aleg, about Matther C.

Question 7.

$$\int_{0}^{1} \frac{f'(x)}{[f(x)]^2 \sqrt{1-[f(x)]^2}} \, dx = \int_{\pi/6}^{\pi/3} \frac{1}{\sin^2 u} \, du$$

let $u = \arcsin f(x) \Rightarrow f(x) = \sin u$

$$du = \frac{f'(x)}{\sqrt{1-[f(x)]^2}} \, dx$$

$$= \int_{\pi/6}^{\pi/3} \csc^2 u \, du$$

if $x=1$, $u = \arcsin[f(1)] =$

$$= -\cot u \Big|_{\pi/6}^{\pi/3}$$

$$= \arcsin \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{3}$$

$$= -\cot(\pi/3) + \cot(\pi/6)$$

$x=0$, $u = \arcsin[f(0)]$

$$= \arcsin \frac{1}{2}$$

$$= -\frac{\sqrt{3}}{3} + \sqrt{3}$$

$$= \frac{\pi}{4}$$

$$\boxed{= \frac{2\sqrt{3}}{3}}$$