Mathematical Analysis IB

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Review on differentiation

Differentiability

Let f be a function on some open interval I containing x. The derivative of f at x, denoted by $f'(x)$, is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiation rules

1. $$\frac{d}{dx}(cf(x)) = cf'(x)$$
2. $$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$$
3. $$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)$$
4. $$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$
5. $$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

Mean value theorem

Let f be a function that is continuous on $[a, b]$ and is differentiable on (a, b). Then there is a number $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Coroalrries of MVT

Zero Derivative

If $f'(x) = 0 \; \forall x$ in interval I, then $f(x) = c \; \forall x \in I$ for some constant C.

Equal derivatives

If $f'(x) - g'(x) \; \forall x$ in an interval I, then $f(x) = g(x) + C$ for some constant C.

Differentiation formulas

1. $$\frac{d}{dx}(e^x) = e^x$$

2. $$\frac{d}{dx}(ln|x|) = \frac{1}{x}$$

3. $$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

4. $$\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$$

5. $$\frac{d}{dx}(sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

Differentials

$$f'(x) = \frac{dy}{dx}$$
$$dy = f'(x)dx$$

Module 1: Indefinite and definite integrals

Antiderivatives

Substitution rule

The area problem

The definite Integrals

The Fundamental Theorem of Calculus

Proof of Fundamental Theorem of Calculus

Module 2: Application I

Areas between curves

Volumes and volumes of revolution using disks and washers

Volumes of solids of revolution using cylindrical shells

Module 3: Techniques of integration

Integration by parts

Trigonometric integrals

Trigonometric Substitution

Partial fractions

Applications II

Arc length

Variable-separable differential equations and models for population growth