AERO 7970 - Multivariable Control of Uncertain Systems
Homework 1

Matt Boler

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1 Assignment

Design a control system using frequency shaping for the following system:

$$G(s) = \frac{10}{(s+1)^2} \tag{1}$$

to satisfy the performance requirements:

- Steady state error to a unit step = 0
- -40dB attenuation in the frequency range [0.01-0.1] rad/sec
- -40dB attenuation in the frequency range [100-1000] rad/sec
- Bandwidth of approximately 10 rad/sec
- Phase margin of 30 degrees

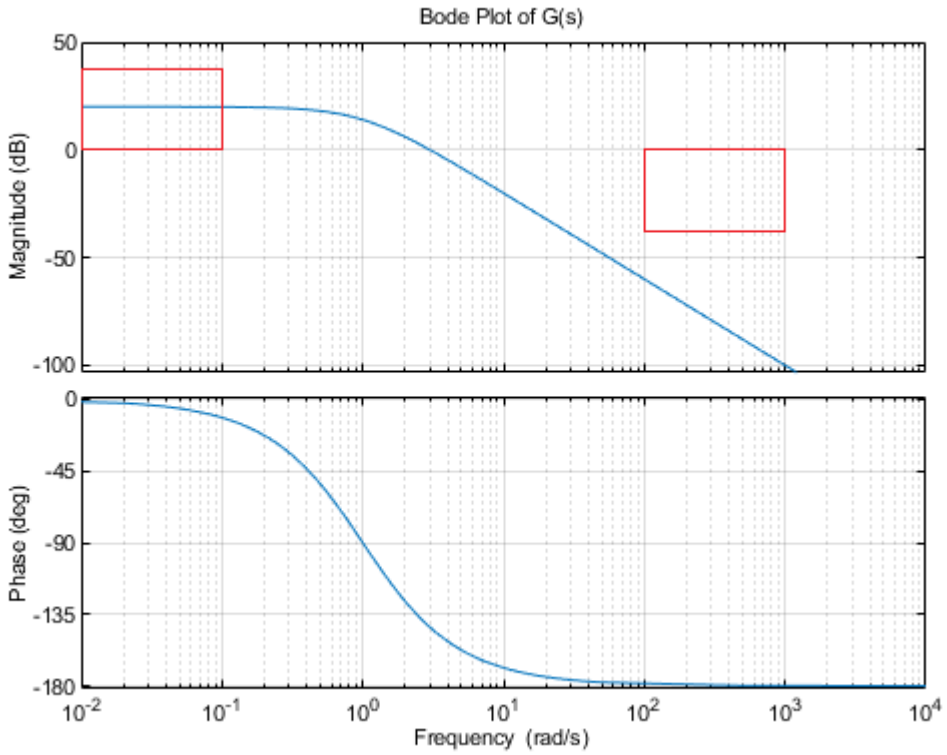
2 Solution

The loop shaping requirements and uncontrolled plant response are shown in Figure 1. From these requirements, $\Lambda(s)$ was designed in Matlab's Bode Design Editor to satisfy the loop shaping requirements. The frequency response of $\Lambda(s)$ is shown in Figure 2.

$$\Lambda(s) = 35.948\frac{(s+0.1049)(s+0.1117)}{(s+9.99)^2} \tag{2}$$

$$W(s) = \Lambda^{-1} = .027818\frac{(s+9.99)^2}{(s+0.1049)(s+0.1117)} \tag{3}$$

Figure 1: Plant Response



As zero steady-state error to a step input is required, it is know that the controller $K(s)$ will include a $\frac{1}{s}$ term. By starting with this, the controller $K(s)$ was designed using Matlab's Bode Design Editor, shown in Figure 3.

$$K(s) = 115.74\frac{(s+1)^2(s+1.825)(s+0.008682)(s^2+1.433s+8583)}{s(s+1613)(s+21.4)^2(s+0.2313)^2} \tag{4}$$

As shown in Figures 4 and 5, the loop transfer and sensitivity resulting from this controller satisfy the requirements:

$$|S(s)W(s)| \leq 1 \forall s \in C_+$$
$$|S(s)| \leq |\Lambda(s)| \forall s \in C_+$$

The resulting closed loop system is

$$\frac{K(s)G(s)}{1+K(s)G(s)} \tag{5}$$

. Figures 6 and 7 show that the system fulfills the margin and closed loop response requirements.

Figure 2: Λ Frequency Shaping

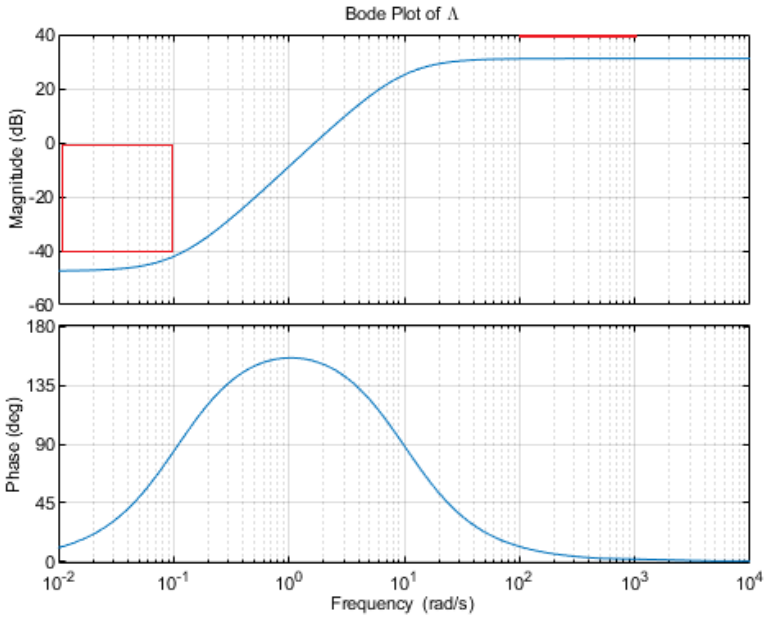


Figure 3: $K(s)$ Frequency Shaping

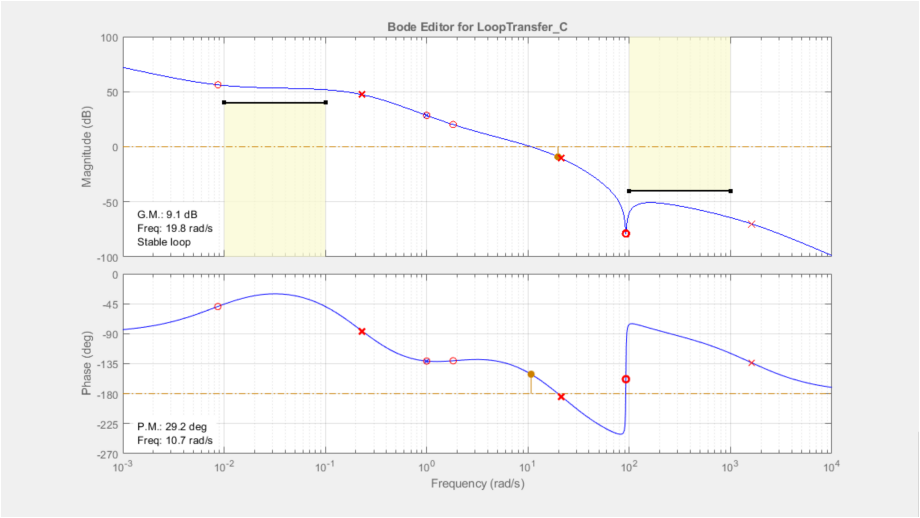


Figure 4: $S(s) * W(s)$

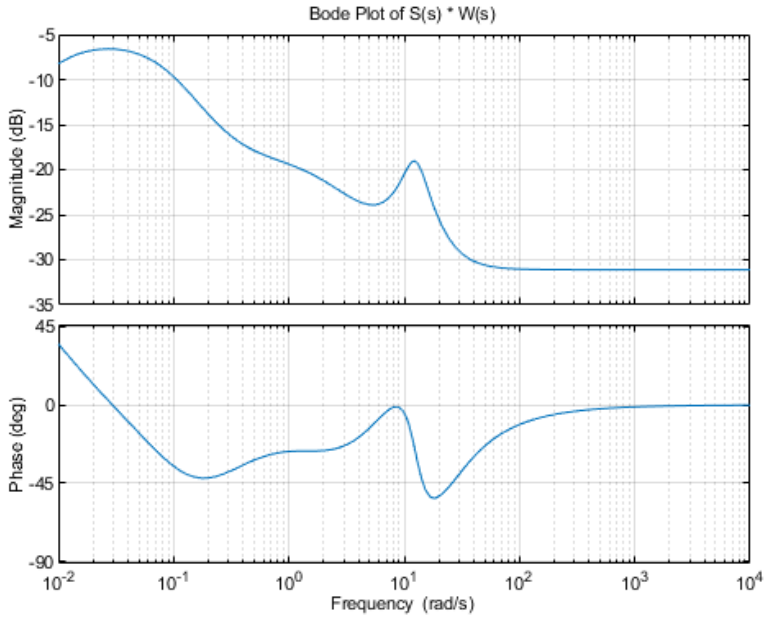


Figure 5: Sensitivity vs Λ

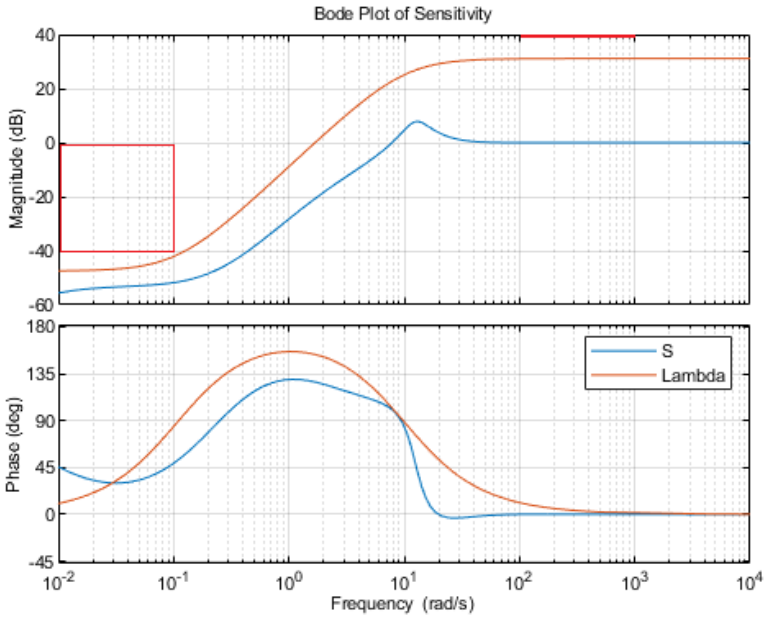


Figure 6: Stability Margins and Frequency Response

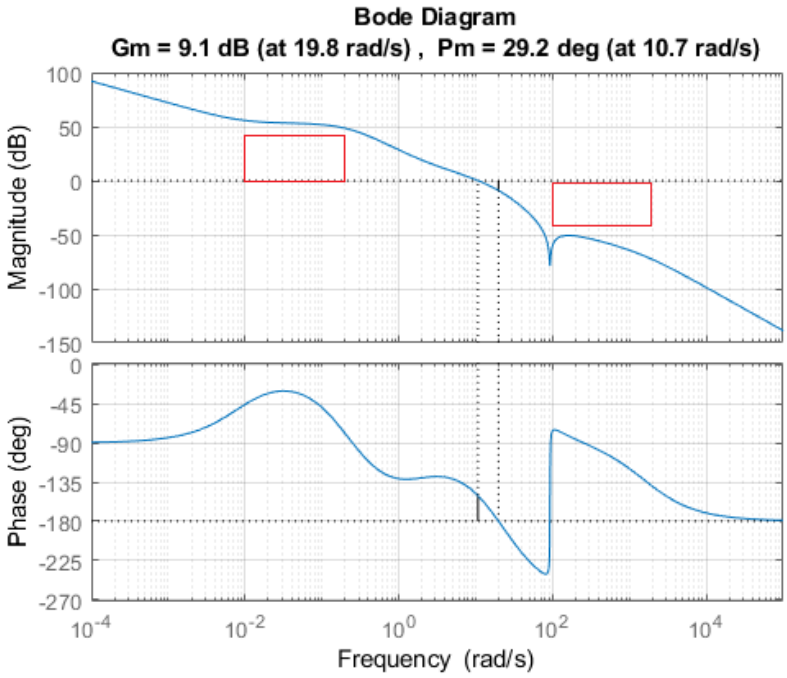


Figure 7: Closed Loop Response