

8. Sampling & Estimation

Spring 2021

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Gov 2002 (Harvard)

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- Last few weeks: probability, learning how to think about r.v.s
- Now: how to estimate features of underlying distributions with data.
- How do we construct estimators? What are their properties?

1/ Point Estimation

Motivating example

- Gerber, Green, and Larimer (APSR, 2008)

Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	_____
9995 JENNIFER KAY SMITH		Voted	_____
9997 RICHARD B JACKSON		Voted	_____
9999 KATHY MARIE JACKSON		Voted	_____

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load("../assets/gerber_green_larimer.RData")  
## turn turnout variable into a numeric  
social$voted <- 1 * (social$voted == "Yes")  
neigh.mean <- mean(social$voted[social$treatment == "Neighbors"])  
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- Is this a “real”? Is it big?

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- Or the **post-stratification estimator**, where we estimate the estimate the difference among two subsets of the data (male and female, for instance) and then take the weighted average of the two (\bar{Z} is the share of women):

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- Which (if either) is better? How would we know?

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 - F represents the **data generating process**, we repeat n times
- **Statistical inference** or **learning** is using data to infer F .

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- **Point estimation:** providing a single “best guess” about these parameters.

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- An **estimate** is one particular realization of the estimator
 - Why is the following statement wrong: “My estimate was the sample mean and my estimator was 0.38”?

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 - $\hat{\theta}_n = 3$ always guess 3

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 - the 0.38 sample mean in the “Neighbors” group is one draw from this distribution

Sampling distribution, in pictures



The diagram consists of two light blue squares with blue borders, positioned side-by-side. The left square contains the mathematical expression $F(x)$ in blue. Below this square, the words "population" and "distribution" are written in black. The right square contains the mathematical expression $\hat{\theta}_n$ in blue. Below this square, the word "estimator" is written in black.

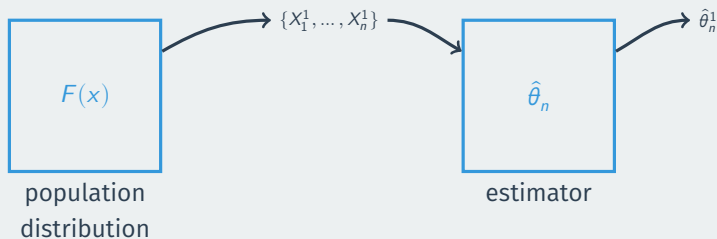
$$F(x)$$

population
distribution

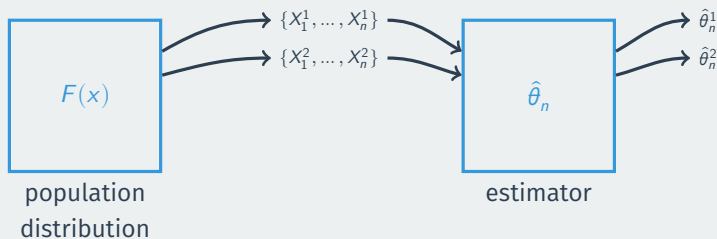
$$\hat{\theta}_n$$

estimator

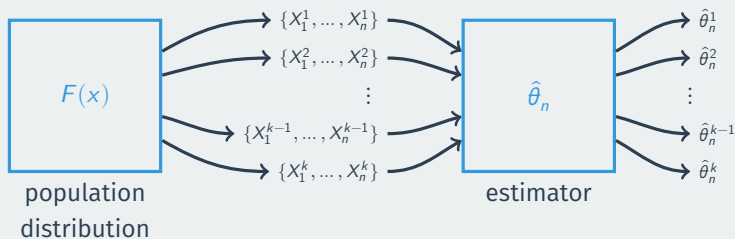
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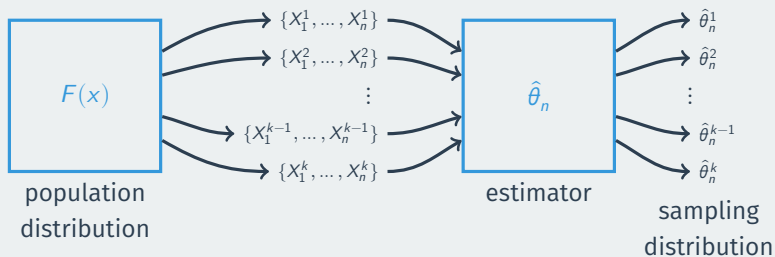
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Sampling distribution

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## now we take the mean of one sample, which is  
## one draw from the **sampling distribution**  
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## let's take another draw from the population dist  
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```
## Let's feed this sample to the sample mean estimator  
## to get another estimate, which is another draw from  
## the sampling distribution  
mean(my.samp.2)
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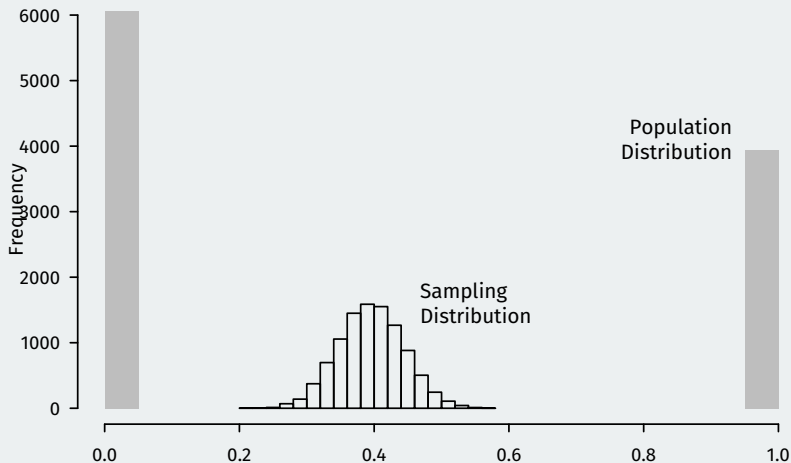
```
## [1] 0.5
```

Sampling distribution by simulation

- Let's generate 10,000 draws from the sampling distribution of the sample mean here when $n = 100$.

```
nsims <- 10000
mean.holder <- rep(NA, times = nsims)
for (i in 1:nsims) {
  my.samp <- rbinom(n = 100, size = 1, prob = 0.4)
  mean.holder[i] <- mean(my.samp) ## sample mean
  first.holder[i] <- my.samp[1] ## first obs
}
```

Sampling distribution versus population distribution



Question The sampling distribution refers to the distribution of θ , true or false.

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$$\hat{F}_n(x) = \frac{\sum_{i=1}^n \mathbb{I}(X_i \leq x)}{n}$$

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- \rightsquigarrow if $\theta = \mathbb{E}[g(X)]$ replace \mathbb{E} sample means: $\hat{\theta} = \frac{1}{n} \sum_{i=1}^n g(X_i)$

Plug-in estimators, examples

- Expectation:

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- Variance:

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- Covariance:

$$\sigma_{xy} = \text{Cov}[X_i, Y_i] = \mathbb{E}[(X_i - \mathbb{E}[X_i])(Y_i - \mathbb{E}[Y_i])] \rightsquigarrow \hat{\sigma}_{xy} = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$$

2/ Finite-Sample Properties of Estimators

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- Want to use estimators whose distribution is “close” to the true value.
- There are two ways we evaluate estimators:
 - **Finite sample:** the properties of its sampling distribution for a fixed sample size n .
 - **Large sample:** the properties of the sampling distribution as we let $n \rightarrow \infty$.

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- Sample mean of i.i.d. X_1, \dots, X_n with $\mathbb{E}[X_i] = \mu$

$$\mathbb{E}[\bar{X}_n] = \frac{1}{n} \sum_{i=1}^n \mathbb{E}[X_i] = \frac{1}{n} \sum_{i=1}^n \mu = \mu$$

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- Might accept some bias for large reductions in variance for lower overall MSE.