

- For the system described,  $y$  has a degree of 3 compared to the output  $u$ , because the first derivative of  $y$  involved only a partial derivative of  $h(x_3)$ , and also  $f_3(x_1, x_2, x_3)$ . The second rule contains partial derivatives of  $f_3(x_1, x_2, x_3)$ , which may include  $u$ , depending on  $f_3$ , because of the first derivative of  $x_1$  (which contains  $g_1(x_1)u$ ). In this second derivative of  $y$ ,  $u$  only appears inside other functions, not explicitly. So, the third derivative of  $y$ , which contains the second derivative of  $x_1$ , where both  $u$  and the first derivative of  $u$  appear, determines the degree above.
- To get our input/output feedback linearizing controller, we need to use the output  $y$  differentiated 3 times, as explained above:

$$y^{(3)} = L_f^3 h(x) + L_g L_f^2 h(x) \cdot u$$

So, our control law for  $u$  can be defined as:

$$u = \frac{1}{L_g L_f^2 h(x)} (-L_f^3 h(x) + v)$$

(assuming  $L_g L_f^2 h(x) \neq 0$  around the origin)

To control the output dynamics, we choose a  $v$  so that:

$$y^{(3)} = v$$

Which would take the form of something like (I'm just expressing in terms of  $y$  instead of introducing a  $z$  coordinate system):

$$v = -\delta_1 y - \delta_2 y' - \delta_3 y''$$

For the zero dynamics, since  $y = h(x_3)$ , we know that as  $x_3 \rightarrow 0$ .  $y \rightarrow 0$ .

So, the remaining states  $x_1, x_2$  must stabilize on their own ( $y' = x_3' = f_3(x_1, x_2, 0) = 0$ )

The controller stabilizes the origin locally asymptotically only if the dynamics of  $x_1, x_2$  are locally asymptotically stable.