text{CO}_2\text{Emis}^{\text{kwyr}}_{\text{natgas}} &= \text{CO}_2\text{Emis}_{\text{natgas}} \cdot CF_1 \cdot CF_2 = 56 \cdot 31.54 \cdot 10^{-9} \approx 1.766 \times 10^{-6}\text{ kg/kW-year} \\
\text{CO}_2\text{Emis}^{\text{kwyr}}_{\text{biomass}} &= \text{CO}_2\text{Emis}_{\text{biomass}} \cdot CF_1 \cdot CF_2 = 100 \cdot 31.54 \cdot 10^{-9} \approx 3.154 \times 10^{-6}\text{ kg/kW-year} \\
\text{CO}_2\text{Emis}^{\text{kwyr}}_{\text{grid}} &= \text{CO}_2\text{Emis}_{\text{grid}} \cdot CF_1 \cdot CF_2 = 90 \cdot 31.54 \cdot 10^{-9} \approx 2.839 \times 10^{-6}\text{ kg/kW-year}
\end{aligned}$$

Fixed Converter Cost

Capacity for each converter

From Table 1, the required input capacities are:

$$\begin{aligned}
C^{\text{input}}_{\text{refrigerator}} &= \frac{\text{Demand}_{\text{refrigeration}}}{\eta_{\text{refrigerator}}} = \frac{1000\text{ kW}}{3.0} = 333.33\text{ kW} \\
C^{\text{input}}_{\text{LED}} &= \frac{\text{Demand}_{\text{lighting}}}{\eta_{\text{LED}}} = \frac{200\text{ kW}}{0.80} = 250\text{ kW} \\
C^{\text{input}}_{\text{heater}} &= \frac{\text{Demand}_{\text{heating}}}{\eta_{\text{heater}}} = \frac{100\text{ kW}}{0.85} = 117.65\text{ kW}
\end{aligned}$$

Total fixed converter cost

$$
\begin{aligned}
Cost_{\text{conv}} &= \sum_{d \in D} \left(\text{Demand}_d \cdot \text{capex}_d + H \cdot C_d^{\text{input}} \cdot \text{opex}_d \right) \\
&= \text{Demand}_{\text{refrigeration}} \cdot \text{capex}_{\text{refrigerator}} + H \cdot C_{\text{refrigerator}} \cdot \text{opex}_{\text{refrigerator}} \\
&\quad + \text{Demand}_{\text{lighting}} \cdot \text{capex}_{\text{LED}} + H \cdot C_{\text{LED}} \cdot \text{opex}_{\text{LED}} \\
&\quad + \text{Demand}_{\text{heating}} \cdot \text{capex}_{\text{heater}} + H \cdot C_{\text{heater}} \cdot \text{opex}_{\text{heater}} \\
&= \$70{,}000 + \$26{,}666.40 + \$2{,}000 + \$5{,}000 + \$3{,}000 + \$7{,}059.00 \\
&= \$113{,}725.40 \text{ (total fixed conversion cost over 20 years)}
\end{aligned}
$$