Alejo, Albert Matthew C.

*

Question 13.

$$\int_0^2 x e^{x^2-|2x^2-1|}\,dx$$

for $x \in [0;2]$, $2x^2-1 \geq 0$ for $x \in [\frac{\sqrt{2}}{2}, 2]$

$2x^2-1 < 0$ for $x \in [0, \frac{\sqrt{2}}{2})$

$$|2x^2-1| = \begin{cases} 2x^2-1 & \text{if } x \geq \frac{\sqrt{2}}{2} \\ -2x+1 & \text{if } x < \frac{\sqrt{2}}{2} \end{cases}$$

$$\int_0^2 x e^{x^2-|2x^2-1|} = \int_0^{\frac{\sqrt{2}}{2}} x e^{x^2-(-2x^2+1)}\,dx + \int_{\frac{\sqrt{2}}{2}}^2 x e^{x^2-(2x^2-1)}\,dx$$

$$= \int_0^{\sqrt{2}/2} x e^{3x^2-1}\,dx + \int_{\sqrt{2}/2}^2 x e^{-x^2+1}\,dx$$

$$= \int_{-1}^{1/2} \frac{1}{4} e^u\,du + \int_{-1/2}^{-3} -\frac{1}{2} e^v\,dv$$

$$= \frac{1}{6}\int_{-1}^{1/2} e^u\,du + \frac{1}{2}\int_{-3}^{-1/2} e^v\,dv$$

$$= \frac{1}{6} e^u \Big|_{-1}^{1/2} + \frac{1}{2} e^v \Big|_{-3}^{-1/2}$$

$$= \frac{1}{6} e^{1/2} - \frac{1}{6} e^{-1} + \frac{1}{2} e^{-1/2} - \frac{1}{2} e^{-3}$$

$$= \frac{\sqrt{e}}{6} - \frac{1}{6e} + \frac{1}{2\sqrt{3}} - \frac{1}{2e^3}$$

$$\approx 1.0129$$

let $u = 3x^2 - 1$

$du = 6x\,dx$

$\frac{du}{6} = x\,dx$

if $x = 0$, $u = 3x^2 - 1 = -1$

$x = \frac{\sqrt{2}}{2}$; $u = 3x^2 - 1 = \frac{1}{2}$

let $v = -x^2 + 1$

$dv = -2x\,dx$

$-\frac{1}{2}\,dv = x\,dx$

if $x = \frac{\sqrt{2}}{2}$; $v = -x^2 + 1 = \frac{1}{2}$

$x = 2$, $v = -x^2 + 1 = -3$