Mathematical Analysis IB

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# 0 - Review on differentiation

## Differentiability

Let be a function on some open interval containing . The derivative of at , denoted by , is

## Differentiation rules

## Differentiation formulas I

## Differentiation formulas II

## Mean value theorem

Let be a function that is continuous on and is differentiable on . Then there is a number such that

## Consequences of MVT

### Zero derivative

If in interval , then for some constant .

### Equal derivatives

If in an interval , then for some constant .

#### Example

Let and

This implies that and

Since for , then for some constant by a corollary.

Substituting , in this case, let’s use ,

## Differentials

# 1 - Indefinite and definite integrals

## Indefinite integral

The main interpretation of derivative is the slope of a tangent line of a curve.

#### Example

At any point on a particular curve , the tangent line has a slope equal to . If the curve contains the point , find .

**Solution.** Since the slope is equal to for any point , then the slope at is .

therefore represents the tangent slope for all values of . So

By intuition, we can conclude that .

However given , remains the same. And so is , , and infinitely more functions. We can arbitrarily assign a constant , so that .

Substituting

So .

### Definition of an antiderivative

A function is called an antiderivative of the function on an interval if .

is a **possible** antiderivative of . is also a **possible** antiderivative of .

### Equal derivatives

If in an interval , then for some constant .

### Integration notation

The collection of all antiderivatives of is denoted by

which is read as “the integral of .”

This collection is also called the **indefinite integral** of .

The reverse process if differentiation is called **antidifferentiation** or **integration**.

for some constant .

is the constant of integration.

### Integration rules

1. , constant

### Integration formulas I

### Integration formulas II

### Integration formulas III

## Substitution rule

### Chain rule for derivatives

If follows that

#### Example

Evaluate .

**Preliminary work.** By intuition, we can get and

**Solution.** Suppose that

Let , then

Let

### Definition of the substitution rule

If is a differentiable function whose range is interval and is continuous on , then

## The Fundamental Theorem of Calculus

### The area problem

Let be a continuous nonnegative function on . Find the area of the regiom bounded by the curve , the lines , , and the -axis.

The area is often coined the **region under the curve**, which generally means the area in between the curve and the -axis

#### Example

Consider on .

**Solution.** Let be the area under the curve\

Using right endpoints (5 rectangles)

Rectangle 1:

Rectangle 2:

Rectangle 3:

Rectangle 4:

Rectangle 5:

is an overestimation of .\

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Rectangle 1:

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We can increase the number of rectangles and compute the area more **accurately** by computing the area as the number of rectangles approach infinity.\

Let the number of rectangles be

Let be the first point:

Consider $y=f(x) $ on .

Let ,

This is also called the **Riemann sum**.

### Definite integral and integrability

The definite integral of from to is

provided that such limit exists.

We say that is integrable on

#### Remarks

1. If a function is continuous on , it is integrable on .
2. If is a nonnegative continuous function on , then is the area under the curve from and

### Conventions on definite integral

### Properties of the definite integral

## Proof of the Fundamental Theorem of Calculus

## The area problem

## The definite Integrals

# 2 - Application I

## Areas between curves

## Volumes and volumes of revolution using disks and washers

## Volumes of solids of revolution using cylindrical shells

# 3 - Techniques of integration

## Integration by parts

## Trigonometric integrals

## Trigonometric substitution

## Partial fractions

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## Arc length

## Variable-separable differential equations and models for population growth