

1. Insert the following keys into an empty red-black tree and show the tree **after each insertion**. The keys should be taken as strings of characters not as months.  
For example, Jul < Jun.

DEC, JAN, APR, MAR, JUL, AUG, OCT, SEP, FEB, NOV, MAY, JUN

2. According to the lower-bound argument for comparison based sorting,  $\lceil \log n! \rceil$  comparisons are necessary for sorting  $n$  elements. For  $n = 4$ , you need minimally  $\lceil \log 4! \rceil = \lceil \log 24 \rceil = 5$  comparisons and for  $n = 5$ , we need minimally,  $\lceil \log 5! \rceil = \lceil \log 120 \rceil = 7$  comparisons. Devise a method that sorts 4 elements using exactly 5 comparisons and a method to sort 5 elements in exactly 7 comparisons.

3. Devise a method to find the median of 5 elements in exactly 6 comparisons
4. Solve the following recurrences. Use the Master theorem wherever possible:

a)  $T(n) = 2 T(n/2) + n^3$

b)  $T(n) = T(9n/10) + n$

c)  $T(n) = 16 T(n/4) + n^2$

d)  $T(n) = 7 T(n/3) + n^2$

e)  $T(n) = 7 T(n/2) + n^2$

f)  $T(n) = 2 T(n/4) + \text{sqrt}(n)$

g)  $T(n) = T(n-1) + n$

h)  $T(n) = T(\text{sqrt}(n)) + 1$