

Modelling Analytics Group Project Report

Introduction

A football club has been promoted to the top tier of English football for the first time in several decades and its stadium has not been developed during this period. The club's board of directors have decided that promotion to the Premier League creates the opportunity for a much-needed stadium expansion due to the huge boost in television, advertising, and attendance revenue (Noll, 2002). This is a common phenomenon among newly promoted clubs.

Objectives

Whilst hoping to maximise the stadium's revenue from tickets and concessions, the board have several other goals they wish to achieve in the stadium expansion. They envision the stadium as a world-class entertainments and events venue. The board aims to diversify its revenue streams by relying on sources other than attendance, this accounts for approximately 60% of world-class stadium revenue (DeSchrive, Rascher and Shapiro, 2016). To achieve this, they must build multi-purpose function rooms and suites. They also consider increasing the seating capacity as vital for long-term success and for creating an enhanced atmosphere during matches. The board understand that due to certain constraints, - such as limited area available due to Planning Permission and compliance with Premier League stadium regulations - these goals may be in competition with each other, in Table 1. Thus, they are looking for a computerised Decision Support System that can help them decide the optimal course of action.

	Area Usage (metres squared)	Construction Costs	CO2 Emissions	Capacity	Profit per month
Unsheltered Stand	4,000	80,000	8,000	1,000	32,000
Sheltered Stand	4,000	200,000	9,000	1,000	44,000
Bars	3,000	60,000	6,000	n/a	33,000
Function Rooms	6,000	250,000	5,000	n/a	45,000

Table 1: Our data was sourced from various websites and datasets and is in accordance with both its realistic context and applicability. See UK Green Building Council, Stadium Solutions and FIFA Publications.

Methods

Model

To solve this problem, we propose Goal Programming (GP) a goal-oriented optimization technique to solve decision problems with a multiplicity of objectives (Chowdhury et al., 2019). GP is useful as it offers a structured and quantitative approach to finding realistic, practical solutions in an environment with various constraints. However, GP as the prioritisation of goals is linear and subjective which can be influenced by the decision-makers biases. Thus, we explored models that offer an objective approach to assigning goal weights considering extensions of AHP processes: Entropy Weights Method (EMW) and CRITIC.

Analytical Hierarchy Process

The Analytical Hierarchy Process (AHP) was developed by Saaty (1988) and designed to solve complex multi-criteria decision problems (Wisniewski et al., 2008). There are specific judgements about the relative importance of each of the four criteria which formed a pairwise comparison matrix to establish priorities for the criteria (Appendix I). While AHP is useful in the selection of goals, it does not consider the relevant constraints for this problem (Badri, 1999). Therefore, in combination with GP, this creates a structured and transparent framework that balances competing objectives.

Goal Programming

Before proceeding with optimisation, we prepare the problem by breaking down the complex GP model into manageable sub-problems. Linearisation of a problem can be used to avoid complexities associated with nonlinearities in GP problems. We linearise by formulating four linear program (LP) models, irrespective of other goals, to determine challenging but achievable aspirational levels for the goals outlined (Appendix II). With the GP model, unwanted deviations from this set of aspiration values are then minimized in an achievement function (Chang, 2008). AHP weights are applied on favourable deviation variables (Appendix I). It was noted that the GP problem could have been linearised further with

conventional Sequential Linear Programming. However, this approach starts from an initial arbitrary point that can be far-off compared to solution values, which affects its convergence to a feasible solution (Sampath et al., 2018). Therefore, we did not include this model. Additionally, the use of the four LP models allows us to improve efficiency, accuracy, and review constraints.

Analysis

Consistency Ratio Calculation

After creating the Pairwise Comparison Matrix for AHP, CRITIC and EMW we performed Synthesisation to calculate the priority of each criterion and their contribution towards the achievement of the overall goal. A consistency check has been performed to avoid any degree of inconsistency. If the consistency ratio is greater than 0.1, then it indicates that there is inconsistency among the pairwise comparisons and needs to be reviewed before proceeding further (Wisniewski, et al., 2008). During this process, it was identified that CRITIC weights were not realistic. EMW was not selected due to its inconsistency and known propensity to distort weighting with dispersion (Zhu *et al.*, 2020). Therefore, the pairwise comparison matrix of AHP used, featured in Appendix 1, Table 2.

$$\begin{aligned}\text{Consistency Index (CI)} &= 0.0616 \\ \text{Random Index (RI)} &= 0.9 \text{ (} n = 4 \text{)} \\ \text{Where, } n &= \text{total number of criteria,} \\ \text{Consistency Ratio (CR)} &= CI / RI = 0.068,\end{aligned}$$

As mentioned, a consistency ratio of $0.068 < 0.1$ is considered acceptable. Hence, we can conclude that the pairwise comparisons made by the decision-makers are consistent. The priorities of each criterion are given by averaging the elements in each row as shown in (Appendix I, table 4).

Criteria	Weight
Total Profit	0.399

Minimisation of CO₂	0.085
Stand Capacity	0.218
Function Room Profit	0.299

Table 2: AHP Results

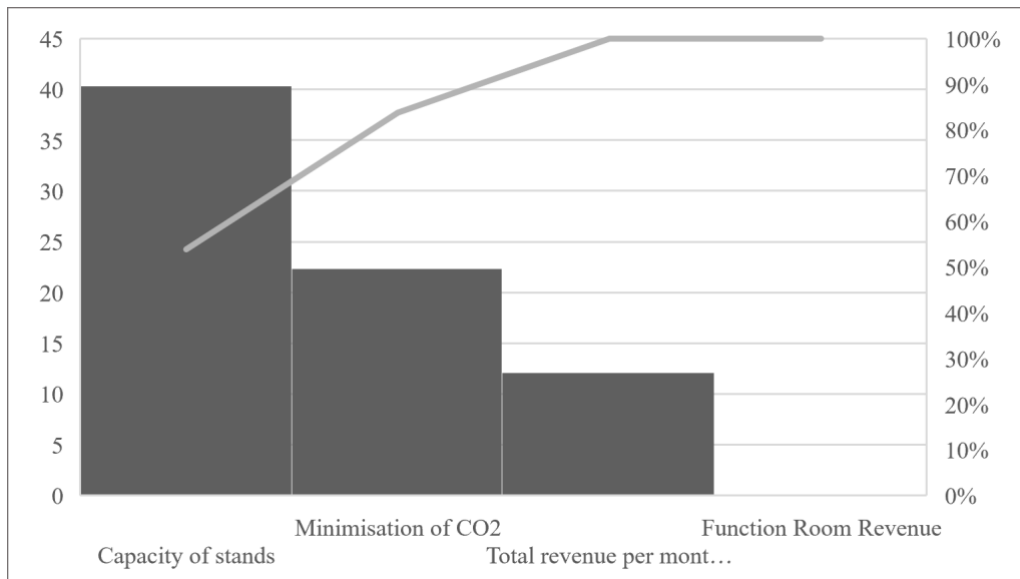
Looking at the results of the AHP model (Table 2) we see that Total Profit is the most important goal, followed by Function Room Profit, then capacity, then CO₂ Minimisation. This makes sense because Total Profit generation is critical to the success of the club. Similarly, building facilities for other types of events, thus diversifying revenue, is an important step to reducing risk (Chavez, *et al.*, 2012).

Results

The competing objectives are Total Profit, Stand Capacity, Function Room Profit and Minimisation of CO₂. The achievement levels of each LP can be found in Appendix II. With these achievement levels, we formulated our objective function by inputting the weight values into our objective function, which looks as follows:

$$\text{Minimise the total Weighted Underachievement of Goals} = 39.8D_{Total Profit}^+ + 8.5D_{CO_2}^- + 21.8D_{Capacity}^+ + 29.9D_{FR Profit}^+$$

Our results (Appendix II) imply that the optimal solution for the football stadium expansion is to build 1 Unsheltered Stand, 3 Sheltered Stands, 1 Bar, and 1.83 Function Rooms. Graph 1 shows the relative deviations as a percentage where Total Profit, CO₂ Minimisation, and Capacity of Stands are underachieved by £38,500, 9,166 tonnes, and 2,706 seats respectively, whereas the Function Room Profit goal is achieved at £82,500.



Graph 1: Deviation Pareto Plot (percentage)

Sensitivity Analysis

To check the robustness of our solution, we propose performing sensitivity analysis. The results for the sensitivity report can be found in Appendix III, and they show that all the constraints in this expansion model are satisfied at their optimal values. The shadow price for the constraints is the amount by which the final value will change if the RHS of the constraint is increased by 1 unit, whereas the allowable increase and decrease columns define the feasibility region of the constraints (Wisniewski et al., 2008). Interestingly, the shadow price of the Area Usage is negative, indicating that a unit increase in the RHS will decrease the achievement function by 515.66. Thus, the area made available by Planning Permission is more than what is required for the optimal solution. Similarly, the shadow price of the Minimisation of CO₂ constraint is -8.5 and the allowable increase is 9166.66. Therefore, to satisfy sustainable preferences in accordance with the management, the RHS can be increased by 9166.66 to minimise CO₂ emission further.

To evaluate how our solution responds to changing weights of objectives over time, we altered the model to place higher weight on the Minimisation of CO₂ constraint. This objective is likely to become increasingly important in the future due to changing regulations and customer preferences. Therefore, we reduced the weights of the Function Room Profit, Stand Capacity and Total Profit objectives. This altered model produces the same solution of 1 Unsheltered Stand, 3 Sheltered Stands, 1 Bar, and 1.83 Function Rooms, demonstrating that

our solution is Pareto Efficient in increasing the importance of minimising carbon emissions over time.

Discussion

The DSS we have developed produces a solution that maximises the weighted achievement of the project's goals and is feasible given the constraints. Decision-makers on the board of directors can implement this solution knowing that it is optimal given the constraints and the assumptions made in the formulation of the model. From the sensitivity analysis, we can see that the total used construction area for the project is far below the maximum allowed by Planning Permission. A smaller construction area is beneficial as there is less impact on the local environment and community, meaning there will be improved public perception of the project and likely more support from the local community. Maintaining unused construction area is beneficial as it allows for the possibility of further expansion. This gives the board of directors flexibility to adapt to expand the stadium in ways that reflect their objectives in the future, which may change. From sensitivity analysis we also see that the CO₂ constraint has a negative shadow price. Thus, CO₂ emissions could increase by 618.8 tonnes, and this would not affect the achievement of the minimisation goal. Our solution currently gives the project leeway, as emissions may turn out higher than projected but would remain within feasibility limits. We would recommend maintaining this safety buffer as it is likely that the Premier League's emissions regulations will become stricter in the future (Ráthonyi-Ódor et al., 2020).

Limitations

A limitation of this approach is that the model's input values for revenue, costs and CO₂ emissions are estimated. Uncertainty over these values could undermine the model's results as small differences can dramatically affect the output values (Wisniewski, et al., 2008). Another major disadvantage of using weighted GP is the problem of incommensurability, when elements in the objective function are measured in different units. This can make it challenging to compare the various elements, which can lead to inappropriate weights and suboptimal solutions.

Conclusion

In conclusion, the proposition for a stadium expansion project has been analysed using a combination of Goal Programming (GP) and Analytical Hierarchy Process (AHP) techniques. These techniques have allowed the board to find a solution that balances their various goals whilst satisfying all constraints. The solution considers the various costs and profits generated from stands, sheltered and unsheltered, function rooms and bars. The problem was highly complex, however by breaking it down into manageable sub-problems, the techniques used offer a reliable, accurate and efficient solution. Furthermore, our analysis highlights the benefits of a project that does not fully exhaust the area available for construction, for efficiency reasons and for flexibility in achieving goals in the future. Our solution also gives the board of directors some margin for error regarding CO₂ emissions, however we nonetheless urge them to maintain this buffer given the likelihood of regulations becoming stricter in the future. The results of this study can assist the board in their decision-making, leading to an optimal outcome for the stadium expansion.

References

- Badri, M. A. (1999): Combining the analytic hierarchy process and goal programming for global facility location-allocation problem, *International Journal of Production Economics* Volume 62, Issue 3, Pages 237-248, [https://doi.org/10.1016/S0925-5273\(98\)00249-7](https://doi.org/10.1016/S0925-5273(98)00249-7)
- DeSchraver, T., Rascher, D.A. and Shapiro, S.L. (2016). If We Build it, Will They Come? Examining the Effect of Expansion Teams and Soccer-Specific Stadiums on Major League Soccer Attendance. *SSRN Electronic Journal*, <https://doi.org/10.2139/ssrn.2924140>
- Chang, C. (2008) Revised multi-choice goal programming, *Applied Mathematical Modelling*, Volume 32, Issue 12, Pages 2587-2595, ISSN 0307-904X, <https://doi.org/10.1016/j.apm.2007.09.008>
- Noll, R.G., (2002). The economics of promotion and relegation in sports leagues: The case of English football. *Journal of Sports Economics*, 3(2), pp.169-203, <https://journals.sagepub.com/doi/pdf/10.1177/152700250200300205>
- Saaty, T.L., (1988) What is the analytic hierarchy process? (pp. 109-121). Springer Berlin Heidelberg.
- Sampath, L. P. M. I., Patil, B. V., Gooi, H. B., Maciejowski, J. M. and Ling, K. V. (2018) 'A Trust-Region based Sequential Linear Programming, *Electric Power Systems Research*, Volume 165, Pages 134-143, ISSN 0378-7796, <https://doi.org/10.1016/j.epsr.2018.09.002>
- Zhu, Y., Tian, D., & Yan, F. (2020). Effectiveness of Entropy Weight Method in Decision-Making. *Mathematical Problems in Engineering*. 202(7), 1-5, <https://doi.org/10.1155/2020/3564835>
- Chowdhury, A.R., Mangaraj, B.K. and Jomon, M.G., 2019. Optimizing Manpower Planning. *Indian Journal of Industrial Relations*, 55(2), pp.355-368.

A Saltelli, Chan, K. and E Marian Scott (2008). *Sensitivity analysis*. Chichester ; New York: Wiley

Thabane, L., Mbuagbaw, L., Zhang, S., Samaan, Z., Marcucci, M., Ye, C., Thabane, M., Giangregorio, L., Dennis, B., Kosa, D., Debono, V.B., Dillenburg, R., Fruci, V., Bawor, M., Lee, J., Wells, G. and Goldsmith, C.H. (2013). A tutorial on sensitivity analyses in clinical trials: the what, why, when and how. *BMC Medical Research Methodology*, [online] 13(1). doi: <https://doi.org/10.1186/1471-2288-13-92>

Jaafar, J.A., Latiff, A.R.A., Daud, Z.M. and Osman, M.N.H., 2023. Does revenue diversification strategy affect the financial sustainability of Malaysian Public Universities? A panel data analysis. *Higher Education Policy*, 36(1), pp.116-143.

Wisniewski, M., Anderson, D. R., Sweeney, D. J., Williams, T. A. (2008) *An Introduction to Management Science Quantitative: Approaches to Decision Making*, CENGAGE Learning EMEA, Pages 692-714, ISBN: 978-1-84480-595-2

Ráthonyi-Ódor, K., Bácsné Bába, É., Müller, A., Bács, Z., Ráthonyi, G. (2020) ‘How Successful Are the Teams of the European Football Elite off the Field?—CSR Activities of the Premier League and the Primera División’, *International Journal of Environmental Research and Public Health*, 17(20), 7534, available: <http://dx.doi.org/10.3390/ijerph17207534>.

Chavez, M. D., Berentsen, P. B., & Lansink, A. O. (2012). Assessment of criteria and farming activities for tobacco diversification using the Analytical Hierarchical Process (AHP) technique. *Agricultural Systems*, 111, 53-62.

Development plans. (2023). *Development plans*. Ashton Gate.
<https://www.ashtongatestadium.co.uk/development-plans/>

Seated Spectator Stands | Football Terracing | Pitch Perimeter Fencing | Dugouts | Stadium Solutions. (2021). Stadiumsolutions.co.uk. <https://www.stadiumsolutions.co.uk/>

FIFA Dimensions and Surrounding Areas. <https://publications.fifa.com/en/football-stadiums-guidelines/technical-guideline/stadium-guidelines/pitch-dimensions-and-surrounding-areas/>

Appendix I – AHP Calculations

Pairwise Comparison	More Important Criterion	Level of Importance	Numerical Rating
Total Revenue – Sales from Bar	Total Revenue	Moderately	2
Total Revenue – CO ₂ Emission	Total Revenue	Equally to Moderately	3
Total Revenue – Sales from Stands	Total Revenue	Equally to Moderately	2
Sales from Bar – CO ₂ Emission	Sales from Bar	Moderately to Strongly	4
Sales from Bar – Sales from Stands	Sales from Bar	Moderately	2
CO ₂ Emission – Sales from Stands	Sales from Stands	Moderately to Strongly	4

Table 1: Pairwise Comparison

In this process, we have normalized the pairwise matrix by adding the values in each column and dividing each element by its column sum.

	Total Revenue	CO₂ Emission	Sales from Stands	Sales from Bar
Total Revenue	1	3	2	2
CO₂ Emission	$\frac{1}{3}$	1	$\frac{1}{4}$	$\frac{1}{4}$
Sales from Stands	$\frac{1}{2}$	4	1	$\frac{1}{2}$
Sales from Bar	$\frac{1}{2}$	4	2	1

Table 2: Pairwise Comparison Matrix

	Total Revenue	CO₂ Emission	Sales from Stands	Sales from Bar
Total Revenue	0.429	0.250	0.381	0.533
CO₂ Emission	0.143	0.083	0.048	0.067
Sales from Stands	0.214	0.333	0.190	0.133
Sales from Bar	0.214	0.333	0.381	0.267

Table 3: Normalized Pairwise Comparison Matrix

	Total Revenue	CO ₂ Emission	Sales from Stands	Sales from Bar	Priority
Total Revenue	0.429	0.250	0.381	0.533	0.398
CO₂ Emission	0.143	0.083	0.048	0.067	0.085
Sales from Stands	0.214	0.333	0.190	0.133	0.218
Sales from Bar	0.214	0.333	0.381	0.267	0.299

Table 4: Priority of Each Criterion

Linear Programming

We define the regular LP problem's goal objectives below which are all subject to the constraints in the second paragraph. These models are designed to find the maximum of each goal while irrespective of other goals.

Maximise Total Revenue = £32,000 US + £44,000 SS + £33,000 B + £45,000 FR

Maximise Seating Capacity = 1,000seats US + 1,000seats SS

Minimise CO₂ Emissions = 8,000t US + 9,000t SS + 6,000t B + 5,000t FR

Maximise FR Revenue = £45,000 FR

All subject to:

$4,000m^2US + 4,000m^2SS + 3,000m^2B + 6,000m^2FR \leq 30,000m^2$ (Area usage metres squared)

$£80,000US + £200,000SS + £60,000B + £250,000FR \leq 1,200,000$ (Construction cost)

$US \leq (US \times SS)0.25$ (75% Sheltered Seating)

$1FR \geq 1$ (At least 1 Bar)

$1US \geq 1$ (At least 1 Unsheltered Stand)

$US, SS, B, FR \geq 0$ (Non-negativity Constraint)

Deviation Variables in Goal Programming

For every goal in the GP model the goal is labelled with an overachieve variable, which is d^+ and an underachieving variable called d^- .

d_T^- = the amount that the total revenue goal is underachieved

d_T^+ = the amount that the total revenue goal is overachieved

$d_{CO_2}^-$ = the amount that the minimisation of CO₂ goal is underachieved

$d_{CO_2}^+$ = the amount that the minimisation of CO₂ goal is overachieved

d_C^- = the amount that the capacity of stands goal is underachieved

d_C^+ = the amount that the capacity of stands goal is overachieved

d_{FR}^- = the amount that the function room revenue goal is underachieved

d_{FR}^+ = the amount that the function room revenue goal is overachieved

Multiple Goal Objectives

We use the previously defined deviation variables to express our goals as mathematical equations. The expressions are detailed below and uses the original expression from the LP model. The output of the LP model is input here as maximum objectives for the Goal Programming model.

- **Goal 1:** the stadium expansion should exceed or equal the total revenue per month goal

$$\text{Maximise Total Revenue} = £32,000US + £44,000SS + £33,000B + £45,000FR + d_T^- - d_T^+$$

- **Goal 2:** the stadium expansion should minimise the amount of CO₂ achieved

$$\text{Minimise CO}_2 \text{ Emissions} = 8,000t \text{ US} + 9,000t \text{ SS} + 6,000t \text{ B} + 5,000t \text{ FR} + d_{CO_2}^- - d_{CO_2}^+$$

- **Goal 3:** the stadium expansion should overachieve the capacity of the stands

$$\text{Maximise Seating Capacity} = 1,000seats \text{ US} + 1,000seats \text{ SS} + d_C^- - d_C^+$$

- **Goal 4:** Function room revenue should generate £45,000

$$\text{Maximise FR Revenue} = £45,000FR + d_{FR}^- - d_{FR}^+$$

Goal Programming

$£32,000US + £44,000SS + £33,000B + £45,000FR + d_T^- + d_T^+ = X_1$ (maximise total revenue goal)

$1,000seats\ US + 1,000seats\ SS + d_C^- + d_C^+ = X_2$ (maximise seating capacity goal)

$8,000t\ US + 9,000t\ SS + 6,000t\ B + 5,000t\ FR + d_{CO_2}^- + d_{CO_2}^+ = X_3$ (minimise CO₂ emissions goal)

$£45,000FR + d_{FR}^- + d_{FR}^+ = X_4$ (maximise function room revenue goal)

Subject to:

$4,000m^2US + 4,000m^2SS + 3,000m^2B + 6,000m^2FR \leq 30,000m^2$ (Area usage metres squared)

$£80,000US + £200,000SS + £60,000B + £250,000FR \leq 1,200,000$ (Construction cost)

$US \leq (US \times SS)0.25$ (75% Sheltered Seating)

$1FR \geq 1$ (At least 1 Bar)

$1US \geq 1$ (At least 1 Unsheltered Stand)

$US, SS, B, FR, d_T^- + d_T^+, d_C^-, d_C^+, d_{CO_2}^-, d_{CO_2}^+, d_{FR}^-, d_{FR}^+ \geq 0$ (Non-negativity Constraint)

Appendix II – Excel Solutions

	US	SS	B	FR			
	Unsheltered Stands	Sheltered Stands	Bar	Function Rooms			
Solution Value	1.00	3.00	4.67	0.00	LP		
Goal						318000.00	
Total revenue per month goal	32000	44000	33000	45000			
Constraints							
Area Usage (m squared)	4000.00	4000.00	3000.00	6000.00	30000	<=	30,000
Construction Cost	80000.00	200000.00	60000.00	250000.00	960000	<=	1200000
75% sheltered seating	1	1			1	<=	1
Atleast 1 Bar			1		4.666666667	>=	1
At least 1 unsheltered stand	1				1	>=	1
					LHS	SIGN	RHS

Exhibit A: LP1 finds the maximum value for the total revenue per month goal

	US	SS	B	FR			
	Unsheltered Stands	Sheltered Stands	Bar	Function Rooms			
Solution Value	1.68	5.03	1.00	0.00	LP		
Goal						6705.88	
Capacity of stands	1000	1000					
Constraints							
Area Usage (m squared)	4000.00	4000.00	3000.00	6000.00	29823.52941	<=	30,000
Construction Cost	80000.00	200000.00	60000.00	250000.00	1200000	<=	1200000
75% sheltered seating	1	1			1.676470588	<=	1.67647059
Atleast 1 Bar			1		1	>=	1
At least 1 unsheltered stand	1				1.676470588	>=	1
					LHS	SIGN	RHS

Exhibit B: LP2 finds the maximum value for the capacity of stands

	US	SS	B	FR	
	Unsheltered Stands	Sheltered Stands	Bar	Function Rooms	
Solution Value	1.00	3.00	1.00	1.83	
Goal					
Conference Room Sales Goal				45000	
Constraints					
Area Usage (m squared)	4000.00	4000.00	3000.00	6000.00	30000 <= 30,000
Construction Cost	80000.00	200000.00	60000.00	250000.00	1198333.333 <= 1200000
75% sheltered seating	1	1			1 <= 1
Atleast 1 Bar			1		1 >= 1
At least 1 unsheltered stand	1				1 >= 1
					LHS SIGN RHS

LP
82500.00

Exhibit C: LP3 finds the maximum value for the function room revenue goal

	US	SS	B	FR	
	Unsheltered Stands	Sheltered Stands	Bar	Function Rooms	
Solution Value	1.00	3.00	1.00	0.00	
Goal					
Minimisation of CO2	8000	9000	6000	5000	
Constraints					
Area Usage (m squared)	4000.00	4000.00	3000.00	6000.00	19000 <= 30,000
Construction Cost	80000.00	200000.00	60000.00	250000.00	740000 <= 1200000
75% sheltered seating	1	1			1 <= 1
Atleast 1 Bar			1		1 >= 1
At least 1 unsheltered stand	1				1 >= 1
					LHS SIGN RHS

LP
41000.00

Exhibit D: LP4 finds the minimum value for CO2

Goal Programming Description:

The next exhibits display the Goal Programming model that we used to determine the optimal solution. When solving our problem, we used the Simplex method through Excel solver. By minimising the deviation variables subject to the constraints and goals.

- The achieved goals denote the value that the solution managed to obtain
- To the left of the achieved goals are the goal (constraints) which were created from the LP models
- The deviation from the goals are located in the Solution Value row
- Below this are the Goal weights input from AHP

	US	SS	B	FR	Dt-	Dt+	Dco2-	Dco2+
		Unsheltered Stands	Sheltered Stands	Bar Rooms	Underachiever Total profit per month goal	Overachiever Total profit per month goal	Underachiever Minimisation of CO2	Overachiever Minimisation of CO2
Solution Value	1.00	3.00	1.00	1.83	38500.00	0.00	0.00	9166.67
Goal Weights					39.8			8.5
Goals								
Total profit per month goal	32000	44000	33000	45000	1	-1		
Minimisation of CO2	8000	9000	6000	5000			1	-1
Capacity of stands	1000	1000						
Function Room Profit				45000				
Constraints								
Area Usage (m squared)	4000.00	4000.00	3000.00	6000.00				
Construction Cost	80000.00	200000.00	60000.00	250000.00				
75% sheltered seating	1	1						
Atleast 1 Bar			1					
At least 1 unsheltered stand	1							

Dc-	Dc+	Dfr-	Dfr+
Underachiever Capacity of Stands			
2705.88	0.00	0.00	0.00
21.8		29.9	
	-1	1	-1
Overachiever Capacity of Stands			
318000	=	318000	=
41000	=	41000	=
6705.882353	=	6705.882353	=
82500	=	82500	=
1669204.90			
Achieved Goals			
279500.00		318000.00	
50166.67		41000.00	
4000.00		6705.88	
82500.00		82500.00	
LHS			
30000	<=	30000	<=
1198333.333	<=	1198333.333	<=
1	<=	1	<=
1	>=	1	>=
1	>=	1	>=
SIGN		RHS	

Exhibit E: GPI

Appendix III – Sensitivity Analysis

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$6	Solution Value Unsheltered Stands	1	0	0	1E+30	1933766.667
\$D\$6	Solution Value Sheltered Stands	3	0	0	9.51237E+21	366166.6667
\$E\$6	Solution Value Bar	1	0	0	1E+30	284600
\$F\$6	Solution Value Function Rooms	1.833333333	0	0	549250	1E+30
\$G\$6	Solution Value Underachieve Total profit per month goal	38500	0	39.8	26.1547619	39.8
\$H\$6	Solution Value Overachieve Total profit per month goal	0	39.8	0	1E+30	39.8
\$I\$6	Solution Value Underachieve Minimisaation of CO2	0	8.5	0	1E+30	8.5
\$J\$6	Solution Value Overachieve Minimisation of CO2	9166.666667	0	8.5	618.8	8.5
\$K\$6	Solution Value Underachieve Capacity of Stands	2705.882353	0	21.8	366.1666667	21.8
\$L\$6	Solution Value Overachieve Capacity of Stands	0	21.8	0	1E+30	21.8
\$M\$6	Solution Value Underachieve FR Profit Goal	2.39808E-14	0	29.9	1E+30	12.20555556
\$N\$6	Solution Value Overachieve FR Profit Goal	0	29.9	0	1E+30	29.9

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$O\$14	Area Usage (m squared) GP	30000	-515.6666667	0	3.19744E-15	11000
\$O\$15	Construction Cost GP	1198333.333	0	0	1E+30	1666.666667
\$O\$16	75% sheltered seating GP	1	-1464666.667	0	1.9984E-19	0.0125
\$O\$17	Atleast 1 Bar GP	1	284600	1	3.666666667	1.06581E-18
\$O\$18	At least 1 unsheltered stand GP	1	1933766.667	1	0.125	1.9984E-19
\$O\$9	Total profit per month goal GP	318000	39.8	318000	1E+30	38500
\$O\$10	Minimisation of CO2 GP	41000	-8.5	41000	9166.666667	1E+30
\$O\$11	Capacity of stands GP	6705.882353	21.8	6705.882353	1E+30	2705.882353
\$O\$12	Function Room Profit GP	82500	29.9	82500	1E+30	2.39808E-14

Table 1: Sensitivity Analysis

Cell	Objective Name	Value
\$O\$7	Goal Weights GP	1669204.90

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
\$C\$6	Solution Value Unshletered Stands	1.00	0.00	85.00	250.00	18835.00
\$D\$6	Solution Value Sheltered Stands	3.00	0.00	110.00	398.50	20035.00
\$E\$6	Solution Value Bar	1.00	0.00	125.00	597.00	21020.00
\$F\$6	Solution Value Function Rooms	1.83				
\$G\$6	Solution Value Underachieve Total profit per month goal	38500.00				
\$H\$6	Solution Value Overachieve Total profit per month goal	0.00				
\$I\$6	Solution Value Underachieve Minimisaation of CO2	0.00				
\$J\$6	Solution Value Overachieve Minimisation of CO2	9166.67				
\$K\$6	Solution Value Underachieve Capacity of Stands	2705.88				
\$L\$6	Solution Value Overachieve Capacity of Stands	0.00				
\$M\$6	Solution Value Underachieve FR Profit Goal	0.00				
\$N\$6	Solution Value Overachieve FR Profit Goal	0.00				

Table 2: Limits Report