

# TMA4315: Compulsory exercise 2: Logistic regression and Poisson regression

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## Part 2

a)

According to the  $\chi^2$ -test performed by the author, there is no evidence against the assumption of independence between goals scored by the home and the away team.

```
filepath <- "https://www.math.ntnu.no/emner/TMA4315/2018h/eliteserien2018"
eliteserie <- read.table(file = filepath, header = TRUE, colClasses = c("character",
  "character", "numeric", "numeric"))
contingency_data <- eliteserie
for (i in 1:length(contingency_data$home)) {
  if (contingency_data[i, ]$yh > 4) {
    contingency_data[i, ]$yh <- 4
  }
  if (contingency_data[i, ]$ya > 4) {
    contingency_data[i, ]$ya <- 4
  }
}
contingency_table <- table(contingency_data$ya, contingency_data$yh)
rownames(contingency_table) <- c("0", "1", "2", "3", "4+")
colnames(contingency_table) <- c("0", "1", "2", "3", "4+")
contingency_table
```

```
##
##      0  1  2  3 4+
## 0   8 19 10 13  8
## 1  18 26 14 10  7
## 2   3 15 13  7  3
## 3   1  5  4  2  1
## 4+  1  3  1  0  0
```

```
chisq.test(contingency_table)
```

```
##
## Pearson's Chi-squared test
##
## data:  contingency_table
## X-squared = 14.156, df = 16, p-value = 0.5871
```

As we can see in our  $\chi^2$ -test, the p-value (0.5871) is high, which supports the  $H_0$ -hypotheses that the goals scored by the home and the away team is independent. We would have to consider some dependency if the p-value had been under our significance value at 0.05. We have clustered goals of 4 or higher in the same row/column as the  $\chi^2$ -test will be more reliable if there is more than 5 occurrences in each row and column.

b)

```
# This function computes the ranking and the goal difference, and
# returns a result table (not ordered)
calculate_points <- function(data_eliteserie) {
  result_table <- data.frame(team <- unique(data_eliteserie$home),
    position <- seq(1, length(team), by = 1), goal_for <- rep(0,
      length(team)), goal_against <- rep(0, length(team)), goal_score <- rep(0,
      length(team)), points <- rep(0, length(team)))
  colnames(result_table) <- c("Team", "Position", "GF", "GA", "GD",
    "Points")
  for (i in 1:length(data_eliteserie$home)) {
    if (data_eliteserie$yh[i] > data_eliteserie$ya[i]) {
      index = which(result_table$Team == data_eliteserie$home[i])
      result_table[index, ]$Points = result_table[index, ]$Points +
        3
    } else if (data_eliteserie$yh[i] < data_eliteserie$ya[i]) {
      index = which(result_table$Team == data_eliteserie$away[i])
      result_table[index, ]$Points = result_table[index, ]$Points +
        3
    } else {
      result_table[which(result_table$Team == data_eliteserie$home[i]),
        ]$Points = result_table[which(result_table$Team == data_eliteserie$home[i]),
        ]$Points + 1
      result_table[which(result_table$Team == data_eliteserie$away[i]),
        ]$Points = result_table[which(result_table$Team == data_eliteserie$away[i]),
        ]$Points + 1
    }
    result_table[which(result_table$Team == data_eliteserie$home[i]),
      ]$GF = result_table[which(result_table$Team == data_eliteserie$home[i]),
      ]$GF + data_eliteserie$yh[i]
    result_table[which(result_table$Team == data_eliteserie$home[i]),
      ]$GA = result_table[which(result_table$Team == data_eliteserie$home[i]),
      ]$GA + data_eliteserie$ya[i]
    result_table[which(result_table$Team == data_eliteserie$away[i]),
      ]$GF = result_table[which(result_table$Team == data_eliteserie$away[i]),
      ]$GF + data_eliteserie$ya[i]
    result_table[which(result_table$Team == data_eliteserie$away[i]),
      ]$GA = result_table[which(result_table$Team == data_eliteserie$away[i]),
      ]$GA + data_eliteserie$yh[i]
  }
  result_table$GD <- result_table$GF - result_table$GA
  ordered_table <- result_table[order(-result_table$Points, -result_table$GD),
    ]
  i <- 1
  for (team in ordered_table$Team) {
    result_table[which(result_table$Team == team), ]$Position <- i
    i <- i + 1
  }
}
```

```

    return(result_table)
}
result_table <- calculate_points(eliteserie)
result_table <- result_table[order(-result_table$Points, -result_table$GD),
]
print(result_table)

```

```

##           Team Position GF GA  GD Points
## 10      Rosenborg       1 43 20  23     52
## 11        Brann       2 36 23  13     48
## 1      Molde       3 48 30  18     43
## 12     Haugesund       4 36 28   8     41
## 8      Ranheim_TF       5 38 40  -2     38
## 13     Vaalerenga       6 35 37  -2     36
## 3           Odd       7 35 29   6     34
## 14     Tromsø       8 35 33   2     33
## 6     Sarpsborg08       9 39 34   5     32
## 7    Kristiansund      10 32 35  -3     31
## 4     Bodø/Glimt      11 28 30  -2     27
## 2    Stjøermsgodset     12 38 38   0     26
## 9      Lillestrøm     13 26 37 -11     25
## 16          Stabæk     14 29 43 -14     23
## 5           Start     15 24 42 -18     23
## 15 Sandefjord_Fotball  16 24 47 -23     15

```

Here is a the table ordered after points, where GF is goals scored, GA is goals against and GD is the goal balance. We can see that Rosenborg is leading with 4 points to Brann and 9 to Molde. The 4 point gap to Brann is not huge, considering there are 6 games left to play, and when Rosenborg and Brann played last time (where Rosenborg had the home advantage), Brann won. They have an unplayed match where Brann has the home advantage, and that match can be deciding.

c

```

# This code generates the design matrix, as well as computing the
# strength parameters with our own function.
library(myglm)
goals <- c(eliteserie$yh, eliteserie$ya)
X <- matrix(data = 0, nrow = 384, ncol = 17)
colnames(X) <- c("Intercept", "HomeAdvantage", unique(eliteserie$home)[-4])
for (i in 1:length(eliteserie$home)) {
  X[i, 1] = 1
  X[i, 2] = 1
  home_index <- which(colnames(X) == eliteserie$home[i])
  away_index <- which(colnames(X) == eliteserie$away[i])
  X[i, home_index] <- 1
  X[i, away_index] <- -1
}
for (i in 1:length(eliteserie$away)) {
  X[i + length(eliteserie$home), 1] = 1
  home_index <- which(colnames(X) == eliteserie$home[i])
  away_index <- which(colnames(X) == eliteserie$away[i])
  X[i + length(eliteserie$home), home_index] <- -1
  X[i + length(eliteserie$home), away_index] <- 1
}

```

```

strength_param <- myglm(goals ~ -1 + X)
strength_param <- strength_param[order(-strength_param)]
names(strength_param) <- substring(names(strength_param), 2)
strength_param

```

```

##      HomeAdvantage      Rosenborg      Molde
##      0.402062206      0.367125310      0.279399199
##      Brann      Haugesund      Intercept
##      0.225775206      0.141301460      0.100321807
##      Odd      Sarpsborg08      Tromsøe
##      0.100120614      0.097677349      0.060581141
##      Stroemsgodset      Vaalerenga      Kristiansund
##      0.049792126      0.014730410      0.012552907
##      Ranheim_TF      Lillestroem      Stabaek
##      0.008502727      -0.132621109      -0.147940567
##      Start Sandefjord_Fotball
##      -0.225757649      -0.291683130

```

Here we see the estimated strength-parameters, with the strength parameter of BodoeGlimt set to zero. The expected number of goals for a home team A is thus  $E[A] = \exp(\beta_0 + \beta_{home} + \beta_A - \beta_B)$  and the away team B has the expected value  $E[B] = \exp(\beta_0 + \beta_B - \beta_A)$ . We see that the intercept is some sort of “default” strength for BodoeGlimt, and if a BodoeGlimt play away against a team with higher strength parameter than the intercept, they would have a expected number of goals less than 1 (the sum of the betas less than zero). Rewriting as  $E[A] = e^{\beta_0} e^{\beta_{home}} e^{\beta_A} / e^{\beta_B}$  we can more easily see that the expected number of goals for a team is e in the power of the intercept, times e in the power of the home parameter (if a home game for the team), times the e to the power of the strength parameter of the team, divided by e to the power of the strength parameter of the opponent. An interesting point is that Molde has a higher strength parameter than Brann, even though Brann has 5 more points so far in the season. That can be a result of Molde meeting better opponents than Brann so far, and the fact that Molde won both the matches between the teams, with 5-1 home and 0-4 away. Ranheim TF had a much lower strength parameter than there position in the table suggests. Looking at their played games, one can see that they are very unstable, loosing to teams that are considered worse, and winning against better teams. Thus it’s hard to analyse their strength, and give it a number. The regression has probably considered some of their wins against better teams like Molde luck, which would explain the pessimistic strength estimate.

```

# This code simulates n seasons, and write the ranking for each
# season to a txt-file.
library("reshape2")
set.seed(42)
filepath <- "https://www.math.ntnu.no/emner/TMA4315/2018h/unplayed2018"
eliteserie_unplayed <- read.table(file = filepath, header = TRUE, colClasses = c("character",
"character"))
simulate_season_end <- function(data_unplayed_matches, strength_param) {
  for (i in 1:length(data_unplayed_matches$home)) {
    intercept <- strength_param["Intercept"]
    home_advantage <- strength_param["HomeAdvantage"]
    strength_hometeam <- strength_param[data_unplayed_matches$home[i]]
    strength_awayteam <- strength_param[data_unplayed_matches$away[i]]
    if (data_unplayed_matches$home[i] == "BodoeGlimt") {
      strength_hometeam <- 0
    } else if (data_unplayed_matches$away[i] == "BodoeGlimt") {
      strength_awayteam <- 0
    }
    data_unplayed_matches$yh[i] <- rpois(1, exp(strength_hometeam +
intercept + home_advantage - strength_awayteam))
  }
}

```

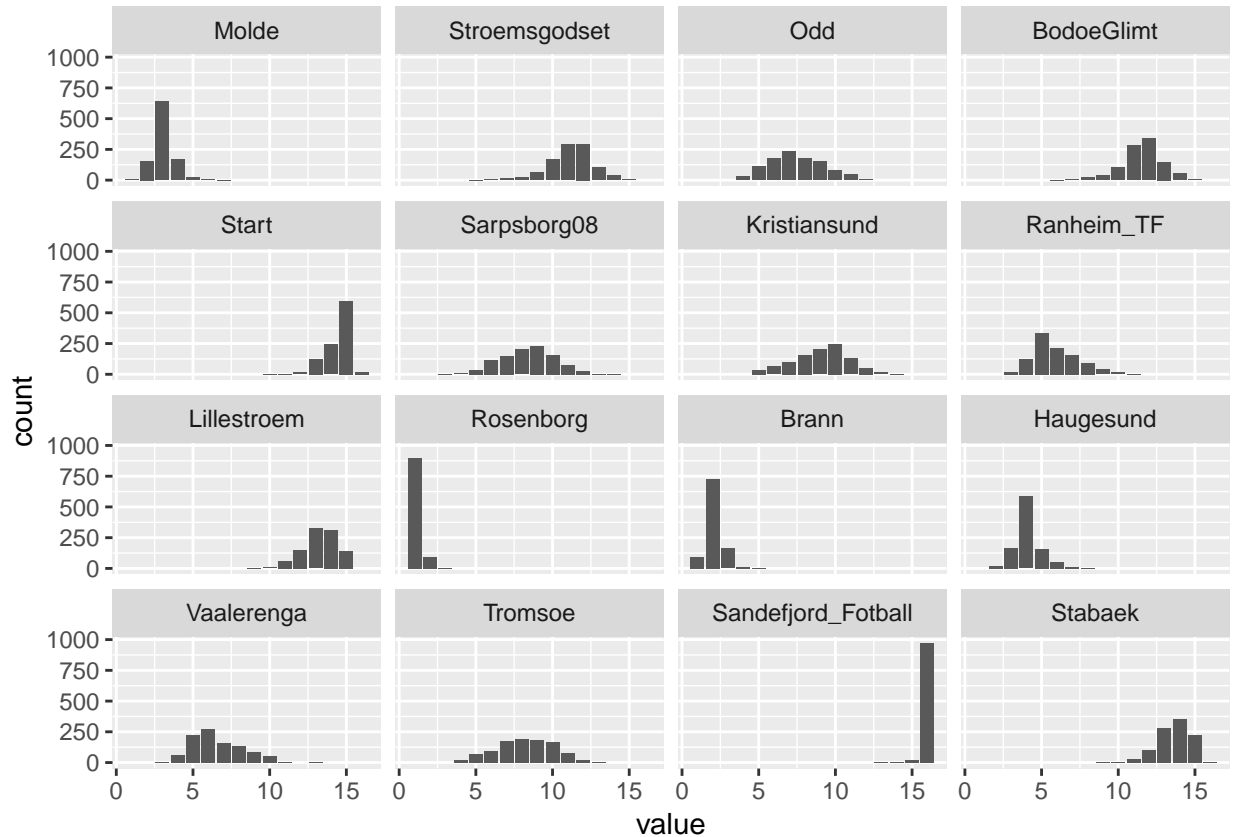
```

        data_unplayed_matches$ya[i] <- rpois(1, exp(strength_awayteam +
            intercept - strength_hometeam))
    }
    return(data_unplayed_matches)
}

Points <- c()
for (i in 1:1000) {
    simulated_results <- simulate_season_end(eliteserie_unplayed, strength_param)
    eliteserie_finished <- rbind(eliteserie, simulated_results)
    standings <- calculate_points(eliteserie_finished)
    write.table(standings, paste("Standings\\Standings_", toString(i),
        ".txt", sep = ""), sep = "\t", quote = FALSE)
}

# This code loads n season simulations, and plots the result
# distribution as bar diagram.
library("ggplot2")
library("reshape2")
n <- 1000
teams <- unique(eliteserie$home)
Positions <- matrix(0, nrow = length(teams), ncol = n)
Points <- matrix(0, nrow = length(teams), ncol = n)
rownames(Positions) <- c(teams)
rownames(Points) <- c(teams)
for (i in 1:n) {
    standings <- read.table(file = paste("Standings\\Standings_", toString(i),
        ".txt", sep = ""), header = TRUE, colClasses = c("numeric", "character",
        "numeric", "numeric", "numeric", "numeric"))
    Positions[, i] <- standings$Position
    Points[, i] <- standings$Points
}
Positions <- melt(Positions, id.vars = c("Teams"))
p <- ggplot(data = data.frame(Positions), aes(x = value)) + geom_bar() +
    facet_wrap(~Var1)
p

```



From this bar diagrams over the different simulations, we can see that Rosenborg wins around 900/1000 simulations, whereas Brann wins the remaining around 100 simulations. Molde wins about 2 or 3 of the simulations, but Rosenborg and Brann are mainly our contestants for the win. The omission of explanatory variables such as Rosenborg are playing Europe League in addition to Eliteserien, and their players will be more exhausted, can cause for a lower variability in the data than we see in our simulations. There are much more variability in the middle of the table, which suggests that the three to teams (Rosenborg and Brann) are a lot better than the others, and Sandefjord being the worst (8 points behind the next team on the table so far). Another point to make is that this regression only set a general value on each teams strength, but following football results one knows that some teams perform better against other specific teams and vice versa. This regression only estimates the general strength of Rosenborg compared to all the other teams, but for a more realistic simulation one would have to estimate the strength difference between each team spesifically.

```
# This code computes result statistics such as mean and standard
# deviation
result_statistics <- data.frame(Team = rownames(Points), Position = seq(1,
  length(Points[, 1]), by = 1), Mean = rowMeans(Points), row.names = NULL)

for (i in 1:nrow(Points)) {
  result_statistics$StdDev[i] <- sqrt(sum((Points[i, ] - result_statistics$Mean[[i]])^2/(length(Points[i, ])-1)))
}
result_statistics <- result_statistics[order(-result_statistics$Mean),
  ]
result_statistics$Position <- seq(1, length(result_statistics$Team),
  by = 1)
result_statistics
```

	Team	Position	Mean	StdDev
## 10	Rosenborg	1	64.588	2.845761
## 11	Brann	2	58.814	3.171666
## 1	Molde	3	53.887	3.136570
## 12	Haugesund	4	50.239	2.857980
## 8	Ranheim_TF	5	46.089	3.080677
## 13	Vaalerenga	6	44.477	3.050701
## 3	Odd	7	42.338	2.972641
## 14	Tromsøe	8	41.268	3.022468
## 6	Sarpsborg08	9	40.696	2.966544
## 7	Kristiansund	10	39.994	3.098639
## 2	Stroemsgodset	11	35.304	3.106321
## 4	BodoeGlimt	12	34.802	3.030508
## 9	Lillestroem	13	30.629	2.951961
## 16	Stabaek	14	29.785	3.061396
## 5	Start	15	27.855	2.703939
## 15	Sandefjord_Fotball	16	19.565	2.637941

Looking at the means, there is a considerable difference in points between Rosenborg and Brann, at 5.774 points. However, the standard deviations for the teams will suggest that if Rosenborg being close to the lower limit and Brann close to the upper limit, would give Brann more points than Rosenborg. If one also take into account that they are playing against each other in the final part of the season, not only giving the winner three points, but also ensures that the looser gets zero points in that game, we have plausible scenarios where Brann wins Eliteserien. As shown in our simulations, they only win around 100/1000, but the match between the two teams will be crucial in the first place battle.

```
# Code from myglm-package
myglm <- function(formula, data = list(), contrasts = NULL, ...) {
  # Extract model matrix & responses
  mf <- model.frame(formula = formula, data = data)
  X <- model.matrix(attr(mf, "terms"), data = mf, contrasts.arg = contrasts)
  y <- model.response(mf)
  terms <- attr(mf, "terms")

  # Add code here to calculate coefficients, residuals, fitted values,
  # etc... and store the results in the list est
  est <- list(terms = terms, model = mf)

  par <- rep(0, ncol(X))
  betahat <- optim(par = par, fn = loglik_poi, x = X, y = y, method = "BFGS")$par
  names(betahat) = colnames(X)
  est$beta <- betahat

  # Store call and formula used
  est$call <- match.call()
  est$formula <- formula

  # Set class name. This is very important!
  class(est) <- "myglm"
  # Return the object with all results
  return(est$beta)
}

loglik_poi <- function(par, x, y) {
  beta <- par
```

```
mu <- beta %*% t(x)
LL <- sum(y * mu - exp(mu))
return(-LL)
}
```