Gov 51: Summarizing Bivariate Relationships: Cross-tabs, Scatterplots, and Correlation

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Effect of assassination attempts

```r
leaders <- read.csv("data/leaders.csv")
head(leaders[, 1:7])
```

```
##   year     country       leadername age politybefore
## 1 1929 Afghanistan Habibullah Ghazi  39           -6
## 2 1933 Afghanistan       Nadir Shah  53           -6
## 3 1934 Afghanistan      Hashim Khan  50           -6
## 4 1924     Albania             Zogu  29            0
## 5 1931     Albania             Zogu  36           -9
## 6 1968     Algeria      Boumedienne  41           -9
##   polityafter interwarbefore
## 1       -6.00              0
## 2       -7.33              0
## 3       -8.00              0
## 4       -9.00              0
## 5       -9.00              0
## 6       -9.00              0
```

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```

```
##        After
## Before   0   1
##      0 177  19
##      1  27  27
```

- Quick summary how the two variables “go together.”

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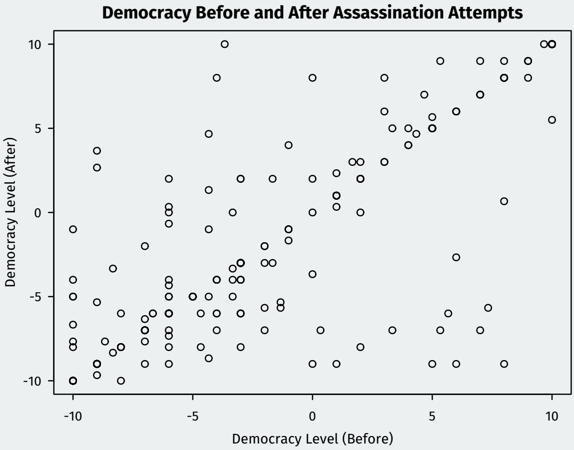

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```
plot(x = leaders$politybefore, y = leaders$polityafter,
     xlab = "Democracy Level (Before)",
     ylab = "Democracy Level (After)",
     main = "Democracy Before and After Assassination Attempts")
```

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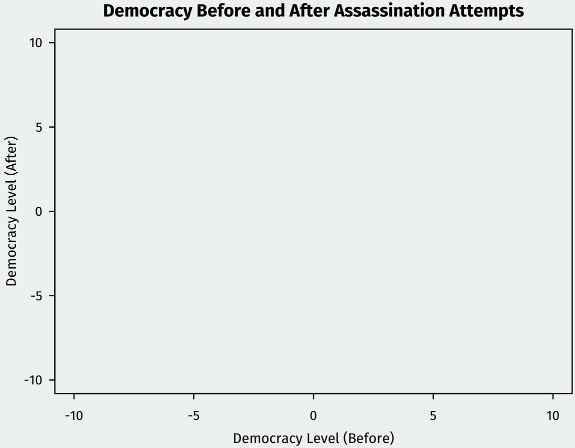

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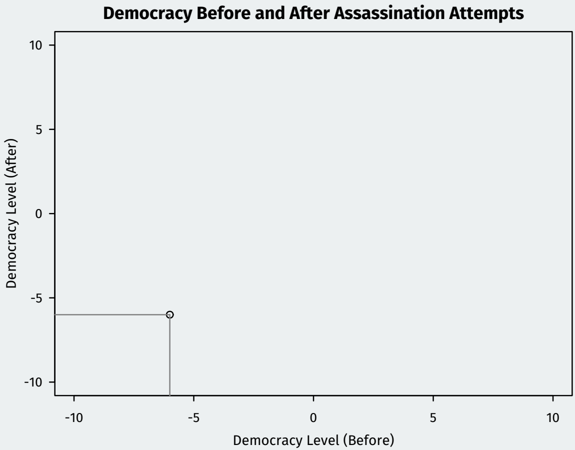
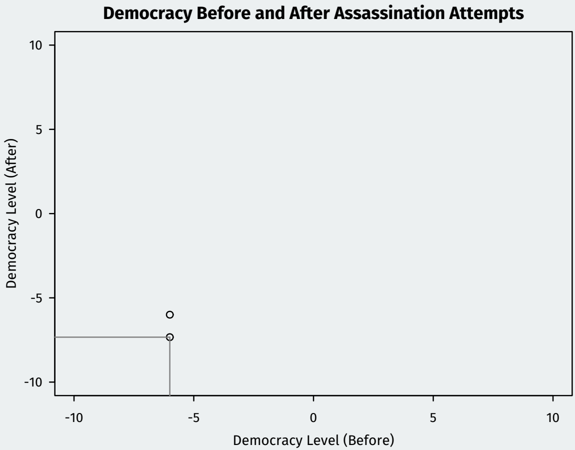


Scatterplot

```
leaders[2, c("politybefore", "polityafter")]
```

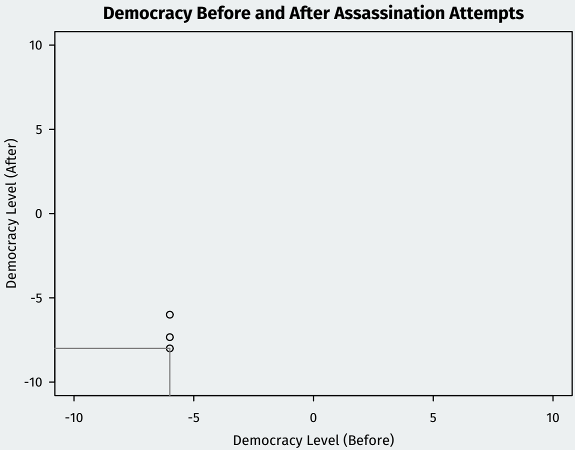
```
##   politybefore polityafter
## 2           -6       -7.33
```



Scatterplot

```
leaders[3, c("politybefore", "polityafter")]
```

```
##   politybefore polityafter
## 3           -6          -8
```



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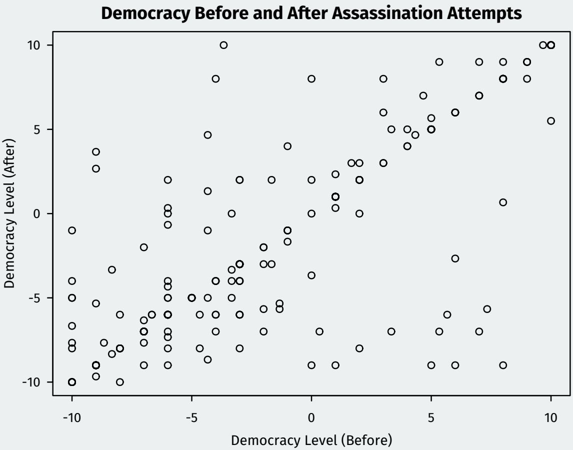

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$$\text{z-score of } x_i = \frac{x_i - \text{mean of } x}{\text{standard deviation of } x}$$

- Crucial property: z-scores don't depend on units

$$\text{z-score of } (ax_i + b) = \text{z-score of } x_i$$

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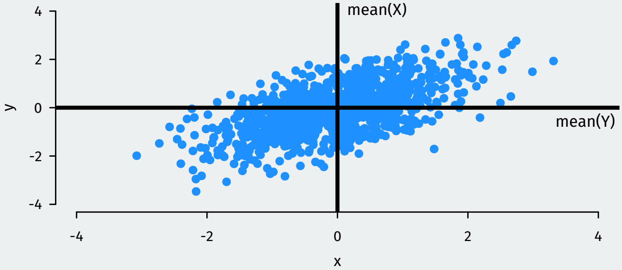
  - High magnitude of correlation: data cluster tightly around a line.
- The technical definition of the **correlation coefficient**:

$$\frac{1}{n-1}\sum_{i=1}^{n}\left[\left(\text{z-score for } x_i\right) \times \left(\text{z-score for } y_i\right)\right]$$

Correlation intuition



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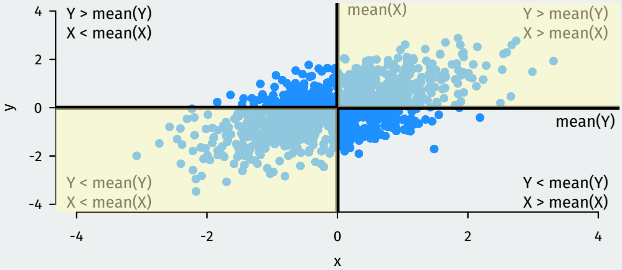

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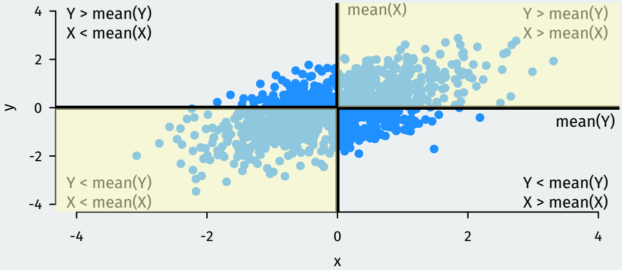

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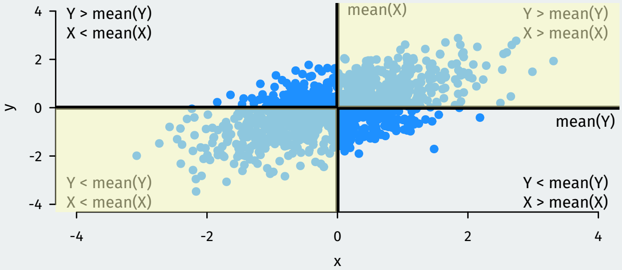

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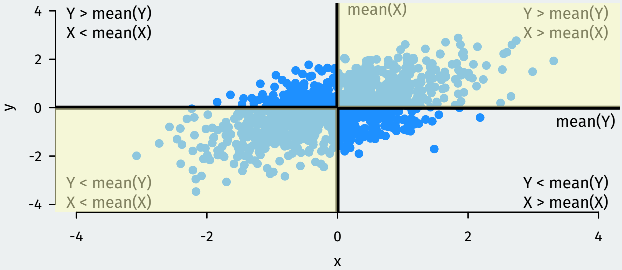

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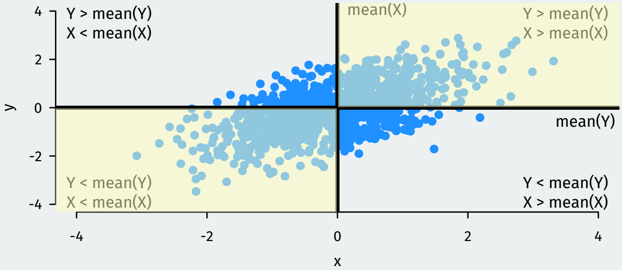

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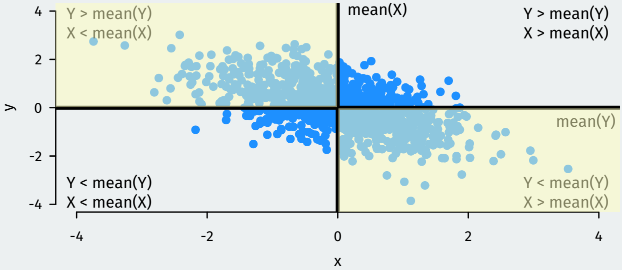

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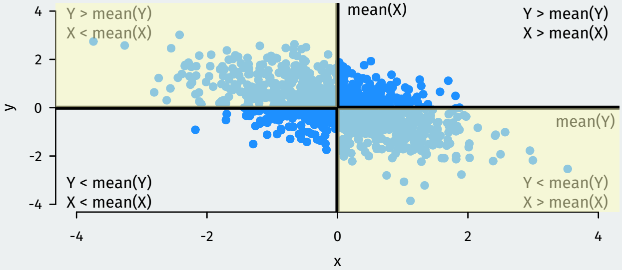

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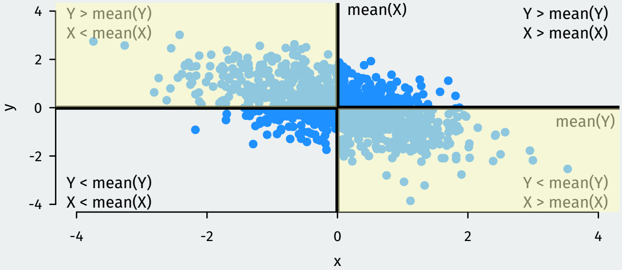

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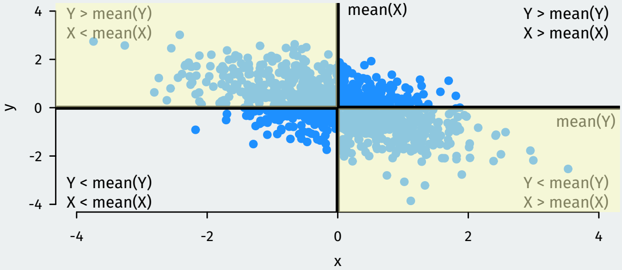

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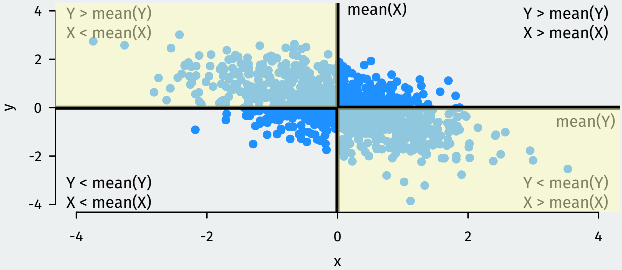

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  - Celsius vs. Fahreneheit; dollars vs. pesos; cm vs. in.

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