The purpose of this assignment is for you to get used to the concept of graphs and also to realize that graphs are fun!

1) Show that in a graph the number of vertices of odd degree is even.

2) Five international delegations meet at a round table on one evening. There are two Americans, two Frenchmen, two Germans, two Englishmen, and two Italians. Each possible pair of nationalities must occur in the seating. That is, there must be an American seated next to a Frenchman, a German seated next to an Italian, and so on. How would you arrange the seating order?

3) Seven students go on vacations. They decide that each will send a postcard to three of the others. Is it possible that every student receives postcards from precisely the three to whom he/she sent postcards?

4) Two men have a full eight-gallon jug of wine, and also two empty jugs of five and three gallon capacity, respectively. What is the simplest way for them to divide the wine equally?

5) A mouse eats his way through a 3X3X3 cube of cheese by tunneling through all of the 27 1X1X1 subcubes. If he starts at one corner and always moves on to an uneaten subcube, can he finish at the center of the cube?

6) A sequence of non-negative non-increasing sequence of integers is *graphic* if there exists a graph whose degree sequence is precisely that sequence. There is an algorithm that checks whether a given sequence is graphic by looking at another, simpler sequence. For example, if the sequence is (6, 5, 5, 4, 3, 3, 2, 2, 2), then we can check whether it is graphic by applying the following algorithm.

(1) 6 5 5 4 3 3 2 2 2
(2) 4 4 3 2 2 1 2 2 (imagine that you are deleting a node with degree 6 from the graph; that means for 6 nodes, the degree will go down by 1)
(3) 4 4 3 2 2 2 2 1 (rearrange the sequence in the non-increasing order)
(4) 3 2 1 1 2 2 1 (now you have deleted a node with degree 4 from the graph; that means for 4 nodes, the degree will go down by 1)
(5) 3 2 2 2 1 1 1 (rearrange the sequence in the non-increasing order)
(6) 1 1 1 1 1 1 (now you have deleted a node with degree 3 from the graph; that means for 3 nodes, the degree will go down by 1)

Now you can construct a graph with 6 nodes each with degree 1 (see above) which means that the sequence (1, 1, 1, 1, 1, 1) is graphic. So the original sequence must be graphic also.

Which of the following sequences are graphic?

a. (7, 6, 5, 4, 3, 3, 2)

b. (6, 6, 5, 4, 3, 3, 1)
c. (5, 5, 4, 4, 3, 2, 2, 1, 1)
d. (6, 5, 4, 3, 2, 2, 2, 2)
e. (4, 4, 4, 4, 3, 3)
f. (7, 6, 5, 4, 4, 3, 2, 1)

7) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.

8) Show that in any graph *G* with 6 vertices either G or its complement contains a triangle.

- A tree is a connected graph with no cycles. If *G* is a tree with *n* vertices then the *e* (number of edges) is always *n* -1.

- Conversely, if *G* is connected and $e = (n - 1)$ where *n* is the number of nodes in the tree, then *G* is a tree.

9) Let *T* be a tree with vertices of degree only 3 or 1. If *T* has 10 vertices of degree 3, how many vertices of degree 1 are in *T*?

10) The average degree of nodes of a tree is 1.99. How many edges does the tree have?

11) A tree has only nodes of degree 5 and 1. If the tree has $4p+2$ nodes, how many have degree 5?