TMA4315: Compulsory exercise 2: Logistic regression and Poisson regression

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Contents

Part 2

a)

According to the χ^2-test performed by the author of (https://www.math.ntnu.no/emner/TMA4315/2017h/Lee1997.pdf), there is no evidence against the assumption of independence between goals scored by the home and the away team. We have of course other data, and can perform our own χ^2-test to evaluate if the assumption of independence is reasonable or not.

```
filepath <- "https://www.math.ntnu.no/emner/TMA4315/2018h/eliteserien2018"
eliteserie <- read.table(file = filepath, header = TRUE, colClasses = c("character",
    "character", "numeric", "numeric"))
contigency_data <- eliteserie
for (i in 1:length(contigency_data$home)) {
    if (contigency_data[i, ]$yh > 4) {
        contigency_data[i, ]$yh <- 4
    }
    if (contigency_data[i, ]$ya > 4) {
        contigency_data[i, ]$ya <- 4
    }
}
contigency_table <- table(contigency_data$ya, contigency_data$yh)
rownames(contigency_table) <- c("0", "1", "2", "3", "4+")
colnames(contigency_table) <- c("0", "1", "2", "3", "4+")
contigency_table
```

```
##
##       0  1  2  3 4+
##   0   8 19 10 13  8
##   1  18 26 14 10  7
##   2   3 15 13  7  3
##   3   1  5  4  2  1
##   4+  1  3  1  0  0
```

```
chisq.test(contigency_table)
```

```
##
##  Pearson's Chi-squared test
##
```

```
## data:  contigency_table
## X-squared = 14.156, df = 16, p-value = 0.5871
```

As we can see in our χ^2-test, the p-value (0.5871) is high, which supports the H_0-hypoteses that the goals scored by the home and the away team is independent. We would have to consider some dependency if the p-value had been under our siginficance value at 0.05. We have clustered goals of 4 or higher in the same row/column as the χ^2-test will be more reliable if there is more than 5 occurrences in each row and column. Thus we can proceed our regression with the assumption of indepencende between goals scored.

b)

```r
# This function computes the ranking and the goal difference, and
# returns a result table (not ordered)
calculate_points <- function(data_eliteserie) {
    result_table <- data.frame(team <- unique(data_eliteserie$home),
        position <- seq(1, length(team), by = 1), goal_for <- rep(0,
            length(team)), goal_against <- rep(0, length(team)), goal_score <- rep(0,
            length(team)), points <- rep(0, length(team)))
    colnames(result_table) <- c("Team", "Position", "GF", "GA", "GD",
        "Points")
    for (i in 1:length(data_eliteserie$home)) {
        if (data_eliteserie$yh[i] > data_eliteserie$ya[i]) {
            index = which(result_table$Team == data_eliteserie$home[i])
            result_table[index, ]$Points = result_table[index, ]$Points +
                3
        } else if (data_eliteserie$yh[i] < data_eliteserie$ya[i]) {
            index = which(result_table$Team == data_eliteserie$away[i])
            result_table[index, ]$Points = result_table[index, ]$Points +
                3
        } else {
            result_table[which(result_table$Team == data_eliteserie$home[i]),
                ]$Points = result_table[which(result_table$Team == data_eliteserie$home[i]),
                ]$Points + 1
            result_table[which(result_table$Team == data_eliteserie$away[i]),
                ]$Points = result_table[which(result_table$Team == data_eliteserie$away[i]),
                ]$Points + 1
        }
        result_table[which(result_table$Team == data_eliteserie$home[i]),
            ]$GF = result_table[which(result_table$Team == data_eliteserie$home[i]),
            ]$GF + data_eliteserie$yh[i]
        result_table[which(result_table$Team == data_eliteserie$home[i]),
            ]$GA = result_table[which(result_table$Team == data_eliteserie$home[i]),
            ]$GA + data_eliteserie$ya[i]
        result_table[which(result_table$Team == data_eliteserie$away[i]),
            ]$GF = result_table[which(result_table$Team == data_eliteserie$away[i]),
            ]$GF + data_eliteserie$ya[i]
        result_table[which(result_table$Team == data_eliteserie$away[i]),
            ]$GA = result_table[which(result_table$Team == data_eliteserie$away[i]),
            ]$GA + data_eliteserie$yh[i]
    }
    result_table$GD <- result_table$GF - result_table$GA
    ordered_table <- result_table[order(-result_table$Points, -result_table$GD),
        ]
    i <- 1
```

```r
    for (team in ordered_table$Team) {
        result_table[which(result_table$Team == team), ]$Position <- i
        i <- i + 1
    }
    return(result_table)
}
result_table <- calculate_points(eliteserie)
result_table <- result_table[order(-result_table$Points, -result_table$GD),
    ]
print(result_table)
```

```
##                   Team Position GF GA  GD Points
## 10           Rosenborg        1 43 20  23     52
## 11               Brann        2 36 23  13     48
## 1                Molde        3 48 30  18     43
## 12           Haugesund        4 36 28   8     41
## 8           Ranheim_TF        5 38 40  -2     38
## 13          Vaalerenga        6 35 37  -2     36
## 3                  Odd        7 35 29   6     34
## 14             Tromsoe        8 35 33   2     33
## 6          Sarpsborg08        9 39 34   5     32
## 7         Kristiansund       10 32 35  -3     31
## 4           BodoeGlimt       11 28 30  -2     27
## 2        Stroemsgodset       12 38 38   0     26
## 9          Lillestroem       13 26 37 -11     25
## 16             Stabaek       14 29 43 -14     23
## 5                Start       15 24 42 -18     23
## 15 Sandefjord_Fotball       16 24 47 -23     15
```

Here is a the table ordered after points, where GF is goales scored, GA is goals against and GD is the goal balance. We can see that Rosenborg is leading with 4 points to Brann and 9 to Molde. The 4 point gap to Brann is not huge, considering there are 6 games left to play, and when Rosenborg and Brann played last time (where Rosenborg had the home advantage), Brann won. They have an unplayed match where Brann has the home advantage, and that match can be deciding.

c

```r
# This code generates the design matrix, as well as computing the
# strength parameters with our own function.
library(myglm)
goals <- c(eliteserie$yh, eliteserie$ya)
X <- matrix(data = 0, nrow = 384, ncol = 17)
colnames(X) <- c("Intercept", "HomeAdvantage", unique(eliteserie$home)[-4])
for (i in 1:length(eliteserie$home)) {
    X[i, 1] = 1
    X[i, 2] = 1
    home_index <- which(colnames(X) == eliteserie$home[i])
    away_index <- which(colnames(X) == eliteserie$away[i])
    X[i, home_index] <- 1
    X[i, away_index] <- -1
}
for (i in 1:length(eliteserie$away)) {
    X[i + length(eliteserie$home), 1] = 1
    home_index <- which(colnames(X) == eliteserie$home[i])
```

```
    away_index <- which(colnames(X) == eliteserie$away[i])
    X[i + length(eliteserie$home), home_index] <- -1
    X[i + length(eliteserie$home), away_index] <- 1
}
strength_param <- myglm(goals ~ -1 + X)
strength_param <- strength_param[order(-strength_param)]
names(strength_param) <- substring(names(strength_param), 2)
strength_param
```

```
##      HomeAdvantage          Rosenborg              Molde
##        0.402062206        0.367125310        0.279399199
##              Brann          Haugesund          Intercept
##        0.225775206        0.141301460        0.100321807
##                Odd        Sarpsborg08            Tromsoe
##        0.100120614        0.097677349        0.060581141
##      Stroemsgodset         Vaalerenga       Kristiansund
##        0.049792126        0.014730410        0.012552907
##         Ranheim_TF        Lillestroem            Stabaek
##        0.008502727       -0.132621109       -0.147940567
##              Start Sandefjord_Fotball
##       -0.225757649       -0.291683130
```

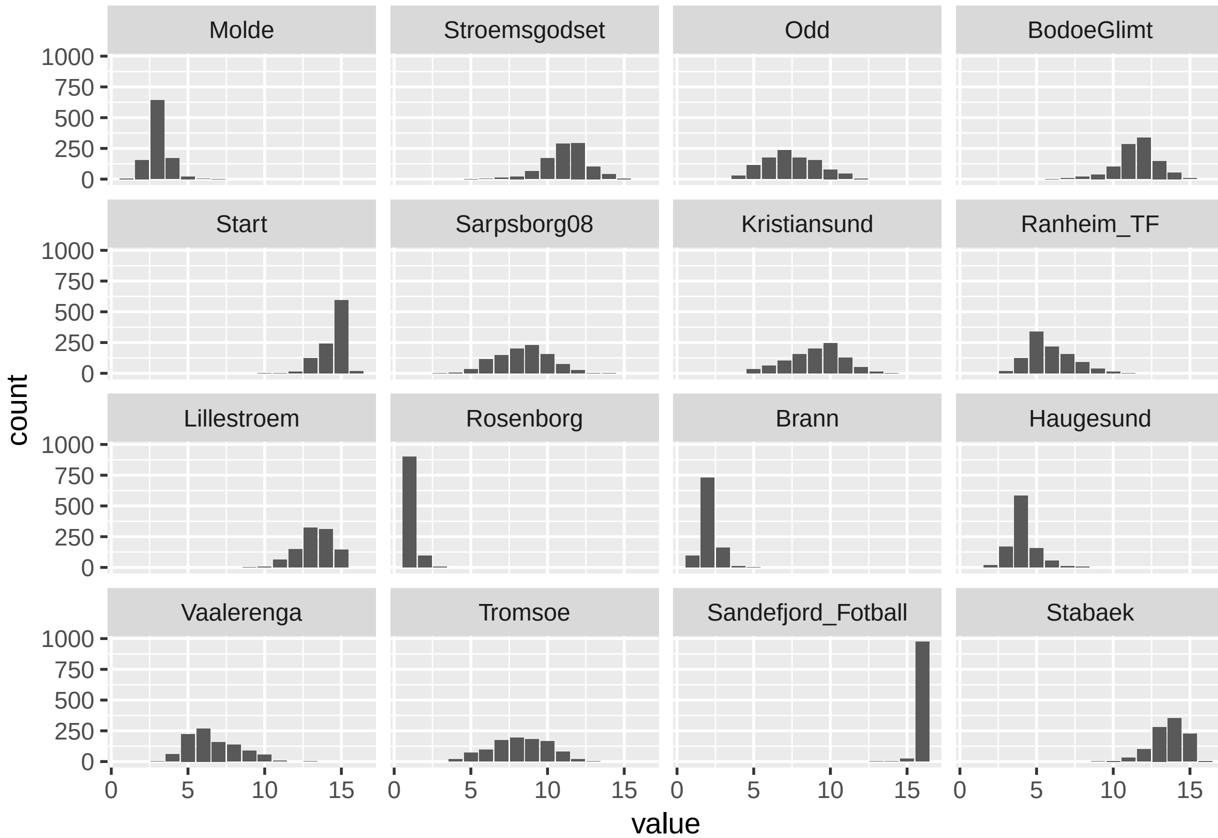
Here we see the estimated strength-parameters, with the stength parameter of BodoeGlimt set to zero. The expexted number of goals for a home team A is thus $E[A] = exp(\beta_0 + \beta_{home} + \beta_A - \beta_B)$ and the away team B has the expected value $E[B] = exp(\beta_0 + \beta_B - \beta_A)$. We see that the intercept is some sort of "default" strength for BodoeGlimt, and if a BodoeGlimt play away against a team with higher strength parameter than the intercept, they would have a expected number of goals less than 1 (the sum of the betas less than zero). Rewriting as $E[A] = e^{\beta_0} e^{\beta_{home}} e^{\beta_A} / e^{\beta_B}$ we can more easily see that the expected number of goals for a team is e in the power of the intercept, times e in the power of the home parameter (if a home game for the team), times the e to the power of the strength parameter of the team, divided by e to the power of the strength parameter of the opponent. An interesting point is that Molde has a higher strength parameter than Brann, even though Brann has 5 more points so far in the season. That can be a result of Molde meeting better opponents than Brann so far, and the fact that Molde won both the matches between the teams, with 5-1 home and 0-4 away. Ranheim TF had a much lower strength parameter than their position in the table suggests. Looking at their played games, one can see that they are very unstable, loosing to teams that are considered worse, and winning against better teams. Thus it's hard to analyse their strength, and give it a number. The regression has probably considered some of their wins against better teams like Molde luck, which would explain the pessimistic strength estimate compared to their position at the table.

```
# This code simulates n seasons, and write the ranking for each
# season to a txt-file.
library("reshape2")
set.seed(42)
filepath <- "https://www.math.ntnu.no/emner/TMA4315/2018h/unplayed2018"
eliteserie_unplayed <- read.table(file = filepath, header = TRUE, colClasses = c("character",
    "character"))
simulate_season_end <- function(data_unplayed_matches, strength_param) {
    for (i in 1:length(data_unplayed_matches$home)) {
        intercept <- strength_param["Intercept"]
        home_advantage <- strength_param["HomeAdvantage"]
        strength_hometeam <- strength_param[data_unplayed_matches$home[i]]
        strength_awayteam <- strength_param[data_unplayed_matches$away[i]]
        if (data_unplayed_matches$home[i] == "BodoeGlimt") {
            strength_hometeam <- 0
        } else if (data_unplayed_matches$away[i] == "BodoeGlimt") {
```

```
            strength_awayteam <- 0
        }
        data_unplayed_matches$yh[i] <- rpois(1, exp(strength_hometeam +
            intercept + home_advantage - strength_awayteam))
        data_unplayed_matches$ya[i] <- rpois(1, exp(strength_awayteam +
            intercept - strength_hometeam))
    }
    return(data_unplayed_matches)
}

Points <- c()
for (i in 1:1000) {
    simulated_results <- simulate_season_end(eliteserie_unplayed, strength_param)
    eliteserie_finished <- rbind(eliteserie, simulated_results)
    standings <- calculate_points(eliteserie_finished)
    write.table(standings, paste("Standings\\Standings_", toString(i),
        ".txt", sep = ""), sep = "\t", quote = FALSE)
}
```

```
# This code loads n season simulations, and plots the result
# distribution as bar diagram.
library("ggplot2")
library("reshape2")
n <- 1000
teams <- unique(eliteserie$home)
Positions <- matrix(0, nrow = length(teams), ncol = n)
Points <- matrix(0, nrow = length(teams), ncol = n)
rownames(Positions) <- c(teams)
rownames(Points) <- c(teams)
for (i in 1:n) {
    standings <- read.table(file = paste("Standings\\Standings_", toString(i),
        ".txt", sep = ""), header = TRUE, colClasses = c("numeric", "character",
        "numeric", "numeric", "numeric", "numeric"))
    Positions[, i] <- standings$Position
    Points[, i] <- standings$Points
}
Positions <- melt(Positions, id.vars = c("Teams"))
p <- ggplot(data = data.frame(Positions), aes(x = value)) + geom_bar() +
    facet_wrap(~Var1)
p
```



From this bar diagrams over the different simulations, we can see that Rosenborg wins around 900/1000 simulations, whereas Brann wins the remaining around 100 simulations. Molde wins about 2 or 3 of the simulations, but Rosenbord and Brann are mainly our contestants for the win. The omission of explanatory variables such as Rosenborg are playing Europe League in addition to Eliteserien, and their players will be more exhausted, can cause for a lower variability in the data than we see in our simulations. There are much more variability in the middel of the table, which suggests that the two top teams (Rosenborg and Brann) are a lot better than the others, and Sandefjord being the far worst (8 points behind the next team on the table so far). Another point to make is that this regression only set a general value on each teams strength, but following football results one knows that some teams perform better against other specific teams and vice versa. This regression only estimates the general strength of Rosenborg compared to all the other teams, but for a more realistic simulation one would have to estimate the strength difference between each team spesifically. And of course take in to account other factors as injuries and such.

```
# This code computes result statistics such as mean and standard
# deviation
result_statistics <- data.frame(Team = rownames(Points), Position = seq(1,
    length(Points[, 1]), by = 1), Mean = rowMeans(Points), row.names = NULL)

for (i in 1:nrow(Points)) {
    result_statistics$StdDev[i] <- sqrt(sum((Points[i, ] - result_statistics$Mean[[i]])^2/(length(Point
        ]) - 1)))
}
result_statistics <- result_statistics[order(-result_statistics$Mean),
    ]
result_statistics$Position <- seq(1, length(result_statistics$Team),
    by = 1)
result_statistics
```

```
##                  Team Position   Mean   StdDev
## 10          Rosenborg        1 64.588 2.845761
## 11              Brann        2 58.814 3.171666
## 1               Molde        3 53.887 3.136570
## 12          Haugesund        4 50.239 2.857980
## 8          Ranheim_TF        5 46.089 3.080677
## 13         Vaalerenga        6 44.477 3.050701
## 3                 Odd        7 42.338 2.972641
## 14            Tromsoe        8 41.268 3.022468
## 6         Sarpsborg08        9 40.696 2.966544
## 7         Kristiansund      10 39.994 3.098639
## 2        Stroemsgodset      11 35.304 3.106321
## 4           BodoeGlimt      12 34.802 3.030508
## 9          Lillestroem      13 30.629 2.951961
## 16            Stabaek       14 29.785 3.061396
## 5               Start       15 27.855 2.703939
## 15 Sandefjord_Fotball       16 19.565 2.637941
```

Looking at the means, there is a considerable difference in points between Rosenborg and Brann, at 5.774 points. However the standard deviations for the teams will suggest that if Rosenborg being close to the lower limit and Brann close to the upper limit, would give Brann more points than Rosenborg. If one also take into acount that they are playing against each other in the final part of the season, not only giving the winner three points, but also ensures that the looser gets zero points in that game, we have plausible scenarios where Brann wins Eliteserien. As shown in our simulations, they only win around 100/1000, but the match between the two teams will be crucial in the first place battle.

```r
# Code from myglm-package
myglm <- function(formula, data = list(), contrasts = NULL, ...) {
    # Extract model matrix & responses
    mf <- model.frame(formula = formula, data = data)
    X <- model.matrix(attr(mf, "terms"), data = mf, contrasts.arg = contrasts)
    y <- model.response(mf)
    terms <- attr(mf, "terms")

    # Add code here to calculate coefficients, residuals, fitted values,
    # etc...  and store the results in the list est
    est <- list(terms = terms, model = mf)

    par <- rep(0, ncol(X))
    betahat <- optim(par = par, fn = loglik_poi, x = X, y = y, method = "BFGS")$par
    names(betahat) = colnames(X)
    est$beta <- betahat

    # Store call and formula used
    est$call <- match.call()
    est$formula <- formula

    # Set class name. This is very important!
    class(est) <- "myglm"
    # Return the object with all results
    return(est$beta)
}

loglik_poi <- function(par, x, y) {
    beta <- par
```

```r
    mu <- beta %*% t(x)
    LL <- sum(y * mu - exp(mu))
    return(-LL)
}
```