ENGY 604 Project - Multi-Period MILP Formulation (Part B - Scheduling)

Team 3

1. Sets

- T: Set of Time Periods, $t \in \{1, 2, \ldots, 24\}$ (hourly periods in a day)
- R: Set of Primary Resources, $r \in$ {Natural Gas, Biomass, Grid Electricity}
- G: Set of Generators, $g \in$ {Biomass ST, NatGas CHP, Solar PV, Wind Farm}
- D: Set of Conversion Devices, $d \in$ {Refrigerator, LED, Heater}

2. Parameters (Base)

General Parameters

- H_{day}: Operating horizon (1 day = 24 hours)
- Δt: Duration of each time period (1 hour)

Resource Parameters

- Price_r: Price of resource r (\$/GJ) - converted to \$/kWh in Appendix B

Generator Parameters

- capex_g: Capital cost for generator g (\$/kW) - *assumed to be sunk cost*
- opex_g: Operational cost for generator g (\$/kW-year) - converted to \$/kWh in Appendix B
- η_g^{elec}: Electrical efficiency of generator g (as fraction)
- η_g^{heat}: Heating efficiency of generator g (as fraction)
- $C_g^{\min}$: Minimum capacity of generator g (kW)
- $C_g^{\max}$: Maximum capacity of generator g (kW)
- $C_g^{\text{installed}}$: Installed (fixed) capacity of generator g (kW) - *pre-determined*

Conversion Devices Parameters

- η_d: Efficiency of conversion device d (as fraction, or COP)
- $\text{Demand}_{d,t}$: Demand for conversion device d at time t (kW)
  - $\text{Demand}_{\text{refrig},t} = 1000$ kW (constant for all t)
  - $\text{Demand}_{\text{led},t}$: Time-varying (see Table 1)
  - $\text{Demand}_{\text{heater},t} = 100$ kW (constant for all t)

3. Parameters (Intermediate)

Total Electrical Demand at Time t (kW)

This is the total power required by the electricity-driven conversion devices at each time period.

$$\text{ElecDemand}_t = \frac{\text{Demand}_{\text{refrig},t}}{\eta_{\text{refrig}}} + \frac{\text{Demand}_{\text{led},t}}{\eta_{\text{led}}} \quad \forall t \in T$$

Total Heat Demand at Time t (kW)

This is the total heat required by the heat-driven conversion devices at each time period.

$$\text{HeatDemand}_t = \frac{\text{Demand}_{\text{heater},t}}{\eta_{\text{heater}}} \quad \forall t \in T$$

4. Mathematical Problem Formulation

4.1 Decision Variables

$P^{\text{elec}}_{g,t}$: Continuous variable for the actual electric power produced by generator g at time t (kW).

$P^{\text{cons}}_{r,t}$: Continuous variable for the amount of primary resource r consumed at time t, in equivalent units of power (kW).

4.2 Objective Function

Objective: Minimize Total Daily Operating Cost (Z_{COST})

$$\min Z_{\text{COST}} = \underbrace{\sum_{t\in T}\sum_{g\in G}\left(P^{\text{elec}}_{g,t}\cdot \Delta t \cdot \text{opex}_g\right)}_{\text{Generator Operating Cost}} + \underbrace{\sum_{t\in T}\sum_{r\in R}\left(P^{\text{cons}}_{r,t}\cdot \Delta t \cdot \text{Price}_r\right)}_{\text{Resource (Fuel) Cost}}$$

where $\Delta t = 1$ hour for each time period.

4.3 Constraints

Note: The installed capacity $C^{\text{installed}}_g$ for each generator is assumed to be pre-determined (a fixed parameter). Wind and solar have no resource costs.

C1: Electricity Balance (for each time period)

$$\sum_{g\in G} P^{\text{elec}}_{g,t} + P^{\text{cons}}_{\text{Grid},t} = \text{ElecDemand}_t \quad \forall t \in T$$

C2: Heat Balance (for each time period)

$$P^{\text{elec}}_{\text{CHP},t}\cdot \frac{\eta^{\text{heat}}_{\text{CHP}}}{\eta^{\text{elec}}_{\text{CHP}}} = \text{HeatDemand}_t \quad \forall t \in T$$

where $P^{\text{heat}}_{\text{CHP},t} = P^{\text{elec}}_{\text{CHP},t}\cdot \frac{\eta^{\text{heat}}_{\text{CHP}}}{\eta^{\text{elec}}_{\text{CHP}}}$

C3: Fuel Balance - Biomass (for each time period)

$$P^{\text{elec}}_{\text{BioST},t} = P^{\text{cons}}_{\text{Biomass},t}\cdot \eta^{\text{elec}}_{\text{BioST}} \quad \forall t \in T$$

C4: Fuel Balance - Natural Gas (for each time period)

$$P^{\text{elec}}_{\text{CHP},t} = P^{\text{cons}}_{\text{NatGas},t}\cdot \eta^{\text{elec}}_{\text{CHP}} \quad \forall t \in T$$

C5: Generation Output Limit (for each time period)

$$P^{\text{elec}}_{g,t} \le C^{\text{installed}}_g \quad \forall g \in G, \forall t \in T$$

C6: Non-Negativity

$$P^{\text{elec}}_{g,t}, P^{\text{cons}}_{r,t} \ge 0 \quad \forall g \in G, \forall r \in R, \forall t \in T$$

Appendix A: Parameter Value Tables

Table 1: Time-varying lighting demand for each hour of the day

Time Period	Lighting Demand [kW]	Time Period	Lighting Demand [kW]
1	91	13	159
2	88	14	163
3	87	15	165
4	88	16	166
5	92	17	168
6	101	18	176
7	112	19	183
8	123	20	180
9	132	21	172
10	141	22	165
11	150	23	156
12	155	24	147

Table 2: Technical and economic parameters of on-site energy conversion technologies

Sources	Refrigerator	LED	Heater
Input Resource	Electricity	Electricity	Heat
Output Resource	Refrigeration	Lighting	Space Heating
η_d [%]	300 (COP)	80	85

Table 3: Technical and economic parameters of energy generation technologies

Processes	BioST	CHP	PV	Wind
Input Resource	Biomass	Natural Gas	Solar	Wind
Output Resource	Electricity	Electricity & Heat	Electricity	Electricity
η_g^{elec} [%]	68	44	9	22
η_g^{heat} [%]	0	28	0	0
$C_g^{\min}$ [kW]	100	800	10	10
$C_g^{\max}$ [kW]	1×10^6	1×10^6	300	500
$C_g^{\text{installed}}$ [kW]	TBD	TBD	TBD	TBD
capex_g [$/kW]	250	500	2000	2000
opex_g [$/kW-year]	15	15	500	1200

Table 4: Prices of the primary energy resources and grid electricity

Sources	Natural Gas	Biomass	Grid Electricity
Price_r [$/GJ]	8.89	9.72	36.11

Table 5: Energy demands for various utilities (constant demands)

Uses	Refrigeration	Space Heating
Demand_d [kW]	1000	100

Appendix B: Sample Intermediate Parameter Calculations

These are sample calculations for selected time periods to illustrate the intermediate parameters.

Resource Price Conversions ($/GJ to $/kWh)

Converting from $/GJ to $/kWh using the conversion factor 1 GJ = 277.78 kWh (or equivalently, divide by 3600 since 1 kWh = 3.6 MJ):

$$\text{Price}_{\text{natgas}} = \frac{8.89\ \$/\text{GJ}}{277.78\ \text{kWh/GJ}} = 0.00247\ \$/\text{kWh}$$

$$\text{Price}_{\text{biomass}} = \frac{9.72\ \$/\text{GJ}}{277.78\ \text{kWh/GJ}} = 0.00270\ \$/\text{kWh}$$

$$\text{Price}_{\text{grid}} = \frac{36.11\ \$/\text{GJ}}{277.78\ \text{kWh/GJ}} = 0.01003\ \$/\text{kWh}$$

Generator OPEX Conversions ($/kW-year to $/kWh)

Converting from $/kW-year to $/kWh by dividing by 8760 hours/year:

$$\text{opex}_{\text{BioST}} = \frac{15\ \$/\text{kW-year}}{8760\ \text{hours/year}} = 0.00171\ \$/\text{kWh}$$

$$\text{opex}_{\text{CHP}} = \frac{15\ \$/\text{kW-year}}{8760\ \text{hours/year}} = 0.00171\ \$/\text{kWh}$$

$$\text{opex}_{\text{PV}} = \frac{500\ \$/\text{kW-year}}{8760\ \text{hours/year}} = 0.05707\ \$/\text{kWh}$$

$$\text{opex}_{\text{Wind}} = \frac{1200\ \$/\text{kW-year}}{8760\ \text{hours/year}} = 0.13699\ \$/\text{kWh}$$

Total Electrical Demand (kW) - Sample Calculations

For $t = 1$ (Time Period 1):

$$\begin{aligned}
\text{ElecDemand}_1 &= \frac{\text{Demand}_{\text{refrig},1}}{\eta_{\text{refrig}}} + \frac{\text{Demand}_{\text{led},1}}{\eta_{\text{led}}} \\
&= \frac{1000\ \text{kW}}{3.0} + \frac{91\ \text{kW}}{0.80} \\
&= 333.33\ \text{kW} + 113.75\ \text{kW} \\
&= 447.08\ \text{kW}
\end{aligned}$$

For $t = 19$ (Time Period 19 - Peak Lighting):

$$\begin{aligned}
\text{ElecDemand}_{19} &= \frac{1000\ \text{kW}}{3.0} + \frac{183\ \text{kW}}{0.80} \\
&= 333.33\ \text{kW} + 228.75\ \text{kW} \\
&= 562.08\ \text{kW}
\end{aligned}$$

Total Heat Demand (kW)

This remains constant for all time periods:

$$\begin{aligned}
\text{HeatDemand}_t &= \frac{\text{Demand}_{\text{heater},t}}{\eta_{\text{heater}}} \\
&= \frac{100\ \text{kW}}{0.85} \\
&= 117.65\ \text{kW} \quad \forall t \in T
\end{aligned}$$

Key Notes for Multi-Period Model

- The installed capacity $C_g^{\text{installed}}$ must be specified before solving the scheduling problem. This represents the generation capacity already built and available for dispatch.
- Capital costs (CAPEX) are not included in the objective function as they are considered sunk costs.
- The model determines the optimal hourly dispatch of generation resources to meet time-varying demands at minimum operating cost.
- Lighting demand varies from 87 kW (minimum at $t = 3$) to 183 kW (maximum at $t = 19$).
- Total electrical demand varies from approximately 442 kW to 562 kW across the 24-hour period.