

Week 3 Exercises

Gov January Linear Algebra Review

2021-01-19

1. (Strang 4.1.13) Put bases for the subspaces V and W into the columns of the matrices V and W . Explain why the test for orthogonal subspaces can be written $V^T W = \text{zero matrix}$. This matches $v^T w = 0$ for orthogonal vectors.
2. (Strang 4.1.25) Find $A^T A$ if the columns of A are unit vectors, all mutually perpendicular.
3. (Strang 4.2.13) Suppose A is the 4 by 4 identity matrix with its last column removed. A is 4 by 3. Project $b = (1, 2, 3, 4)$ onto the column space A . What shape is the projection matrix P and what is P ?
4. (Strang 4.2.21) Multiply the matrix $P = A(A^T A)^{-1} A^T$ by itself. Cancel to prove that $P^2 = P$. Explain by $P(Pb)$ always equals Pb . The vector Pb is in the column space of A so its projection onto that column space is _____.
5. (Strang 4.2.22) Prove that $P = A(A^T A)^{-1} A^T$ is symmetric by computing P^T . Remember that the inverse of a symmetric matrix is symmetric.
6. (Strang 4.2.23) Is the error vector e orthogonal to b or p or e or \hat{x} ? Show that $\|e\|^2$ equals $e^T b$ which equals $b^T b - p^T b$.