Statistical analysis of super-resolution SPTs

Dataset

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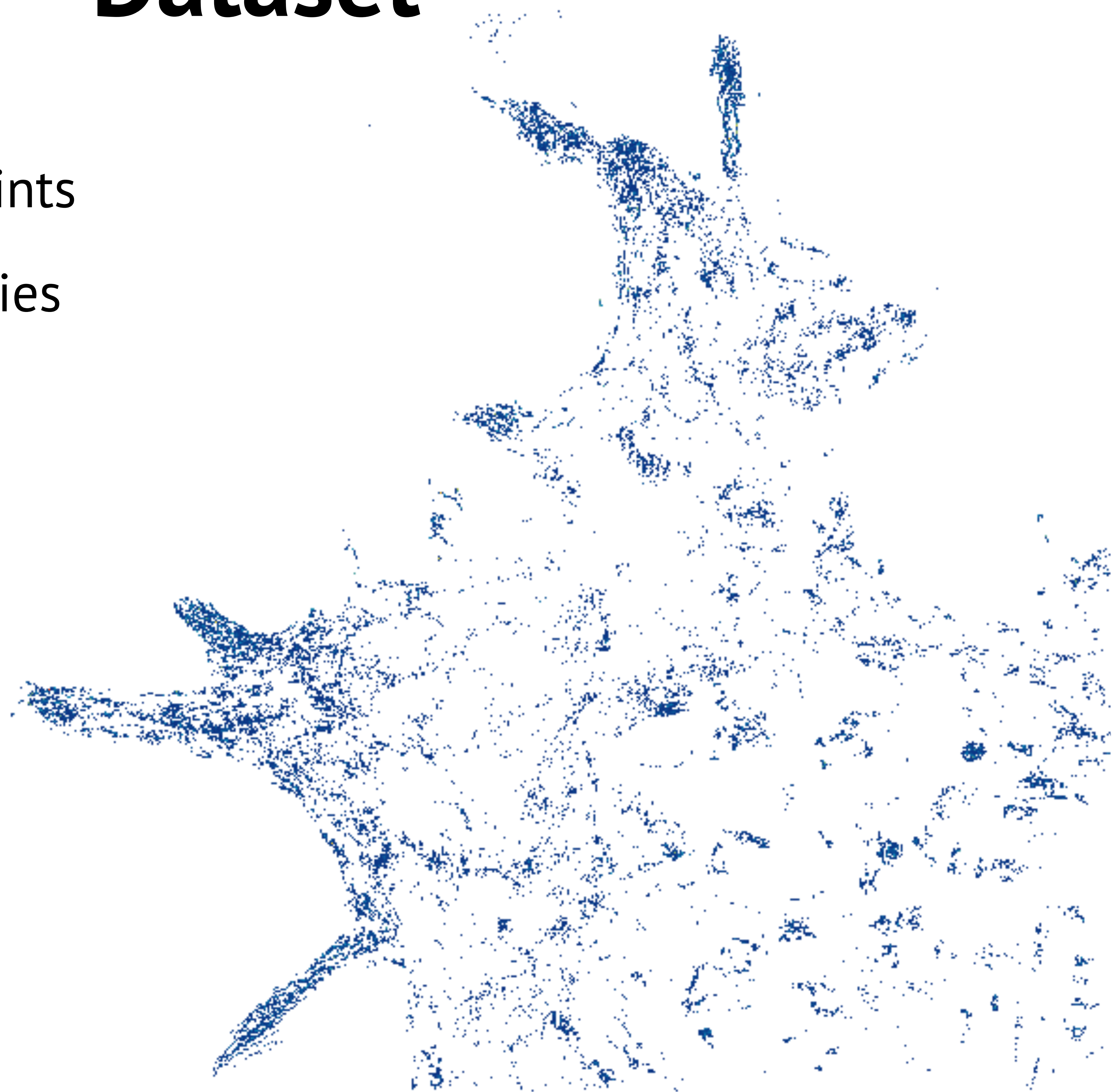
- ~ 20 000 trajectories



Dataset

- ~ 350 000 datapoints
- ~ 20 000 trajectories
- Δt = 125 ms

Langevin’s dynamics

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Effective model

$$\dot{\boldsymbol{x}} = \boldsymbol{a}(\boldsymbol{x}) + \sqrt{2}\boldsymbol{B}\dot{\boldsymbol{w}}$$

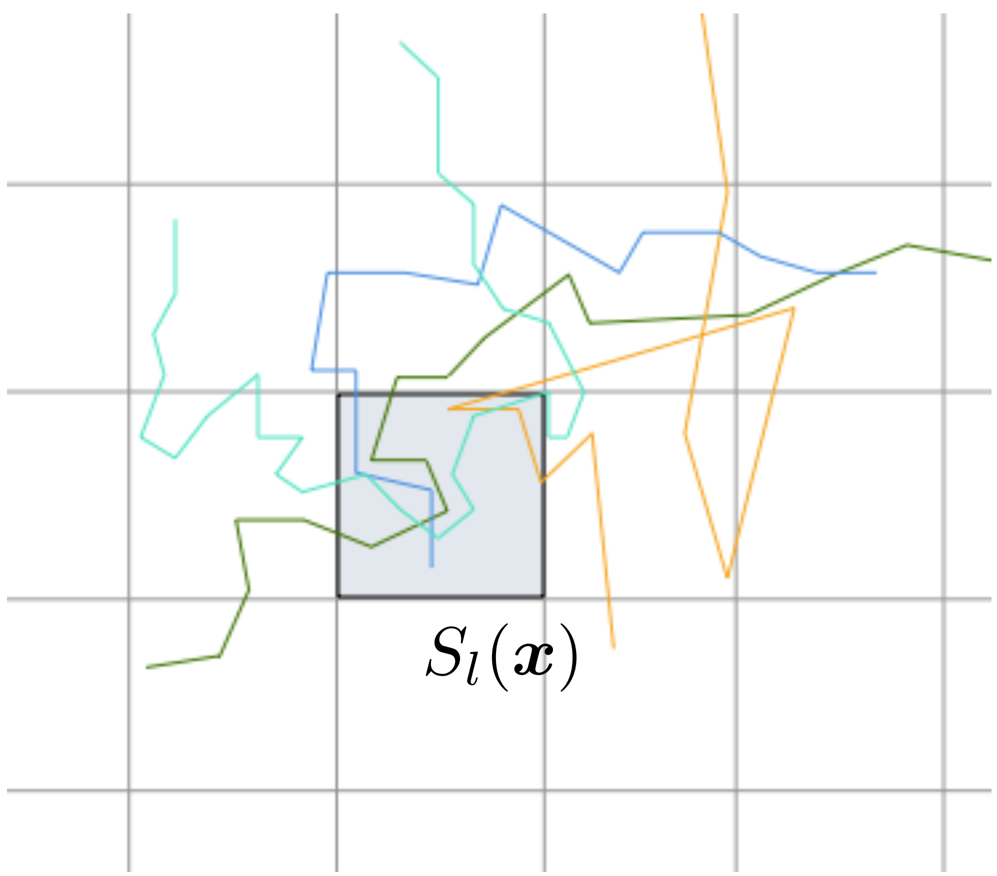
Drift and diffusion

$$a^i(\boldsymbol{x}) = \lim_{\Delta t \to 0} \frac{\mathbb{E}\left[x^i(t+\Delta t) - x^i(t) \mid \boldsymbol{x}(t) = \boldsymbol{x}\right]}{\Delta t}$$

$$2D^{ij}(\boldsymbol{x}) = \lim_{\Delta t \to 0} \frac{\mathbb{E}\left[\left(x^i(t+\Delta t) - x^i(t)\right)\left(x^j(t+\Delta t) - x^j(t)\right) \mid \boldsymbol{x}(t) = \boldsymbol{x}\right]}{\Delta t}$$

Statistical estimators

- Divide the space with a grid of square bins
- Perform averages on the steps that fall inside each bin



$$\hat{a}^i(\boldsymbol{x}) = \frac{1}{N} \sum_{\boldsymbol{x}_k(t_m) \in S_l(\boldsymbol{x})} \frac{\Delta x_k^i(t_m)}{\Delta t}$$

$$\hat{D}^{ij} = \frac{1}{2} \frac{1}{N} \sum_{\boldsymbol{x}_k(t_m) \in S_l(\boldsymbol{x})} \frac{\Delta x_k^i(t_m) \Delta x_k^j(t_m)}{\Delta t}$$

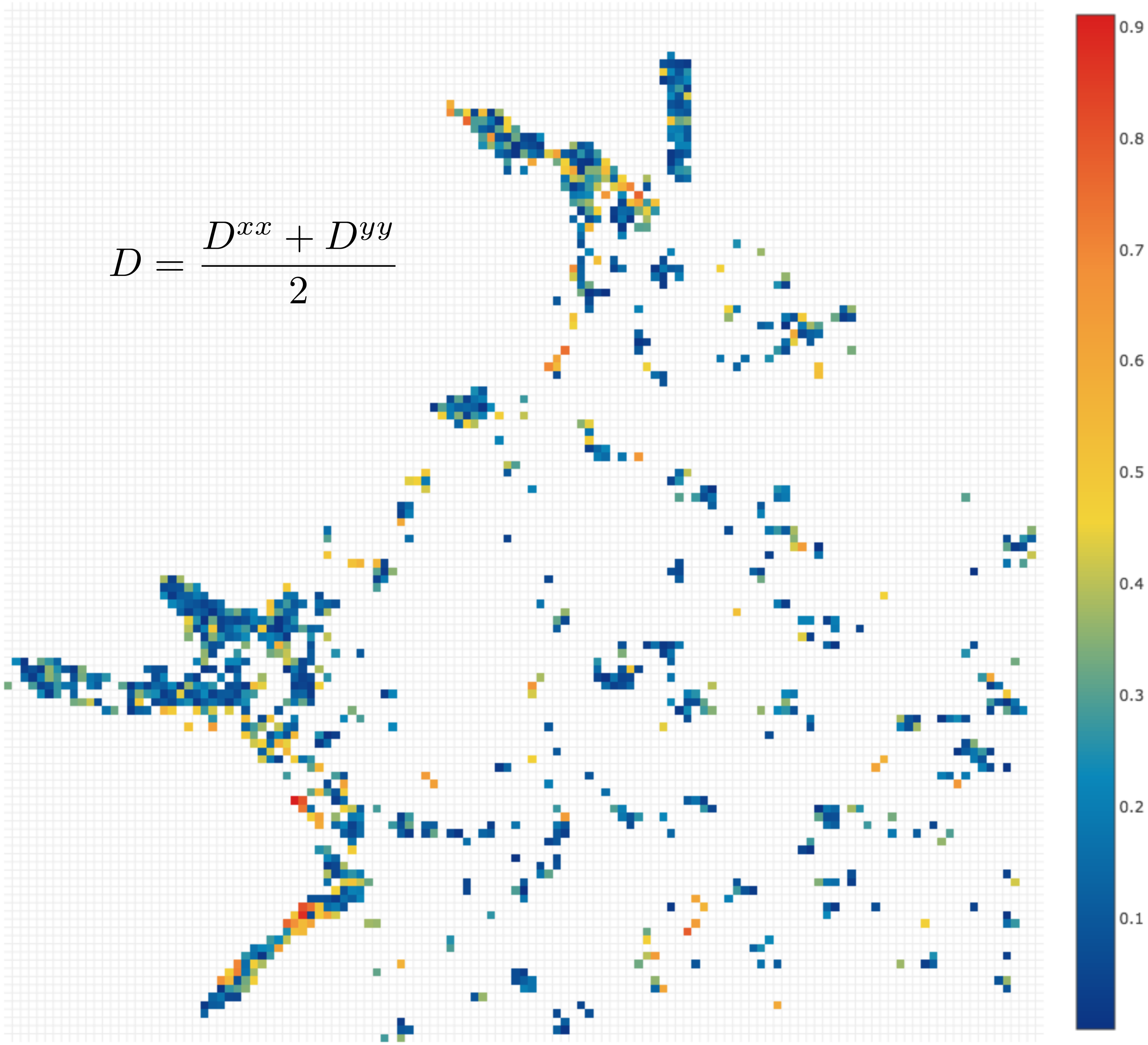
Standard error

$$err \sim \frac{1}{\sqrt{N}}$$

We only consider bins where N > 100

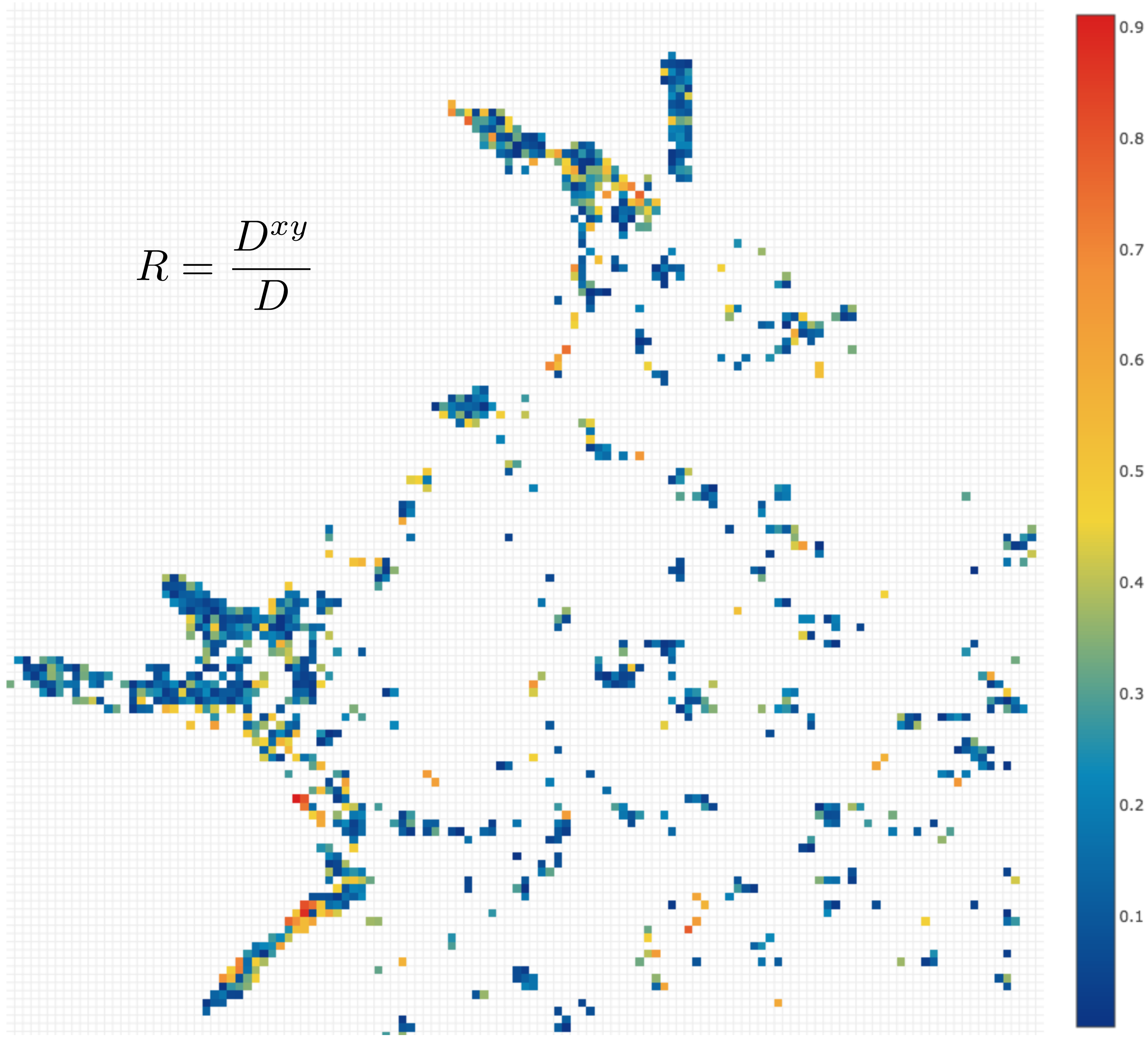
Diffusion coefficient

$$D = \frac{D^{xx} + D^{yy}}{2}$$

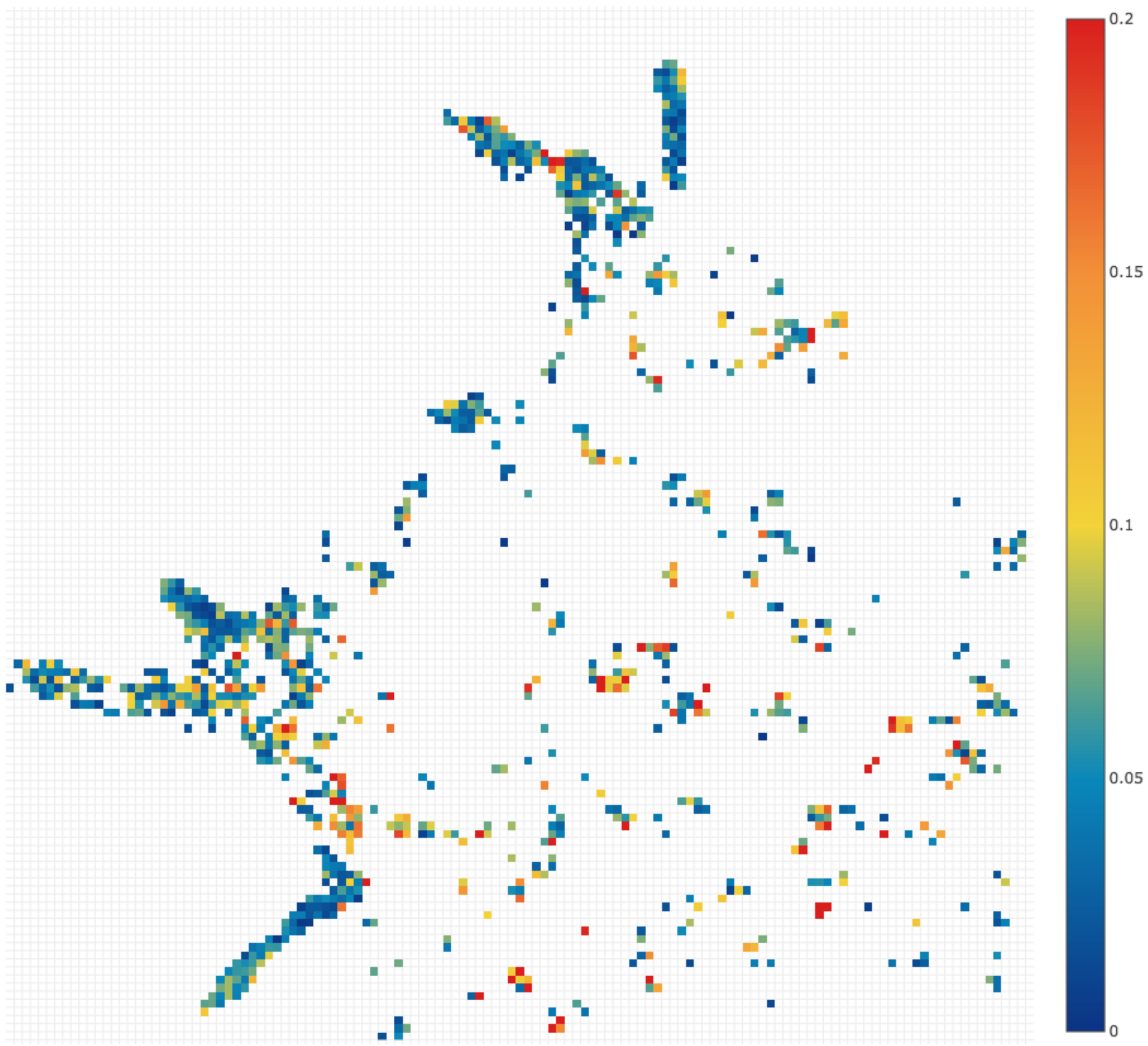


Diffusion isotropy

$$R = \frac{D^{xy}}{D}$$



Drift coefficient



Attractors

The drift field

$$\boldsymbol{a}(\boldsymbol{x}) = -\nabla U(\boldsymbol{x}).$$

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(assumption)

Potential wells

Local minima of the potential

$$U(x,y) = U_0 + \frac{W}{r^2}\left(x - x_0\right)^2 + \frac{W}{r^2}\left(y - y_0\right)^2 + \text{higher order terms}$$

The coefficient W indicates the strength of the potential.

Fitting

Fit to the empirical potential with least squares method

$$W = -\frac{r^2}{2}\frac{\sum_{k=1}^{N} a^x(x_k, y_k)x_k + a^y(x_k, y_k)y_k}{\sum_{k=1}^{N} x_k^2 + y_k^2}$$

Location of potential wells

Numerical simulation

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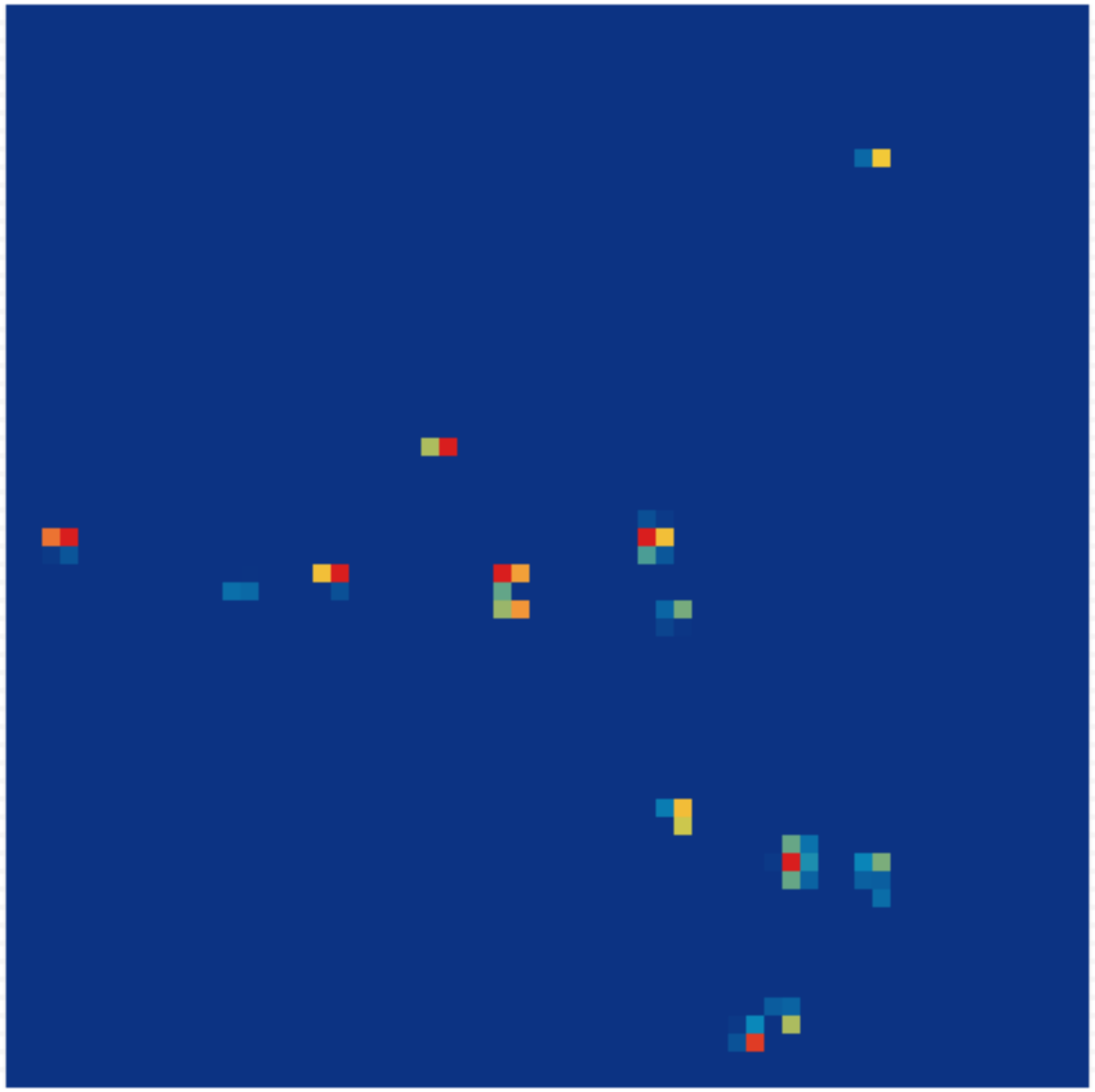
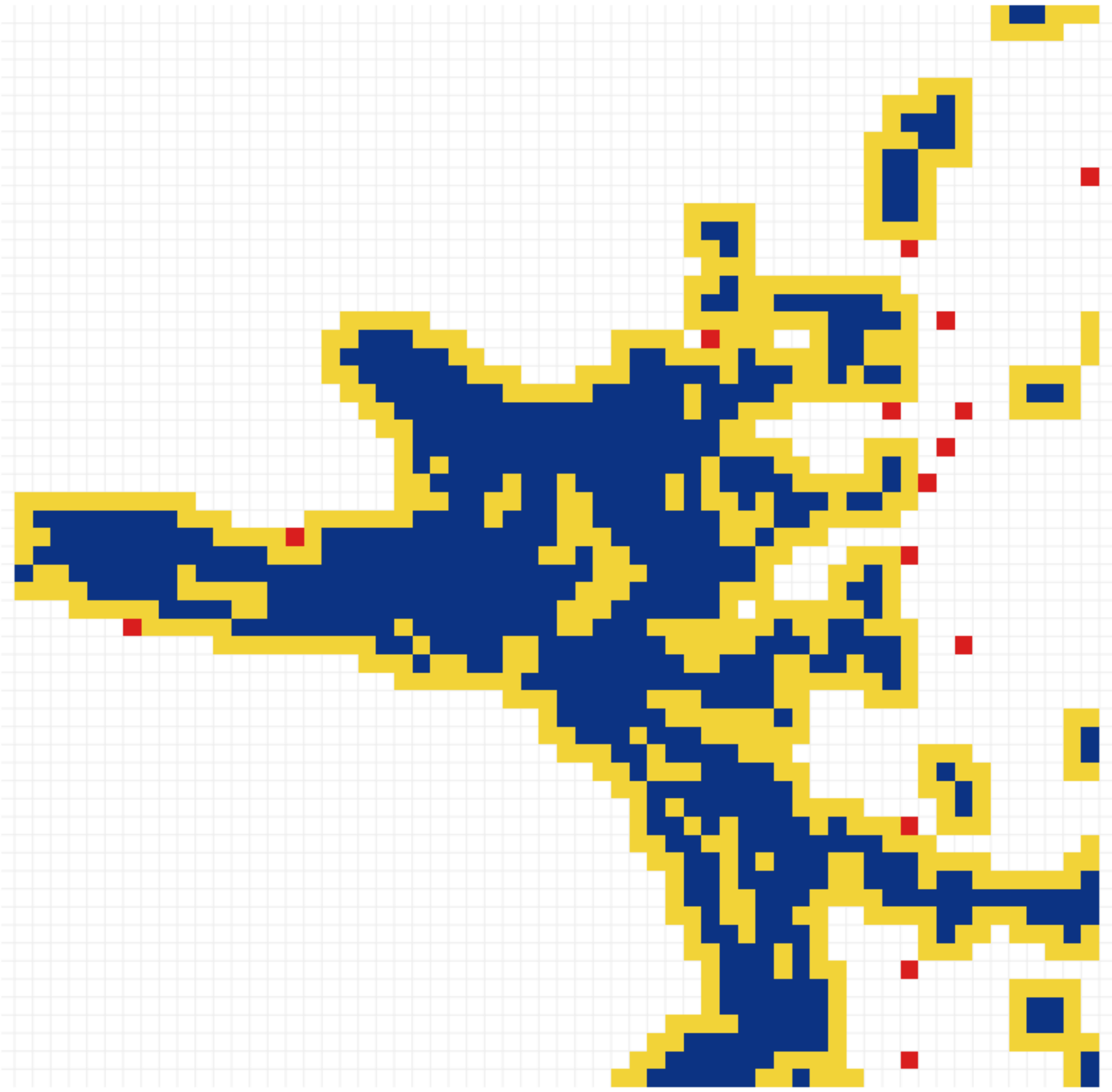
Numerical simulation



Preliminary work

Exclude isolated bins



Preliminary work

Smoothen the drift field $K = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

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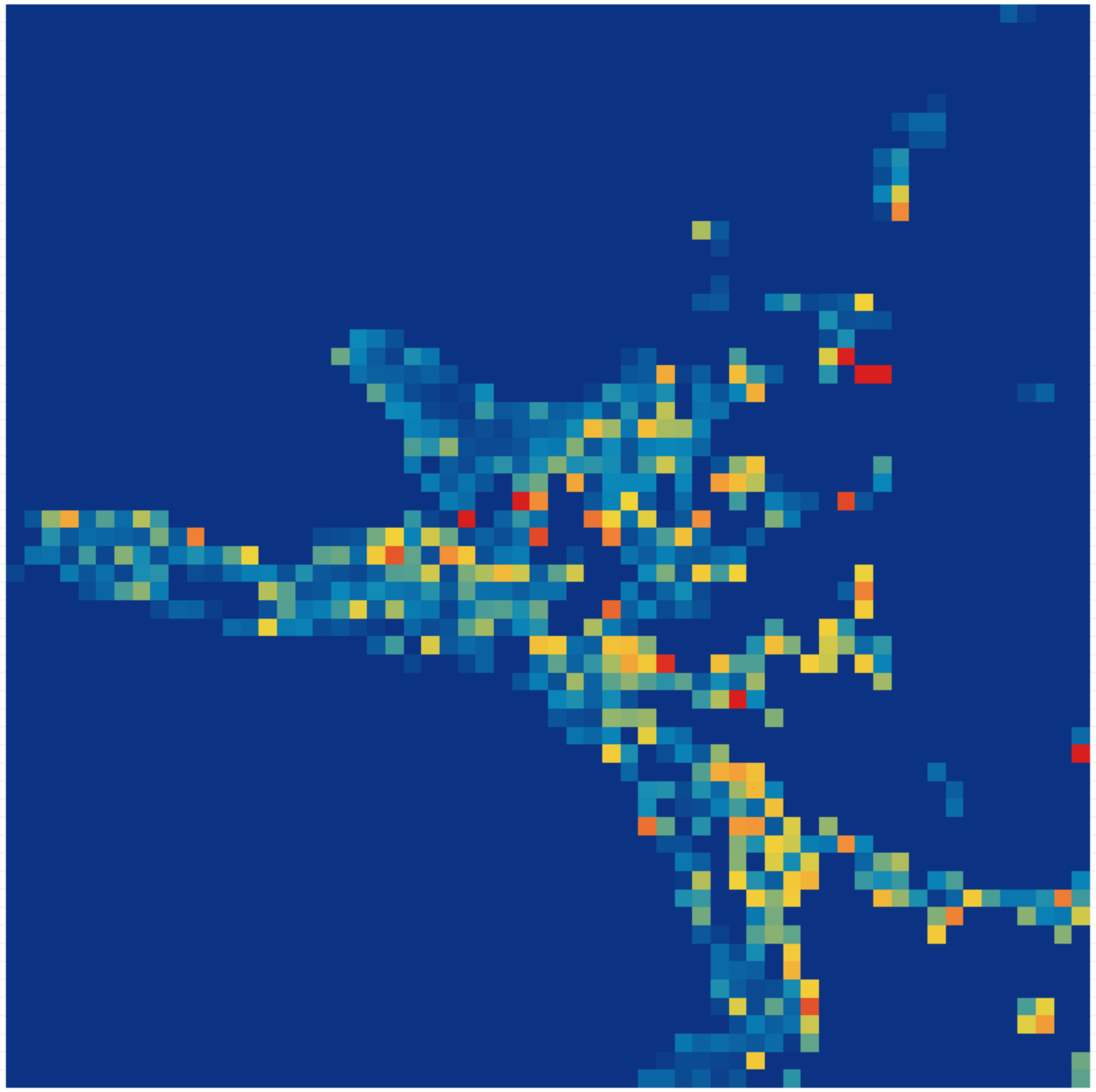

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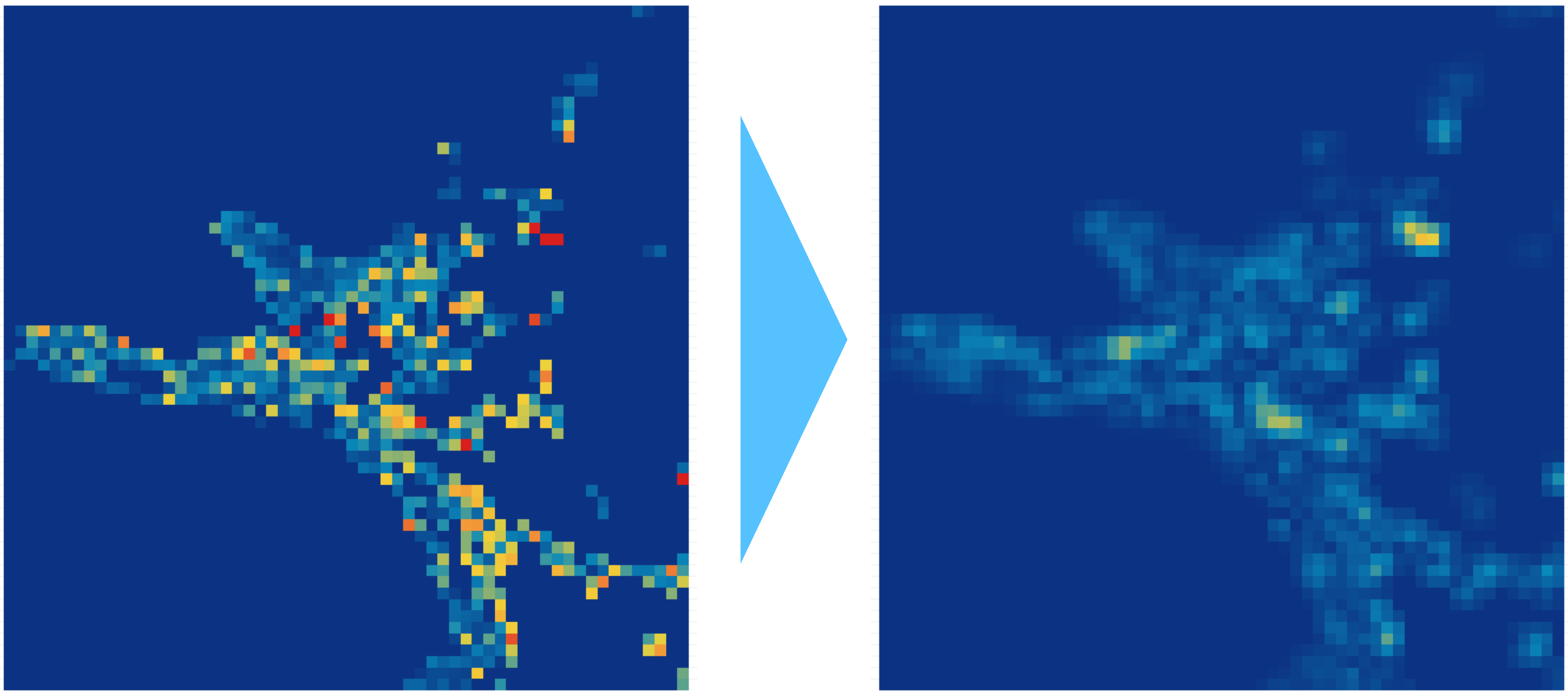

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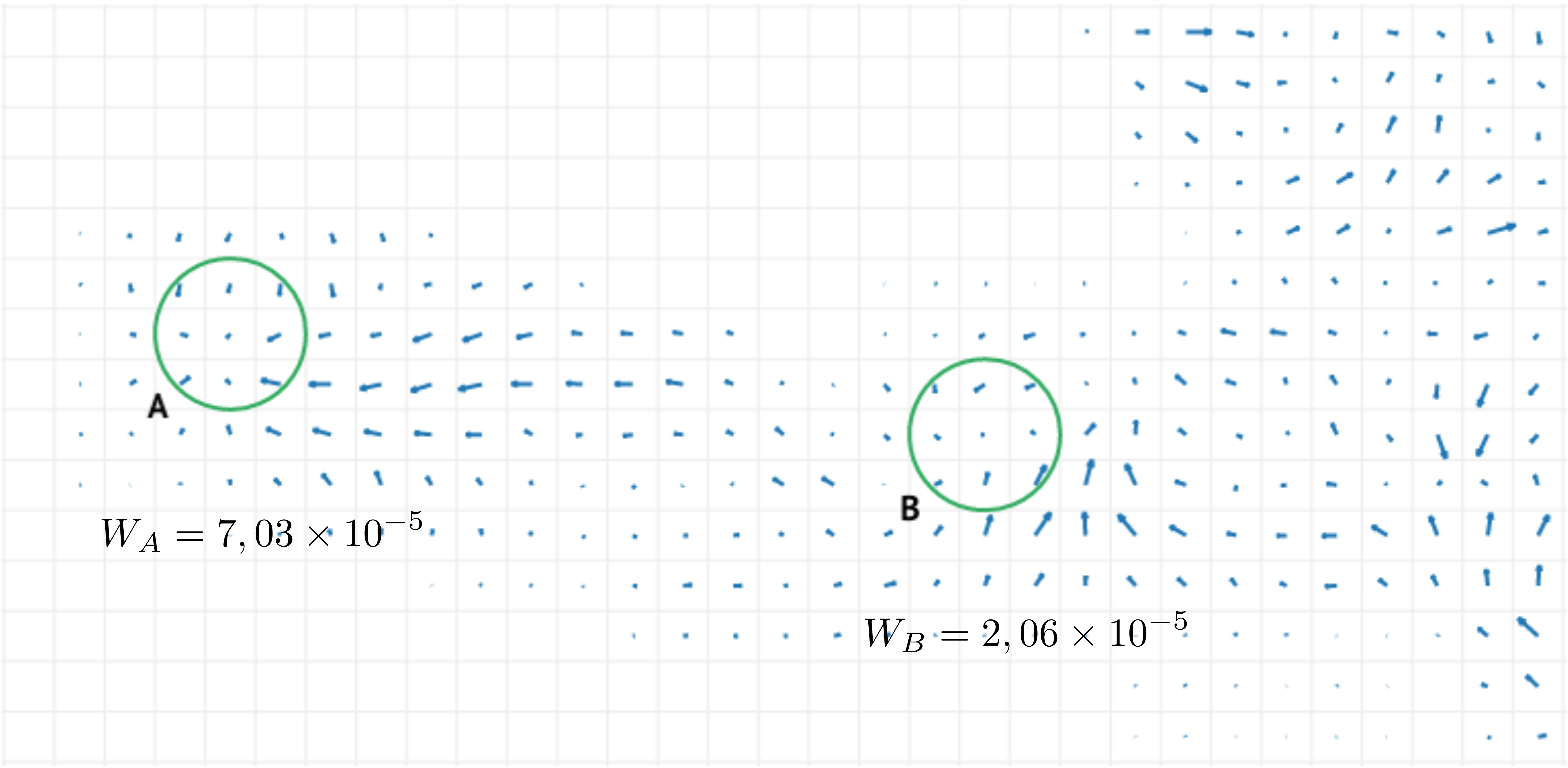
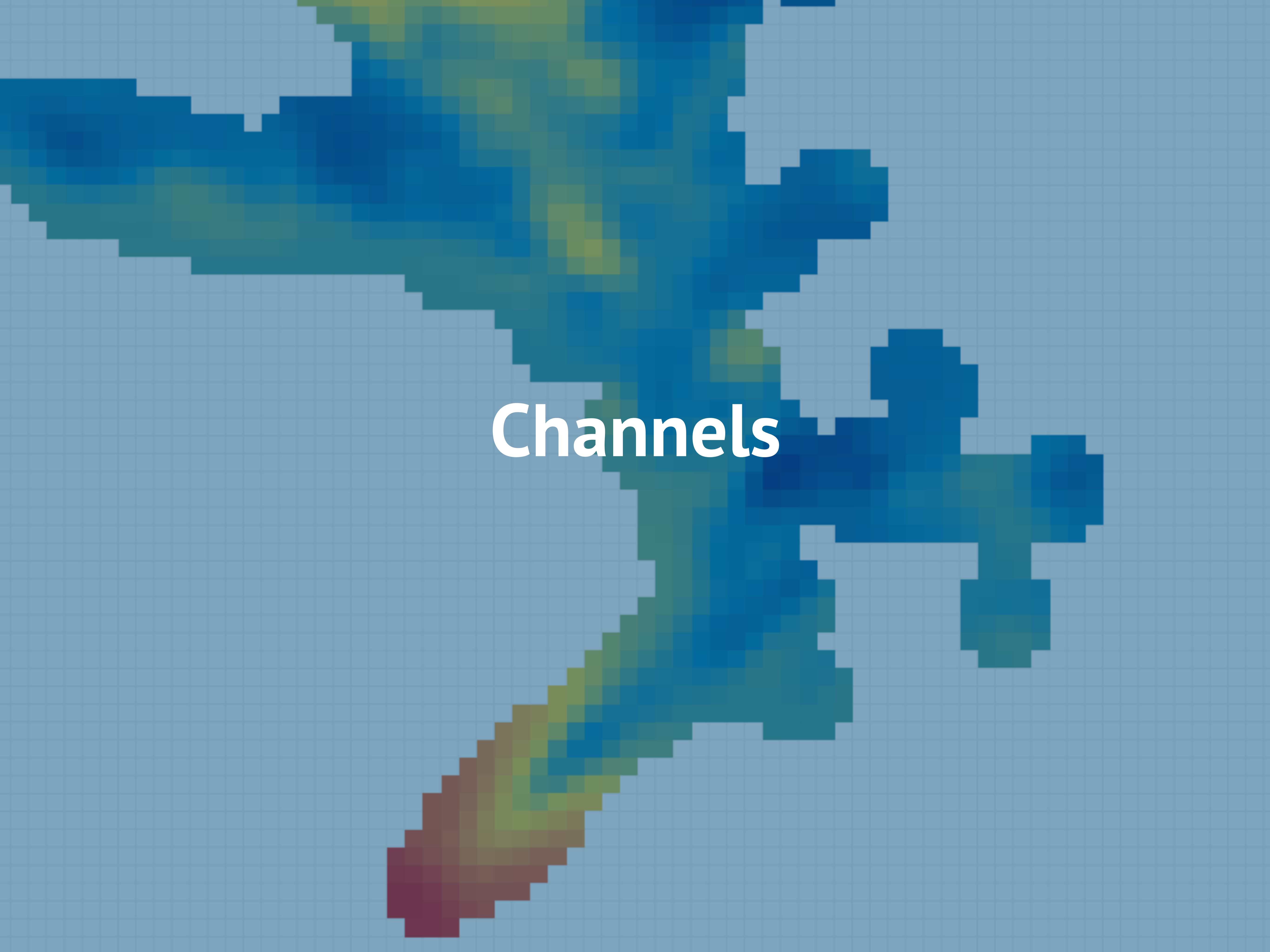


Channels



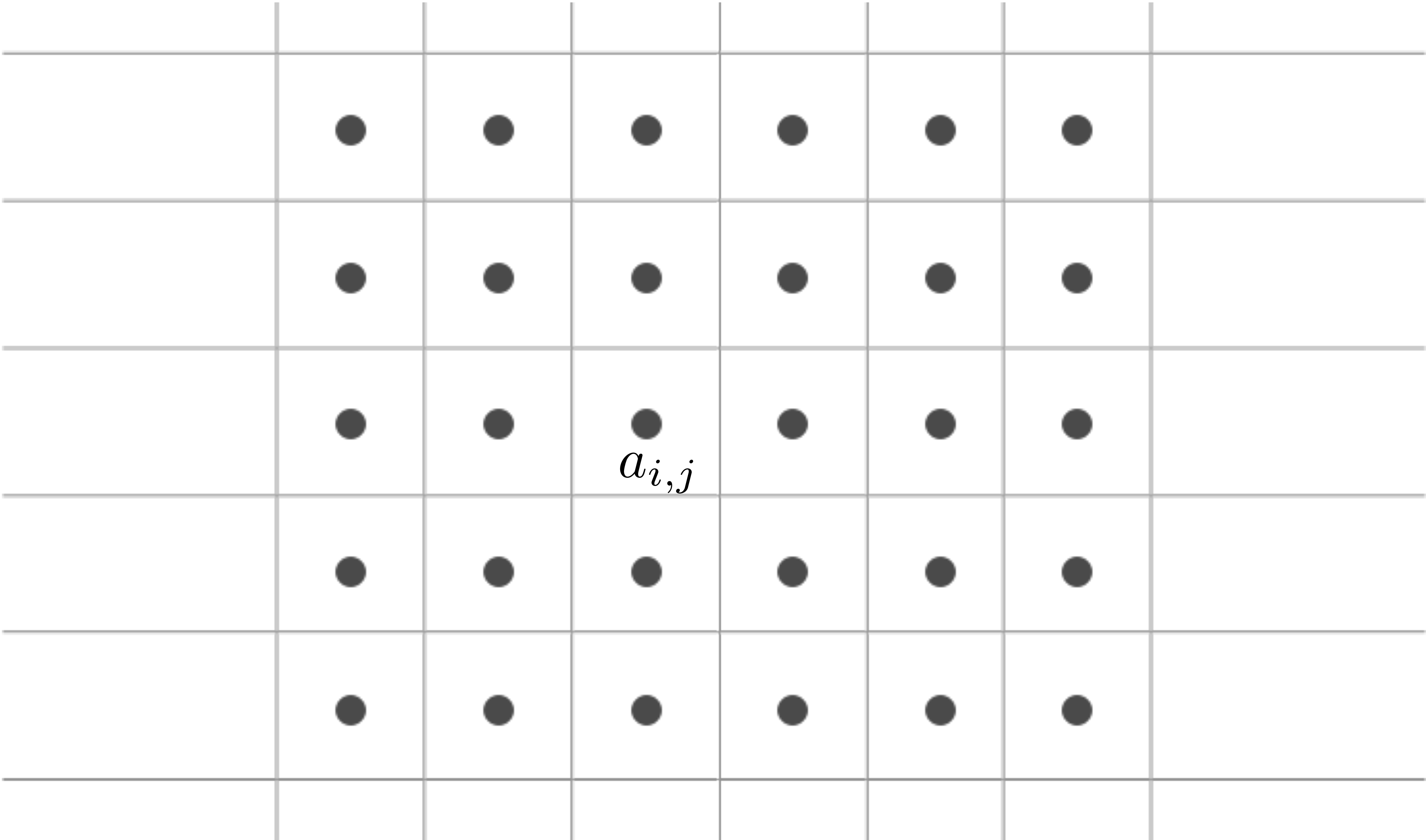
The drift field

$$\boldsymbol{a}(\boldsymbol{x}) = -\nabla U(\boldsymbol{x}).$$

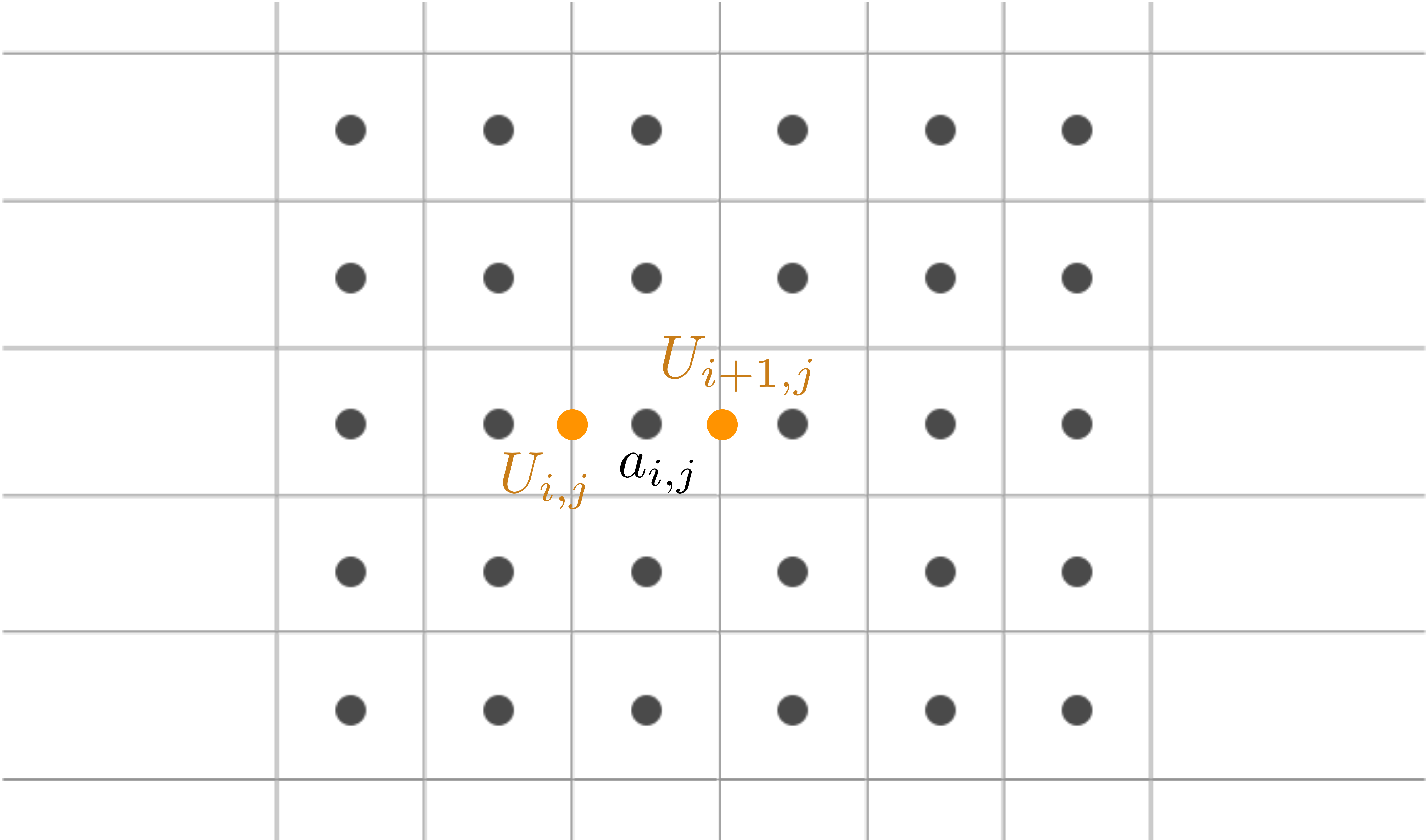
Channels

- Channels as potential valleys
- Computer vision → ridge detection
- We have to reconstruct the potential

Reconstructing the potential



Reconstructing the potential



Reconstructing the potential

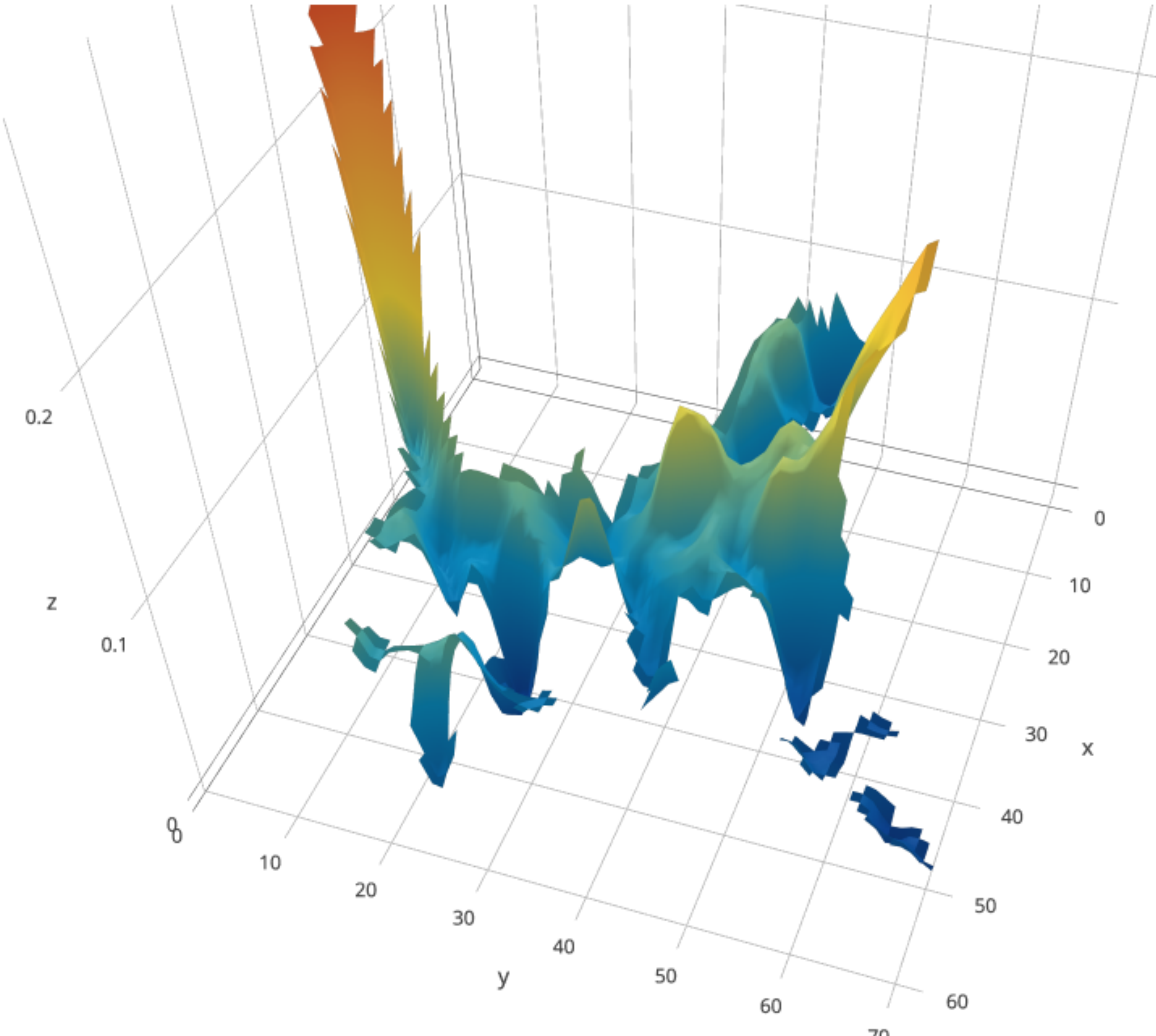
$$a^{x}_{i,j} = -\frac{U_{i+1,j} - U_{i,j}}{l}$$

$$a^{y}_{i,j} = -\frac{U_{i,j+1} - U_{i,j}}{l}$$

Reconstructing the potential

- Remove isolated bins (not useful)
- Smoothen the drift
- Calculate the potential for a cluster of contiguous bins

Reconstructing the potential



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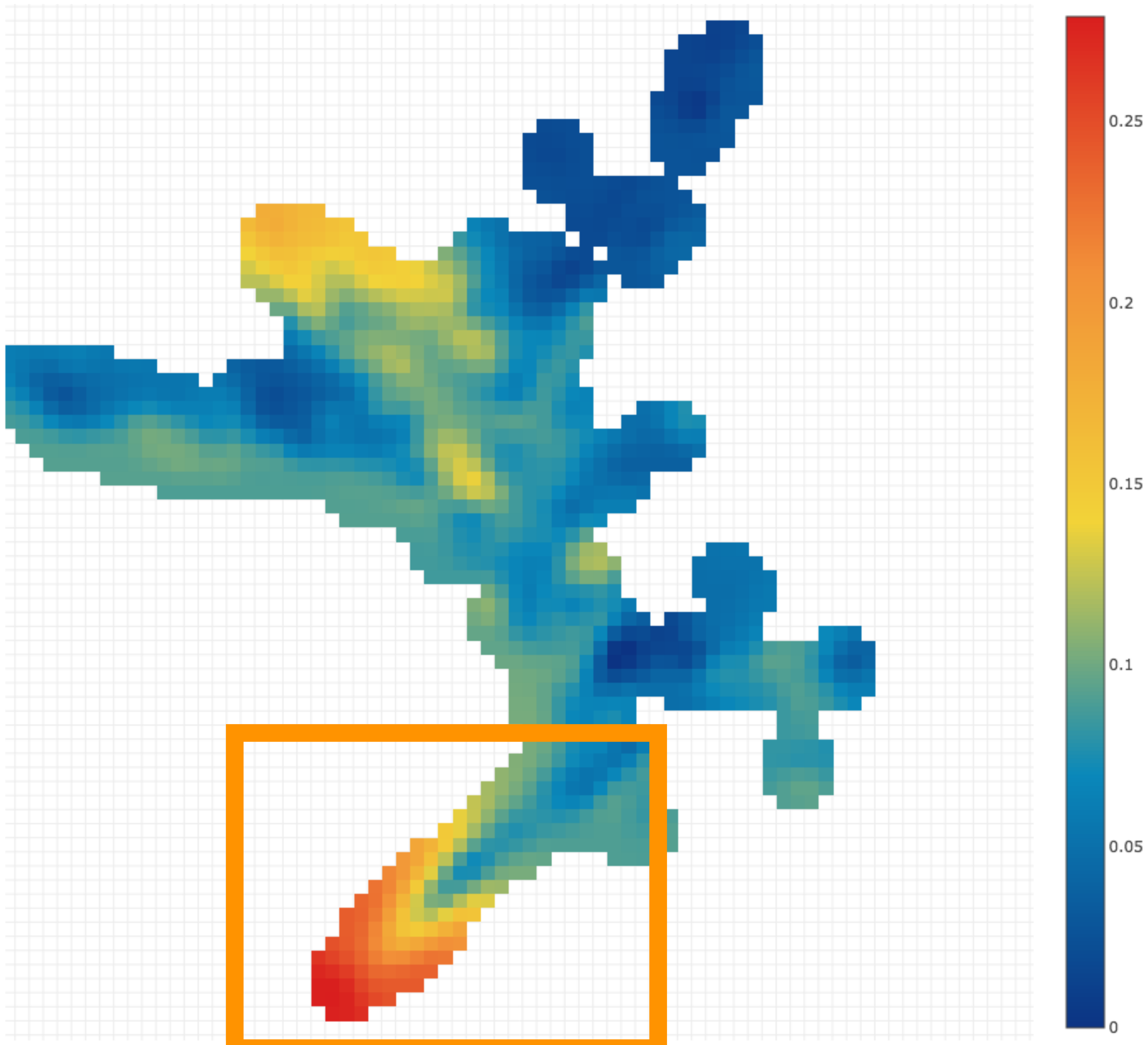
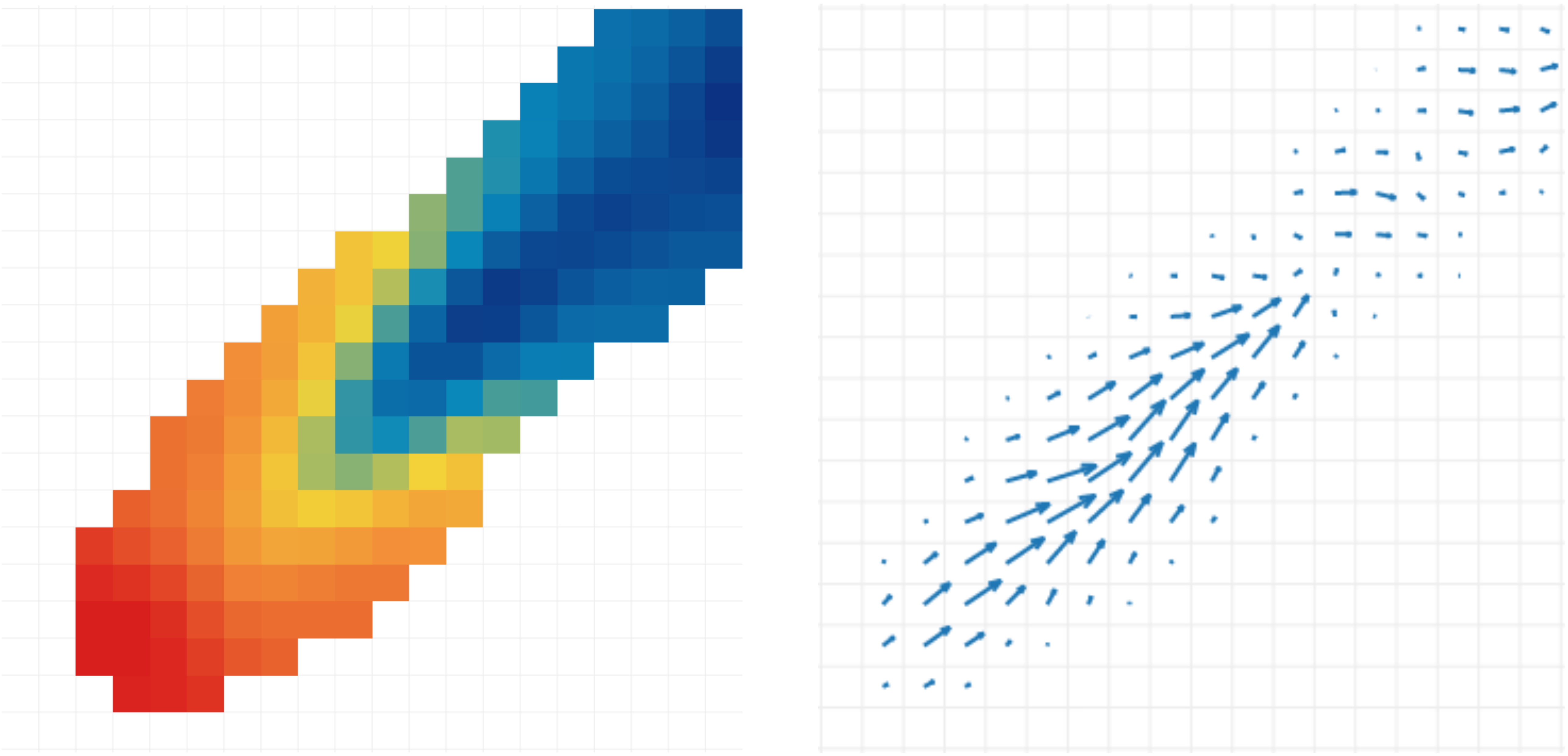
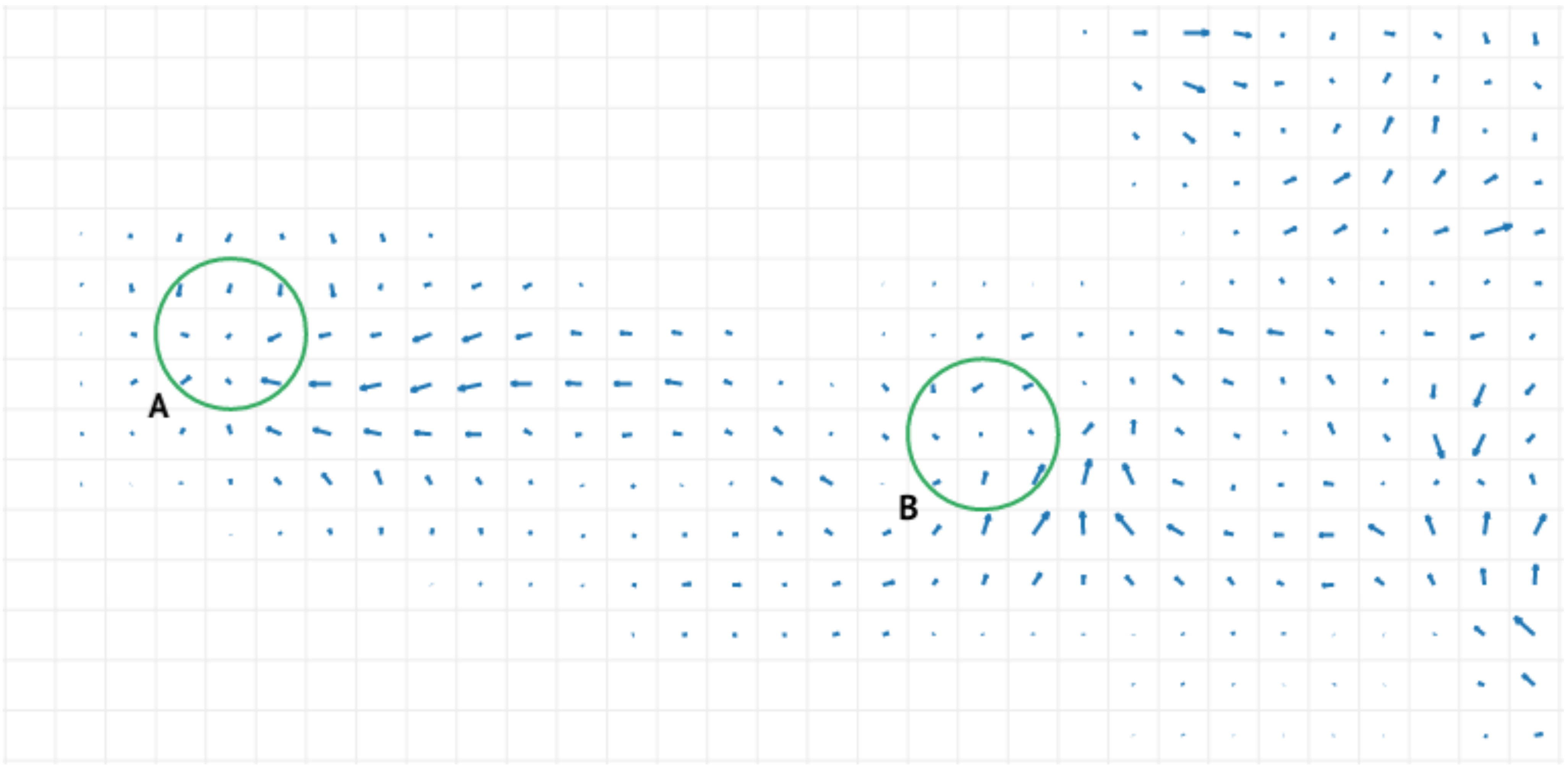


Channel



Bonus: potential wells

We can compare with the results of the numerical simulation



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