

Week 2 Exercises

Gov January Linear Algebra Review

2021-01-11

1. (Strang 3.1.19) Describe the column spaces (lines or planes) of these particular matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}$$

2. (Strang 3.1.23) If we add an extra column b to a matrix A , then the column space gets larger unless _____. Give an example where the column space gets larger and an example where it doesn't. Why is $Ax = b$ solvable exactly when the column space *doesn't* get larger—it is the same for A and $[A \ b]$.
3. (Strang 3.2.22) If $AB = 0$ then the column space of B is contained in the _____ of A . Why?
4. (Strang 3.2.39) Fill out these matrices so that they have rank 1:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & & \\ 4 & & \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} & 9 & \\ 1 & & \\ 2 & 6 & -3 \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} a & b \\ c & \end{bmatrix}$$

5. (Strang 3.3.19) Find the rank of A and also the rank of $A^T A$ and also the rank of AA^T :

$$A = \begin{bmatrix} 1 & 1 & 5 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$$

6. (Strang 3.4.2) Find the largest possible number of independent vectors among:

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

7. (Strang 3.4.7) If w_1, w_2, w_3 are independent vectors, show that the differences $v_1 = w_2 - w_3$ and $v_2 = w_1 - w_3$ and $v_3 = w_1 - w_2$ are *dependent*. Find a combination of the v 's that gives zero.