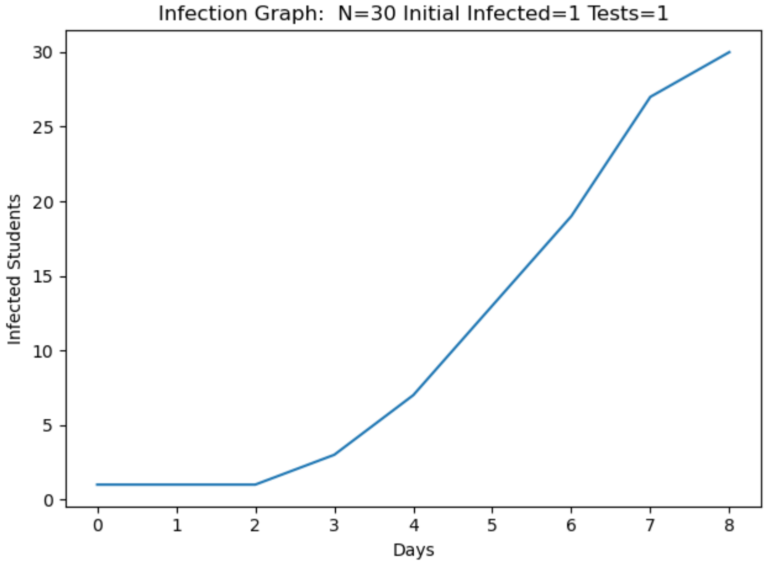
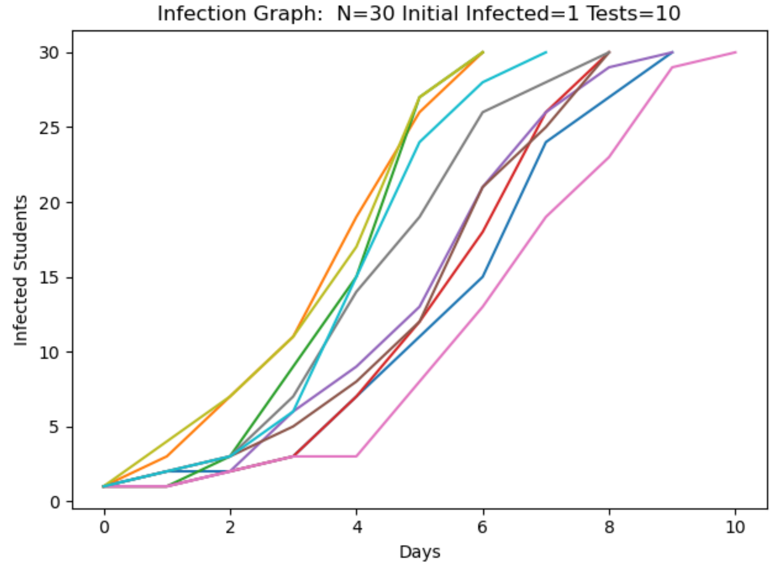
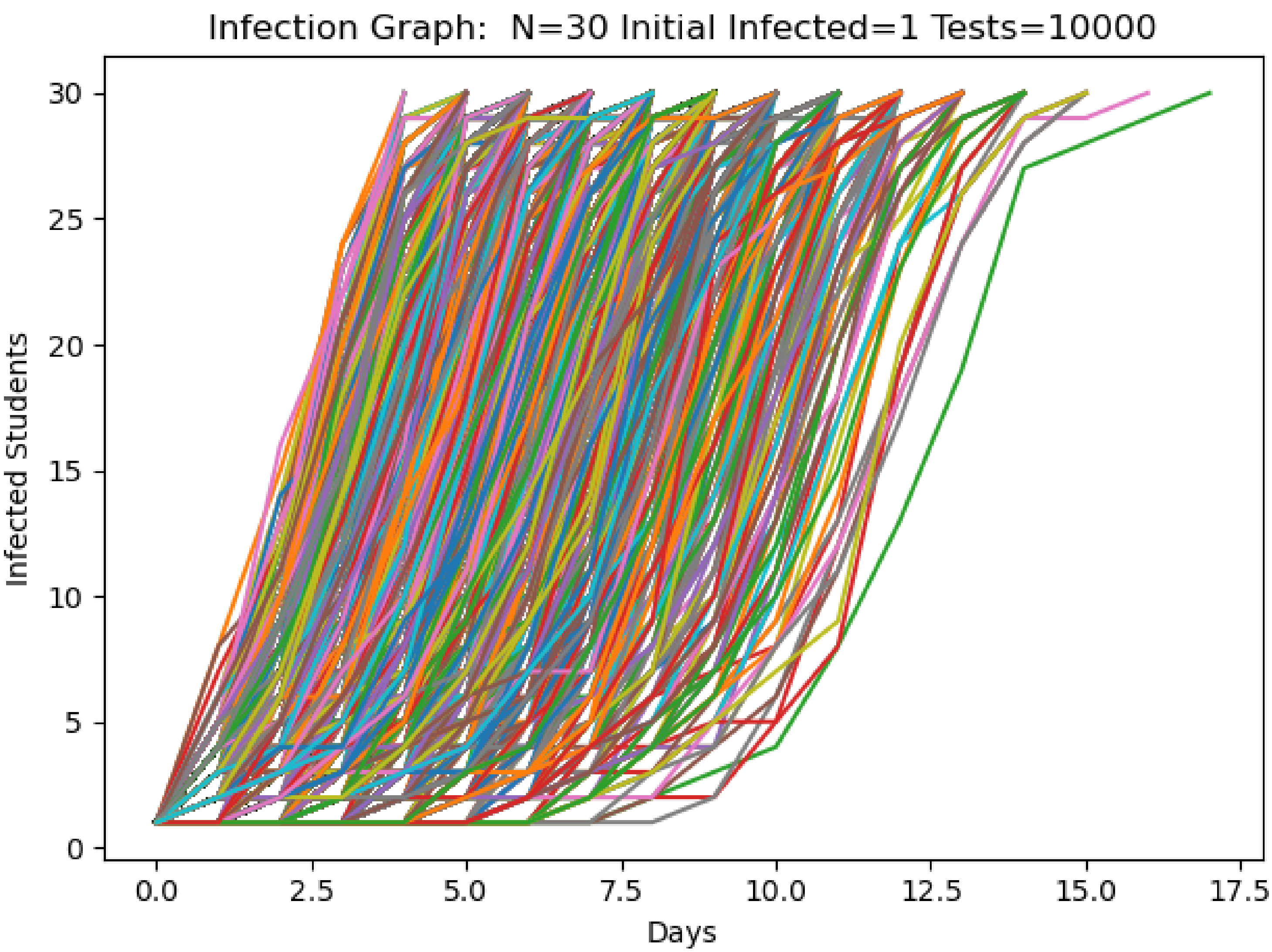
Matthew Benvenuto Modeling project

1.







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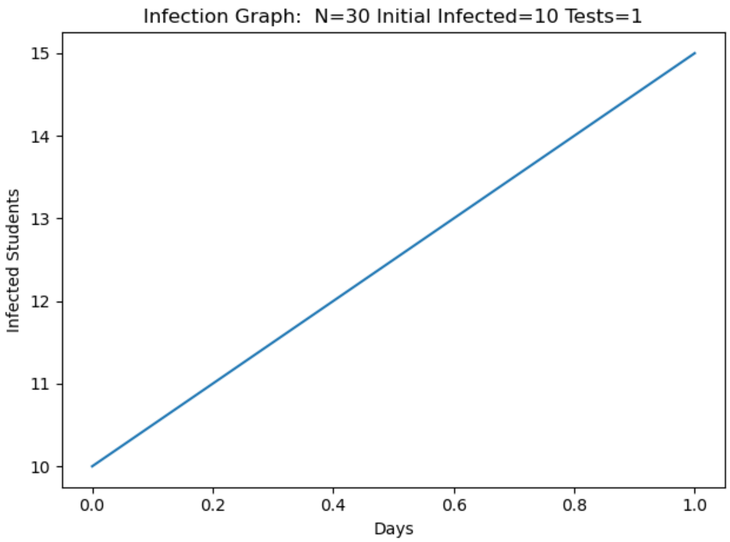
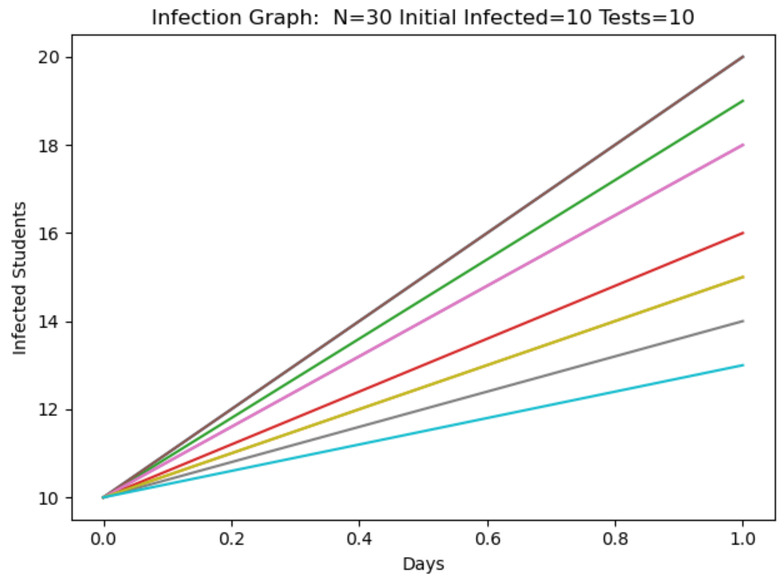
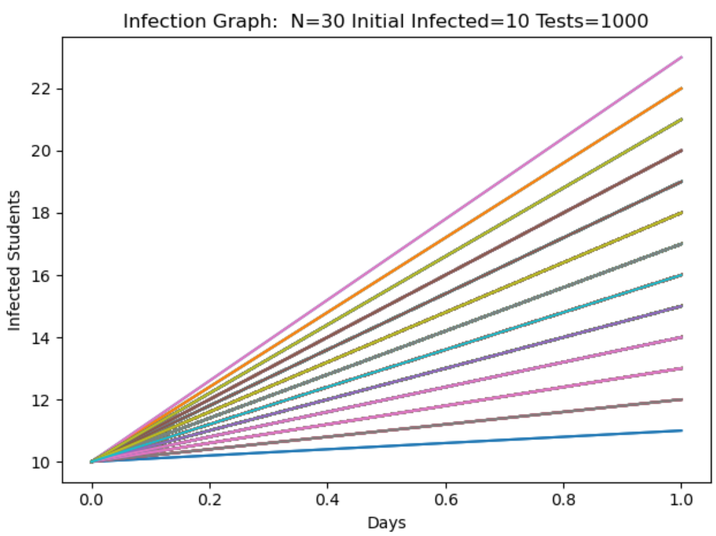
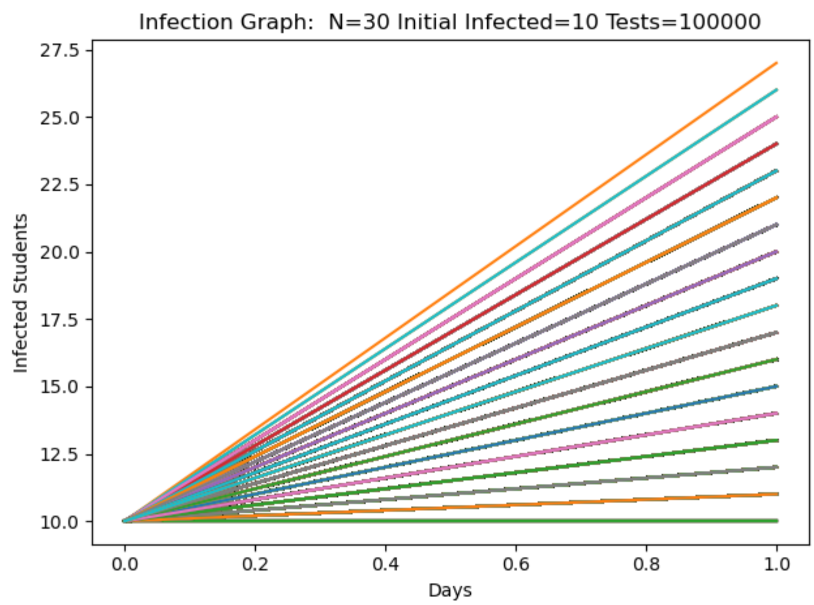
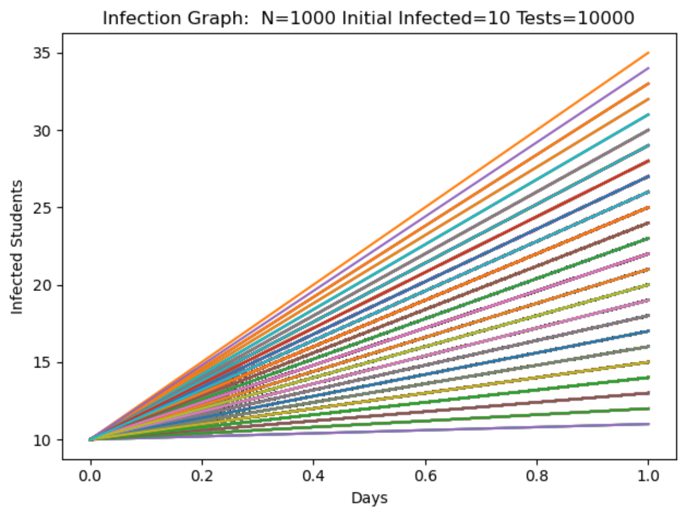

Infection Graph: N=30 Initial Infected=10 Tests=1
Infected Students
Days

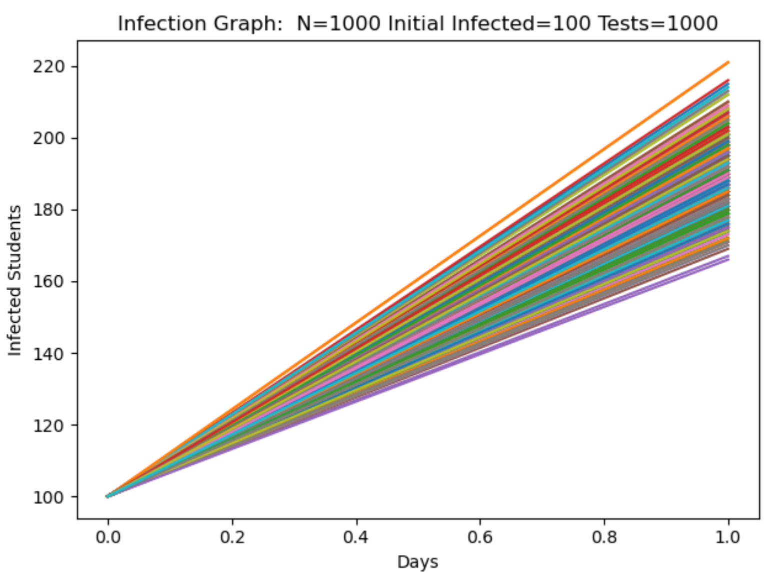

Infection Graph: N=30 Initial Infected=10 Tests=10
Infected Students
Days

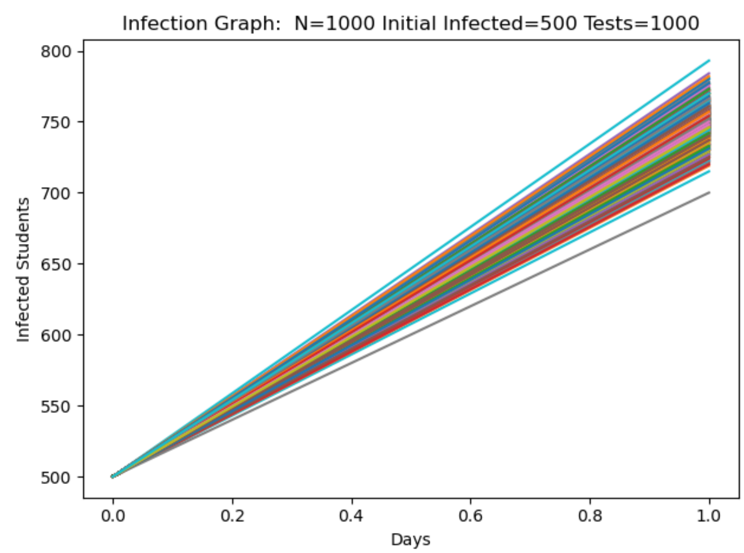

Infection Graph: N=30 Initial Infected=10 Tests=1000
Infected Students
Days

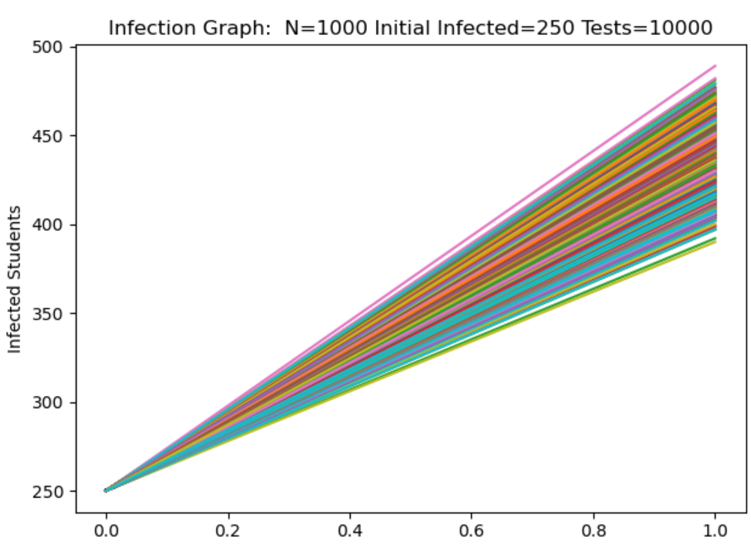

Infection Graph: N=30 Initial Infected=10 Tests=100000
Infected Students
Days

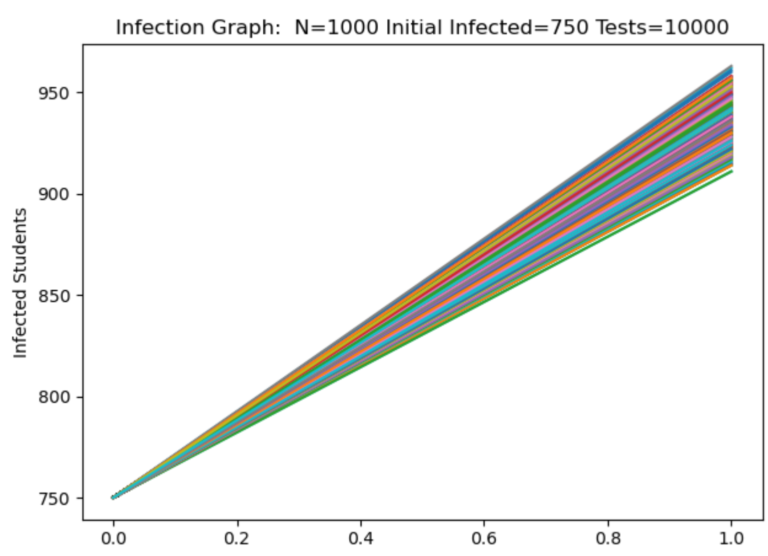
3.


Infection Graph: N=1000 Initial Infected=10 Tests=10000
Infected Students
Days
10
15
20
25
30
35
0.0
0.2
0.4
0.6
0.8
1.0

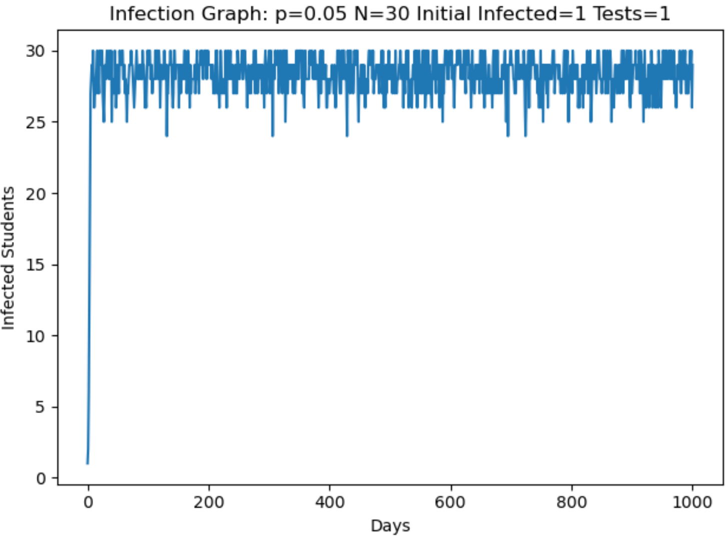

Infection Graph: N=1000 Initial Infected=100 Tests=1000
Infected Students
Days
100
120
140
160
180
200
220
0.0
0.2
0.4
0.6
0.8
1.0

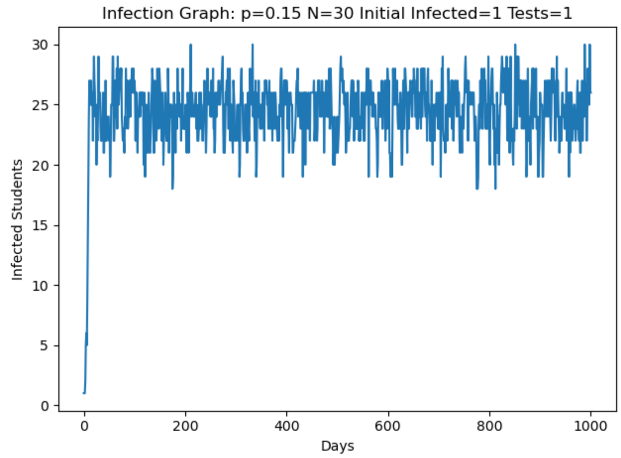

Infection Graph: N=1000 Initial Infected=500 Tests=1000
Infected Students
Days
500
550
600
650
700
750
800
0.0
0.2
0.4
0.6
0.8
1.0

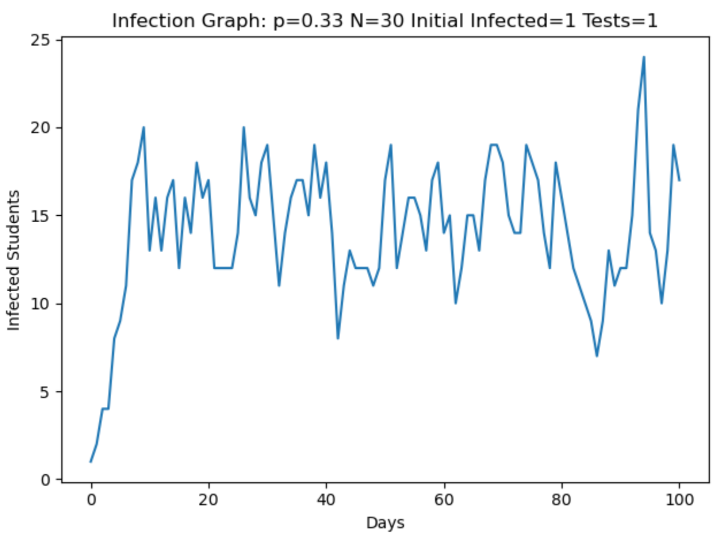

Infection Graph: N=1000 Initial Infected=250 Tests=10000
Infected Students
Days
250
300
350
400
450
500
0.0
0.2
0.4
0.6
0.8
1.0

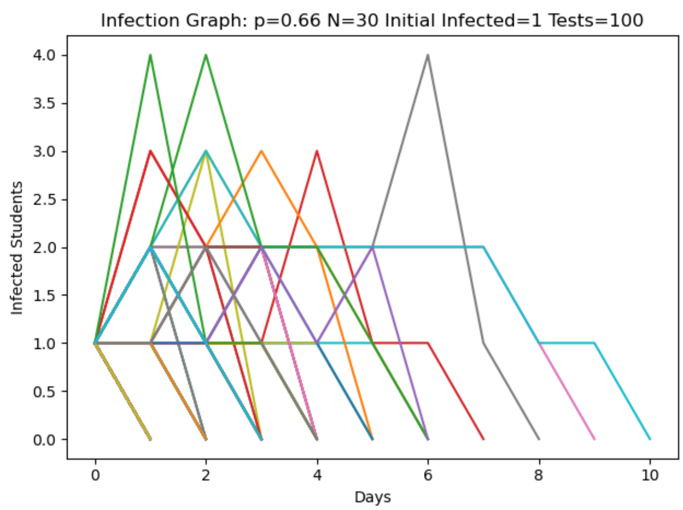

Infection Graph: N=1000 Initial Infected=750 Tests=10000
Infected Students
Days
750
800
850
900
950
0.0
0.2
0.4
0.6
0.8
1.0

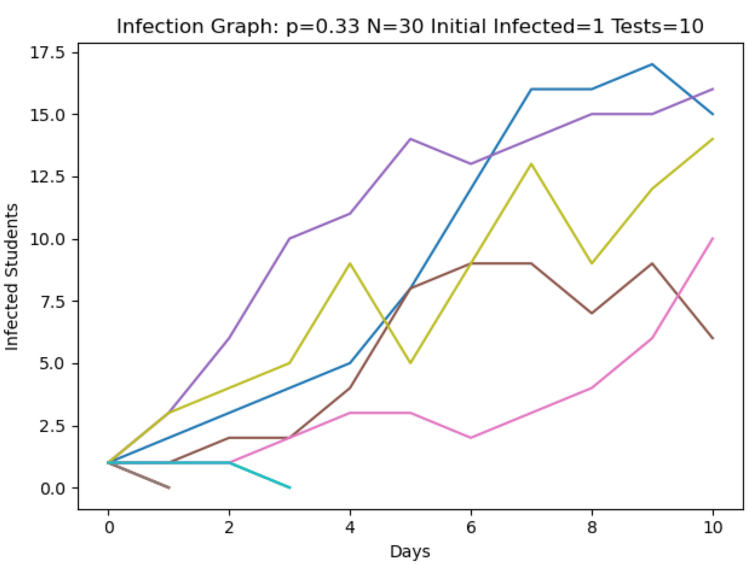
4.

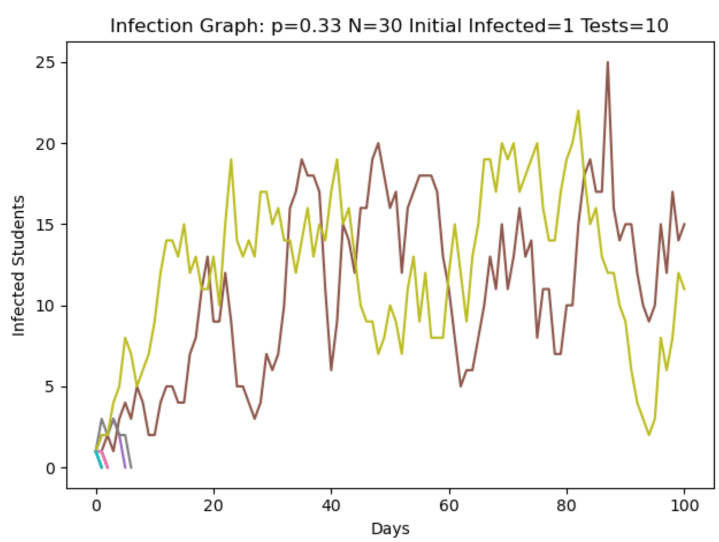

Infection Graph: p=0.05 N=30 Initial Infected=1 Tests=1
Infected Students
Days


Infection Graph: p=0.15 N=30 Initial Infected=1 Tests=1
Infected Students
Days

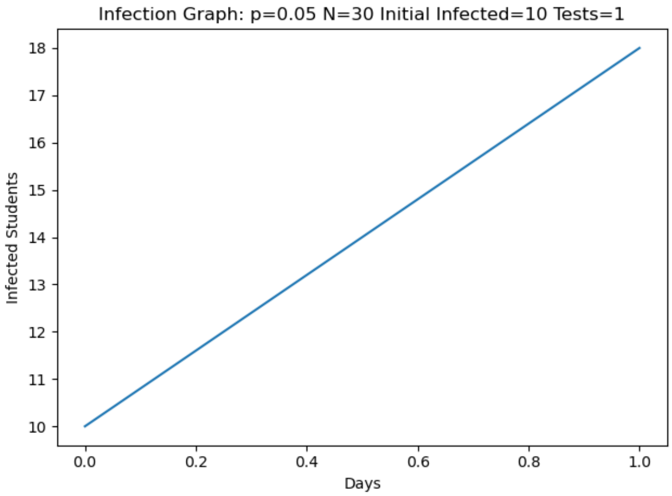

Infection Graph: p=0.33 N=30 Initial Infected=1 Tests=1
Infected Students
Days


Infection Graph: p=0.66 N=30 Initial Infected=1 Tests=100
Infected Students
Days

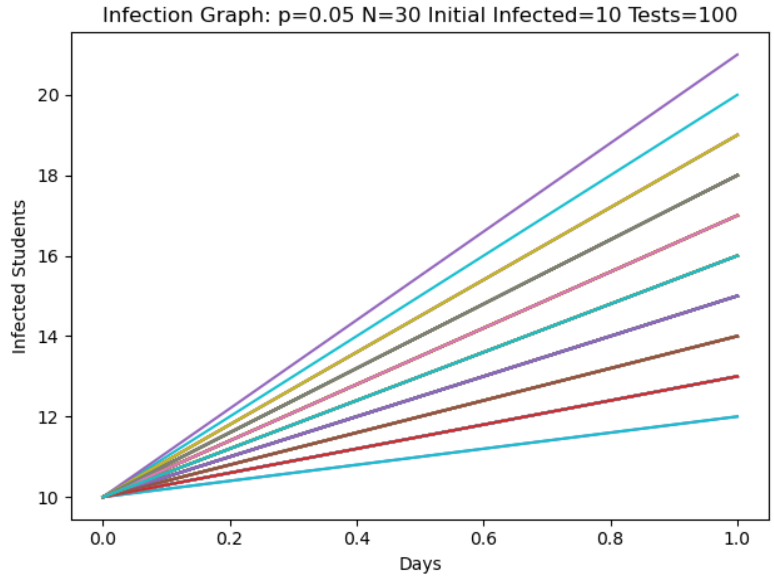

Infection Graph: p=0.33 N=30 Initial Infected=1 Tests=10
Infected Students
Days


Infection Graph: p=0.33 N=30 Initial Infected=1 Tests=10
Infected Students
Days

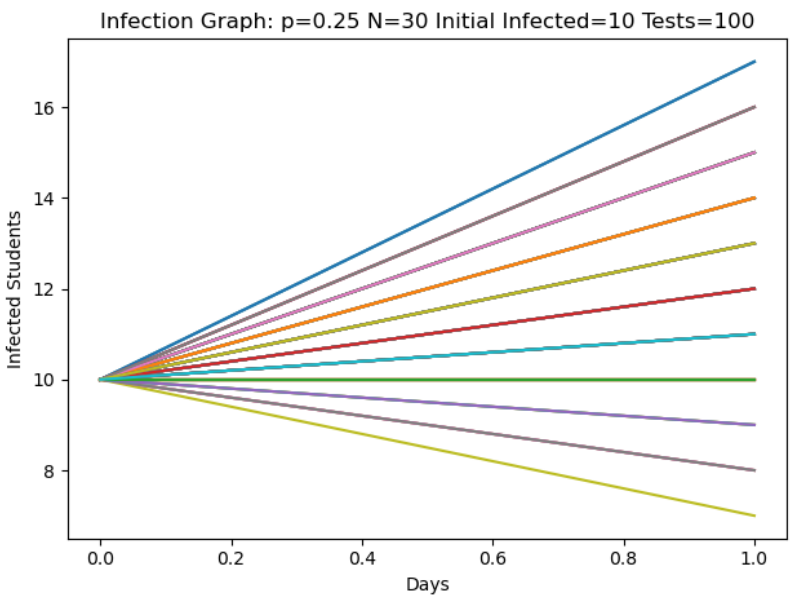
5.



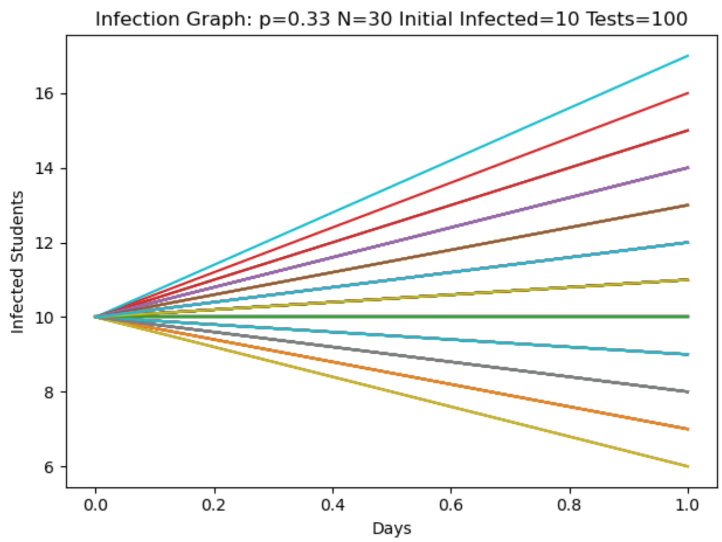
Infection Graph: p=0.05 N=30 Initial Infected=10 Tests=1
Infected Students
Days
10
11
12
13
14
15
16
17
18
0.0
0.2
0.4
0.6
0.8
1.0



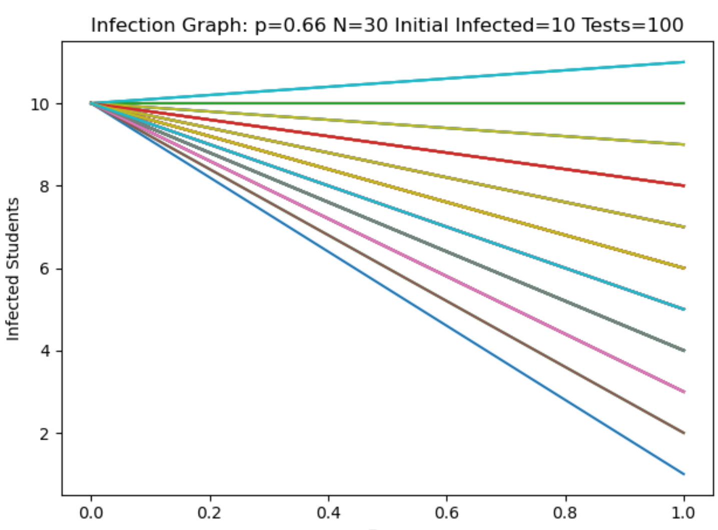
Infection Graph: p=0.05 N=30 Initial Infected=10 Tests=100
Infected Students
Days
10
12
14
16
18
20
0.0
0.2
0.4
0.6
0.8
1.0



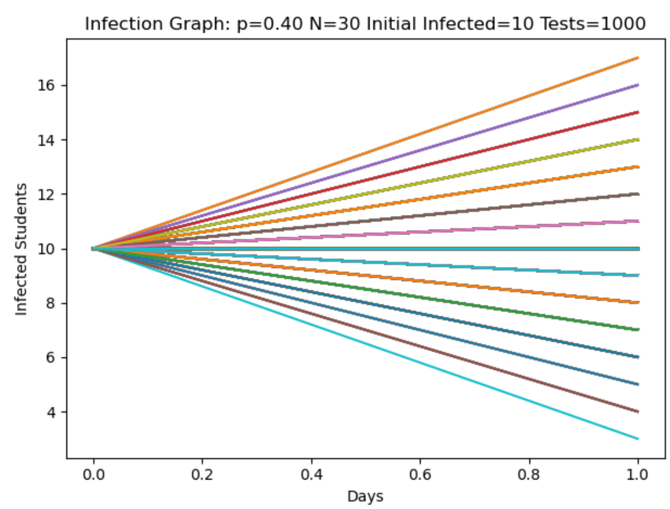
Infection Graph: p=0.25 N=30 Initial Infected=10 Tests=100
Infected Students
Days
8
10
12
14
16
0.0
0.2
0.4
0.6
0.8
1.0



Infection Graph: p=0.33 N=30 Initial Infected=10 Tests=100
Infected Students
Days
6
8
10
12
14
16
0.0
0.2
0.4
0.6
0.8
1.0

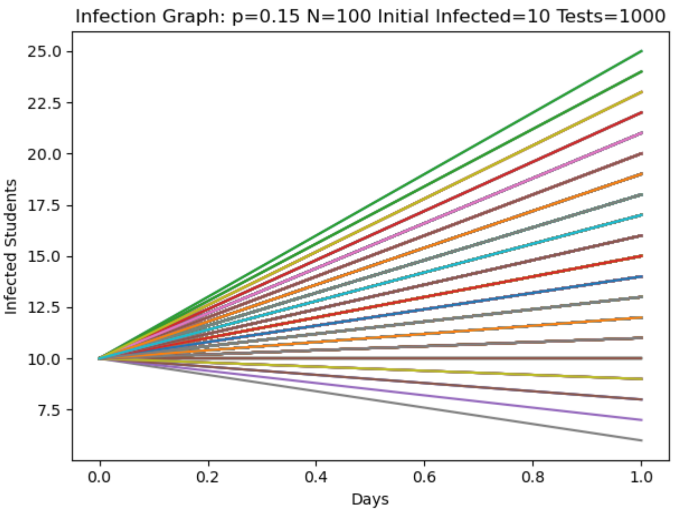


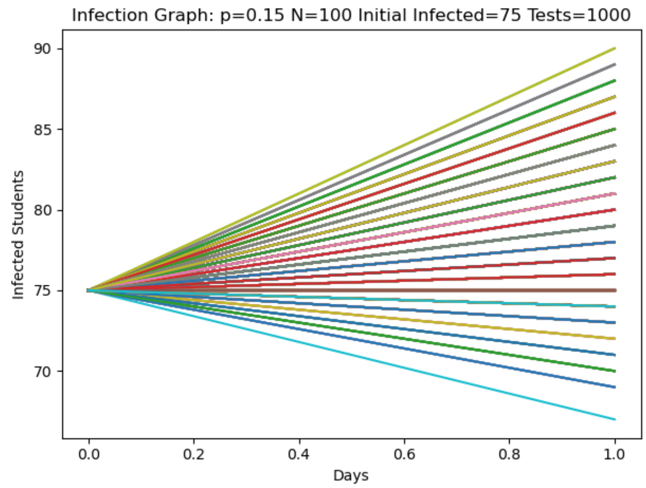
Infection Graph: p=0.66 N=30 Initial Infected=10 Tests=100
Infected Students
Days
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0

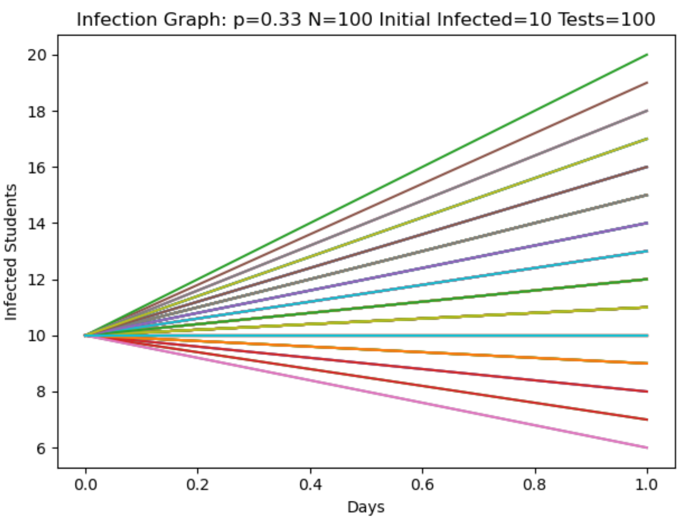


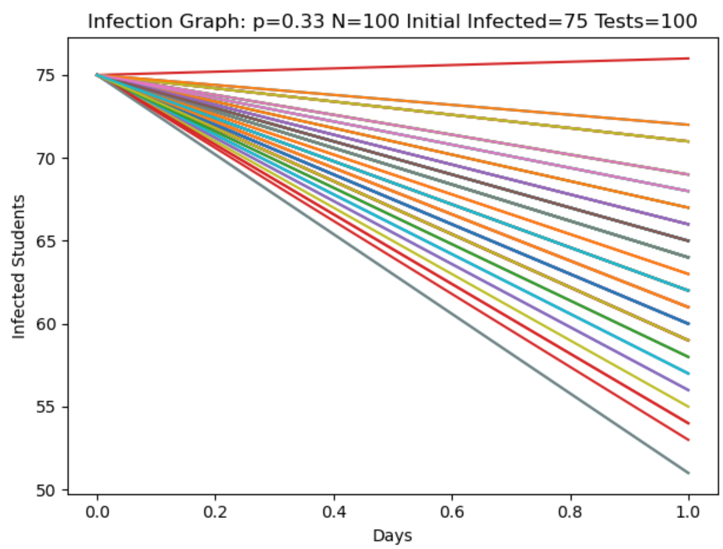
Infection Graph: p=0.40 N=30 Initial Infected=10 Tests=1000
Infected Students
Days
4
6
8
10
12
14
16
0.0
0.2
0.4
0.6
0.8
1.0

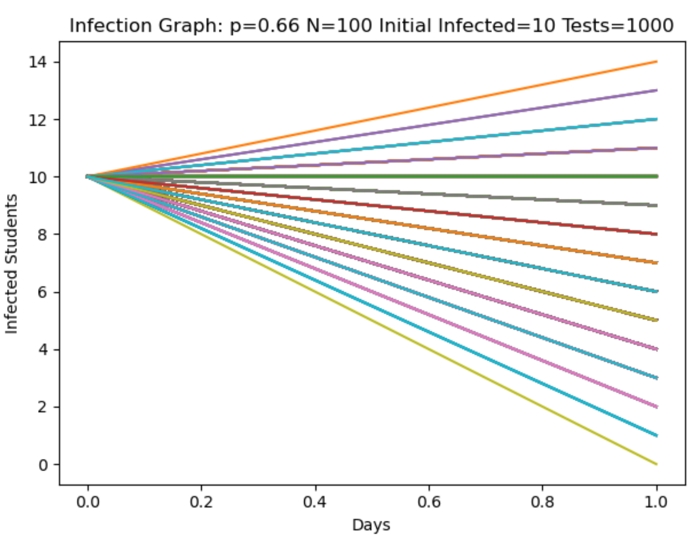
6.

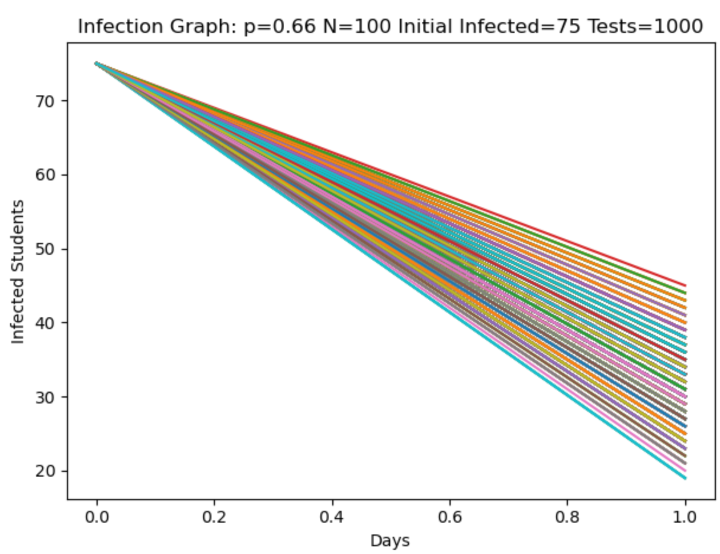

Infection Graph: p=0.15 N=100 Initial Infected=10 Tests=1000
Infected Students
Days


Infection Graph: p=0.15 N=100 Initial Infected=75 Tests=1000
Infected Students
Days


Infection Graph: p=0.33 N=100 Initial Infected=10 Tests=100
Infected Students
Days


Infection Graph: p=0.33 N=100 Initial Infected=75 Tests=100
Infected Students
Days


Infection Graph: p=0.66 N=100 Initial Infected=10 Tests=1000
Infected Students
Days


Infection Graph: p=0.66 N=100 Initial Infected=75 Tests=1000
Infected Students
Days

Scenario 1

1. Sketch of what infected students over time could be. There is a single test, which represents a simulation, and multiple simulations. The graph is exponential because the probability of infection is dependent on the number of students who are infected, stopping at the whole class being infected. It can be noted that the more tests you run, there are some simulations that hit 30 infected students faster, and there are simulations where it takes longer (days) to hit 30 infected students. Interpolating these lines, gives the median, and is the average of the simulations.
2. After a closer examination with the initial infected students=10 and one day passing, I ran 1 simulation, 10 simulations, 1,000 simulations, and 100,000 simulations, the spread of the number of infected students after one day increases. The more simulations I run I can suspect that after one day the variance is between 0 people infected and 30 people infected. Again interpolating this data gives the average of these simulations.
3. Increasing the class size to 1000 and running 10,000 simulations for 10, 100,250,500,750 initially infected I would describe the graphs as depicted. As you increase the graph's initial value, the spread narrows, and the probability of infection spreading increases because more students in the class are infected to choose from.

Scenario 2:

4. I changed the probabilities .05,.15, .33, .66 of students getting healthy, kept the initially infected constant across all graphs and sketched 6 graphs with one test. Then I graphed 2 runs of 10 simulations side by side of probability .33.
5. I narrowed the scope, keeping the initially infected students constant @ 10, and found that after one day for .05 probability student getting healthy, the number of infected students increased. For .25, .33, and .40 probability of a student getting healthy, the spread was neutral, some simulations increasing and some decreasing number infected students after one day. For .66 probability of student getting healthy the number of infected students decreased, but some simulations still were increasing number of infected students, which makes sense because even though there is a higher probability of students getting healthy, there is still a probability of each student getting infected of .33.
6. I ran a sample with .15, .33, and .66 probability of students healthy, and ran two graphs for each. For all the graphs on the left side, I found as you increase the probability of students getting healthy, the interpolated line, which is the average of all the tests, gets more and more negative. For the graphs on the right side, this is seen more clearly as you increase the probability of the students getting healthy, the interpolated line gets more and more negative.

Modeling Project Part 2:

Consider, for example, the outcome of rolling a fair (six sided die). You could describe this as 3.5 (the expected outcome), but this doesn't really seem to capture the physical situation of rolling a die very well. To some extent this is a matter of opinion, but I think at least a description that does not include the randomness of the scenario should have a good explanation for why it does not need to be included.

“Nice simulations. I wonder, can you give a theoretical explanation for what you observed in the simulations? In 4, do you think you ran the simulations for long enough to accurately observe the long term behavior? Why or why not?”

Theoretical explanation for the simulations:

In the first scenario, the simulation was static, or increasing exponentially across time. This is because the simulation was a function of each day a student getting infected and being added to the infectious pool. This scenario will always be exponential because there was no parameter for recovery, the question is how long until the simulation becomes exponential.

Scenario 2, there are more interesting theoretical observations to make,because each day there is some probability of students becoming healthy, so the graph at each day is in flux given the probability is a function of two probabilities. Theoretically speaking, the simulations tend to follow patterns over time. Interestingly enough the range of convergence seems to be whatever the probability of getting healthy is, multiplied by the total number of students. When you subtract that number, that seems to be the interpolated lines magic number, obviously deviating up or down, randomly, but over time that is the most likely outcome. My explanation for this is because the probability of selecting an infected student increased to maximum capacity but was pushed down by the probability of recovering, so intuitively this matches theory nicely.

In Scenario 2, I believe that the experiments are run for long enough, especially with only two variables effecting the simulation. If I were to introduce more variables, like every 7 days everyone is healthy, or decrease amount of students in the class, or increase ability to get healthy over time and not keep constant across the simulation, then this unpredictably would create more flux, and disturbance, that the time I ran for 1000 days would not be sufficient. Given that there are two simple variables, it seems like the simulations were run, in my opinion for long enough. I ran the tests for 100,000 days and the pattern was seen the same, I even ran the test for 1,000,000 days and the same trend existed. Obviously there is randomness, so different things happened, some outliers, but the general pattern, “the magic number” seemed to be the long term effect, and with only one variable applying pressure up, and one down, they converge over time, and they did so the same way over 10 times and 100 times as many days.

Modeling Project Part 3:

Matthew Benvenuto — Modeling Project 3

Scenario 1:

1. My descriptions of Part 1 & 2 were simple programming simulations & qualitative assesments of such programs, so the expected value from Parts 1 & 2 would be the interpolated line of the number of tests.

→ A more accurate representation would be the theoretical sense, such would be a Binomial Random variable'.

- $E[X] = \sum_{i=0}^{N-A} i \binom{N-A}{i} \left(\frac{A}{N}\right)^i \left(1-\frac{A}{N}\right)^{N-A-i} = \frac{AN-A^2}{N}$
- @ $t=1$ $+A$ → $\frac{2AN-A^2}{N}$, where A is # of kids came in w/ flue.
- $Var(X) = E[X^2]-(E[X])^2$:

$$\left(\frac{AN-A^2}{N}\left((N-A-1)\frac{A}{N}+1\right)\right)-\left(\frac{AN-A^2}{N}\right)^2 = \frac{AN-A^2}{N}\left(\frac{A(N-A-1)}{N}+1\right)-\frac{AN-A^2}{N}\Big) = \frac{A(N-A)^2}{N^2}.$$

2. $A=10$, for each k, find $\lim_{N\to\infty}$. Do you recognize limit?

- $\lim_{N\to\infty} E[X_n] = \lim_{N\to\infty} \frac{10(N-10)}{N} = \lim_{N\to\infty} 10-\frac{10}{N} = 10.$
- $\lim_{N\to\infty} Var(X_n) = \lim_{N\to\infty} \frac{10(N-10)^2}{N^2} = \lim_{N\to\infty} \frac{10N^2-200N+1000}{N^2} = \frac{10N^2}{N^2} = 10.$, $\sigma = \sqrt{10}.$
- Continuity correction $-\frac{1}{2}, +\frac{1}{2}$. : $\underbrace{\lim_{N\to\infty} P\left\{\frac{k-9.5}{\sqrt{10}} \le \frac{X_n-10}{\sqrt{10}} \le \frac{k-10.5}{\sqrt{10}}\right\}}_{\text{Central limit theorem.}} = \Phi\left(\frac{k-9.5}{3.162}\right)-\Phi\left(\frac{k-10.5}{3.162}\right).$

3. Suppose $N=1000$ & $A=100$ approx what probability # of infected students @ end day 1 is greater than 210?

- $\mu = E[X] = (1000-100)\cdot\frac{100}{1000} = 90.$ $\sigma = \sqrt{np} = \sqrt{90\cdot.9} = \sqrt{81} = 9.$
- $P\{X \ge 110\} = P\left\{\frac{X-\mu}{\sigma} \le \frac{110-90}{9}\right\} = P\left\{Z \le \frac{20}{9}\right\} = 1-\Phi(2.22) = 1-.9868 = .0132$

* Since, initial sick is 100, & seeking greater than 210, offset $X \ge 110$.

Scenario 2: 4. @ end of class healthy w/ p, $0<p<1$. No immunity, just return to Population & continue simulation.

- let X be # of new infected students @ middle of day.
- let H be # of healthy students @ end of day who were infected.
- let Y be # of infected left students.

So, $E[Y] = \sum_{i=0}^{N} i\, P(X=0)\cdot P(H=|A-i|)+P(X=1)\cdot P(H=|A+1-i|)+\dots+P(X=N-A)\cdot P(H=|N-i|)$, $E[Y^2] = \sum_{i=0}^{N} i^2\, P(X=0)\cdot P(H=|A-i|)+P(X=1)\cdot P(H=|A+1-i| + \dots + P(X=N-A)+\dots P(H=|N-i|)$.

$Var[Y] = E[Y^2]-(E[Y])^2$; Let $E[Y]=E$, let $E[Y^2]=B$

$Var[Y] = B - E^2$ ~~[illegible]~~

5. find probability mass function?

$P(Y=i) = P(X=0)\cdot P(H=|A-i|)+P(X=1)\cdot P(H=|A+1-i|)+\dots P(X=N-A)\cdot P(H=|N-i|)$, $i=0,\dots,N$.

$P(X=j) =$ same as before, $P(X=j)=0$ if $j>A$ & $j<i-A$.

$P(H=i) = \binom{j+A}{i} p^i (1-p)^{j+A-i}$

Modeling Project Part 4:

Matthew Benvenuto

Modeling Project Part 4

Scenario 1:

#1: $\lim_{N\to\infty} N^{-1} E[X_i^*] = \lim_{N\to\infty} \dfrac{(N-A_N)\cdot\frac{A_N}{N} + A_N}{N} = \lim_{N\to\infty} \dfrac{(N-A_N)}{N}\cdot a + \dfrac{A_N}{N}$

$\boxed{\lim_{N\to\infty} \frac{A_N}{N} = a}$

$= \lim_{N\to\infty} \dfrac{Na - A_N\cdot a}{N} + a$

$= \lim_{N\to\infty} \dfrac{N\cdot a}{N} - \dfrac{A_N\cdot a}{N} + a$

$= \lim_{N\to\infty} a - a\cdot a + a$

$= 2a - a^2$

#2: Recurrence Relation : , $(a_i)_{i=0}^{\infty}$ $\boxed{a_{i+1} = 2a_i + a_i^2}$

#3: Program : Code! (Plot)

#4: The plot in the code directly follows the recurrence relation guess!

Scenario 2:

#5: $\lim_{N\to\infty} N^{-1}\cdot E[X_i^*] = \lim_{N\to\infty} \left(\dfrac{(N-A_N)\cdot\frac{A_N}{N} + A_N}{N}\right)(1-P)$, P is constant, $0<P<1$

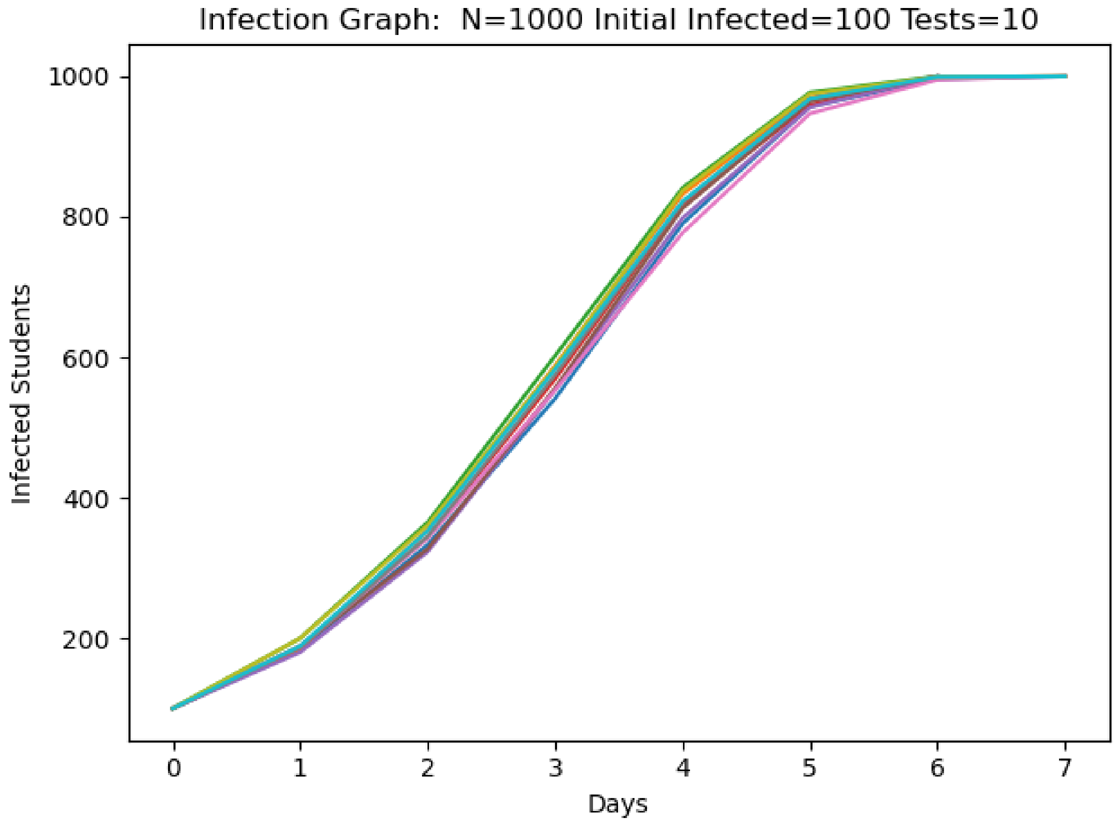
$= 2a - a^2(1-P)$

#6: Recurrence Relation: $\boxed{a_{i+1} = (2a_i + a_i^2)(1-P)}$

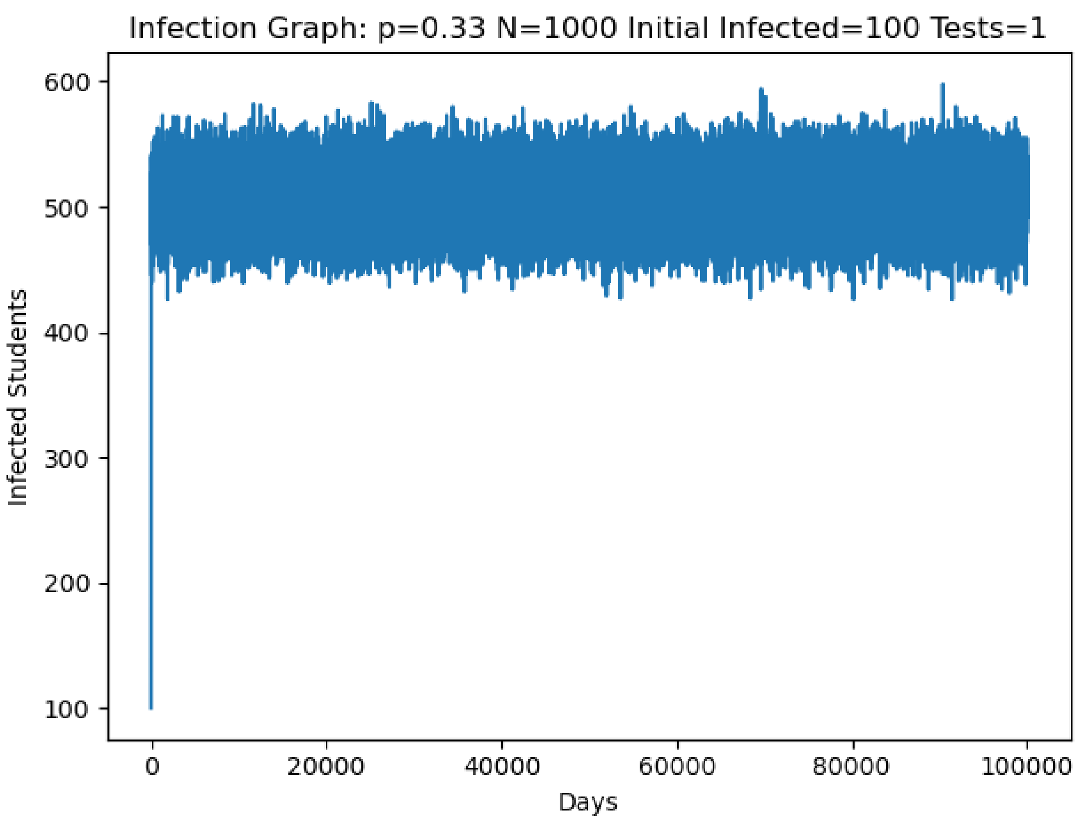
#7: Program: Code! (Plot)

#8: The plot in the code directly follows recurrence relation guess!

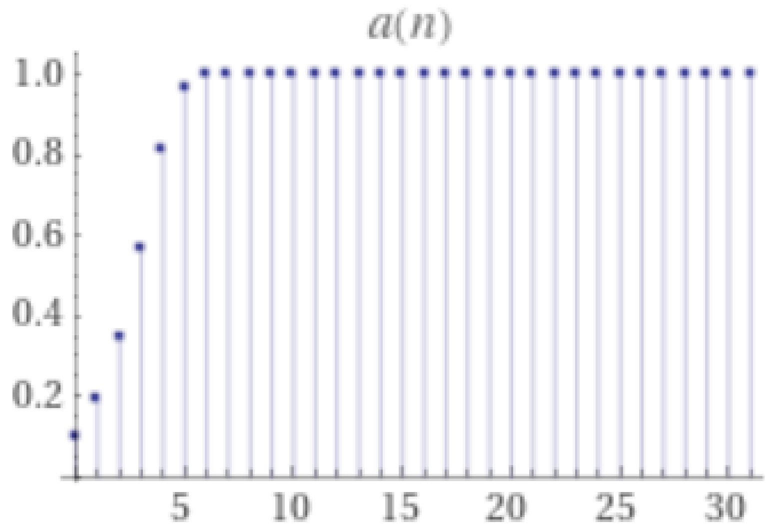
3


Infection Graph: N=1000 Initial Infected=100 Tests=10
1000
800
600
400
200
Infected Students
0
1
2
3
4
5
6
7
Days

8


Infection Graph: p=0.33 N=1000 Initial Infected=100 Tests=1
600
500
400
300
200
100
Infected Students
0
20000
40000
60000
80000
100000
Days

2. Recurrence relation



6. Recurrence relation p=.33