AERO 7970 - Multivariable Control of Uncertain Systems
Homework 1

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1 Assignment

Design a control system using frequency shaping for the following system:

$$G(s) = \frac{10}{(s+1)^2} \tag{1}$$

to satisfy the performance requirements:

- Steady state error to a unit step = 0
- -40dB attenuation in the frequency range [0.01-0.1] rad/sec
- -40dB attenuation in the frequency range [100-1000] rad/sec
- Bandwidth of approximately 10 rad/sec
- Phase margin of 30 degrees

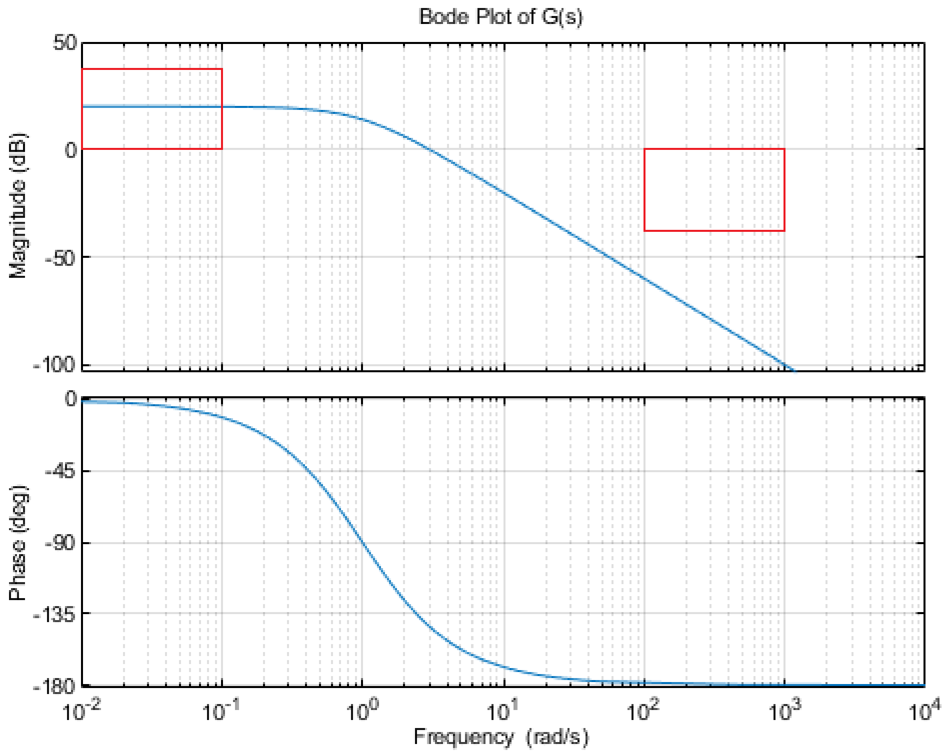
2 Solution

The loop shaping requirements and uncontrolled plant response are shown in Figure 1. From these requirements, $\Lambda(s)$ was designed in Matlab's Bode Design Editor to satisfy the loop shaping requirements. The frequency response of $\Lambda(s)$ is shown in Figure 2.

$$\Lambda(s) = 35.948\frac{(s+0.1049)(s+0.1117)}{(s+9.99)^2} \tag{2}$$

$$W(s) = \Lambda^{-1} = .027818\frac{(s+9.99)^2}{(s+0.1049)(s+0.1117)} \tag{3}$$

Figure 1: Plant Response



As zero steady-state error to a step input is required, it is know that the controller $K(s)$ will include a $\frac{1}{s}$ term. By starting with this, the controller $K(s)$ was designed using Matlab's Bode Design Editor, shown in Figure 3.

$$K(s) = 115.74\frac{(s+1)^2(s+1.825)(s+0.008682)(s^2+1.433s+8583)}{s(s+1613)(s+21.4)^2(s+0.2313)^2} \tag{4}$$

As shown in Figures 4 and 5, the loop transfer and sensitivity resulting from this controller satisfy the requirements:

$$|S(s)W(s)| \leq 1 \forall s \in C_+$$
$$|S(s)| \leq |\Lambda(s)| \forall s \in C_+$$

The resulting closed loop system is

$$\frac{K(s)G(s)}{1+K(s)G(s)} \tag{5}$$

. Figures 6 and 7 show that the system fulfills the margin and closed loop response requirements.

Figure 2: Λ Frequency Shaping

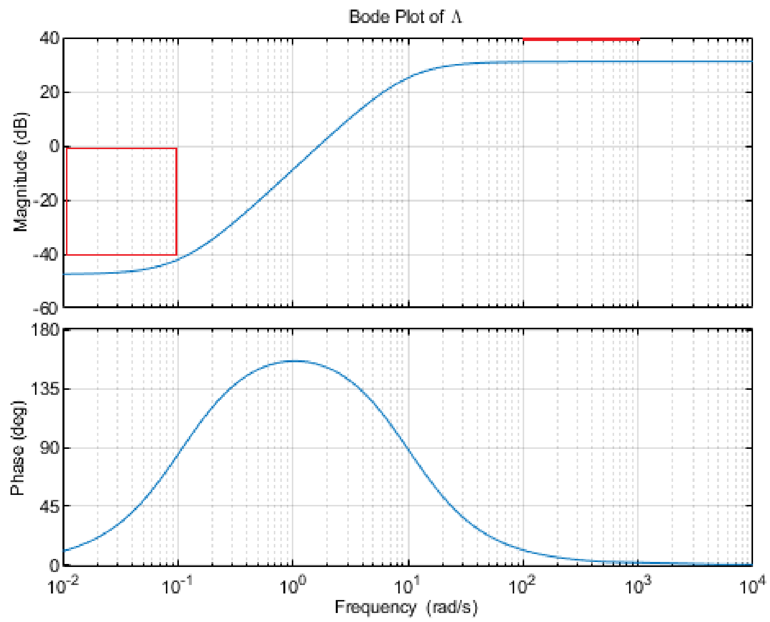


Figure 3: $K(s)$ Frequency Shaping

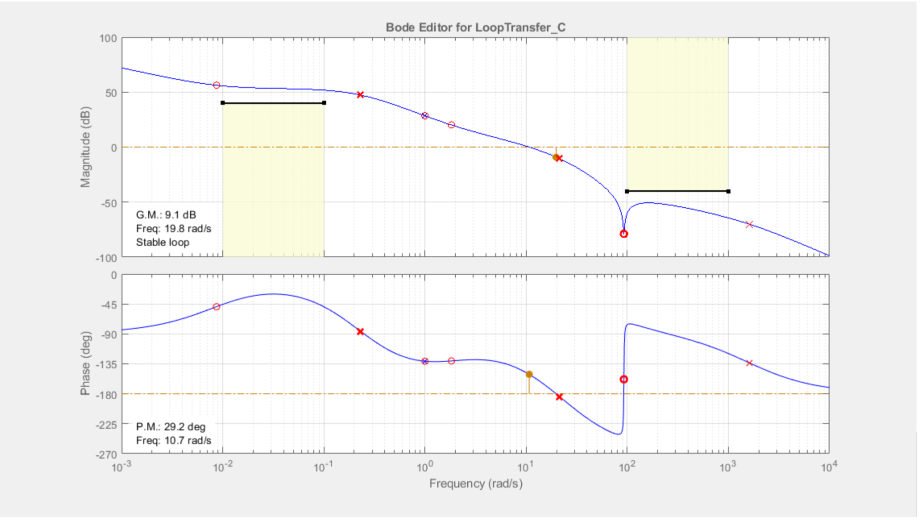


Figure 4: $S(s) * W(s)$

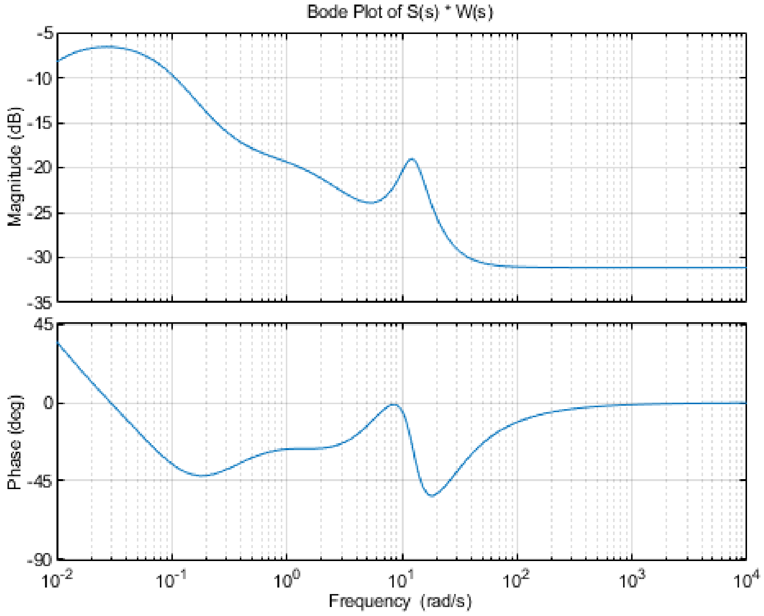


Figure 5: Sensitivity vs Λ

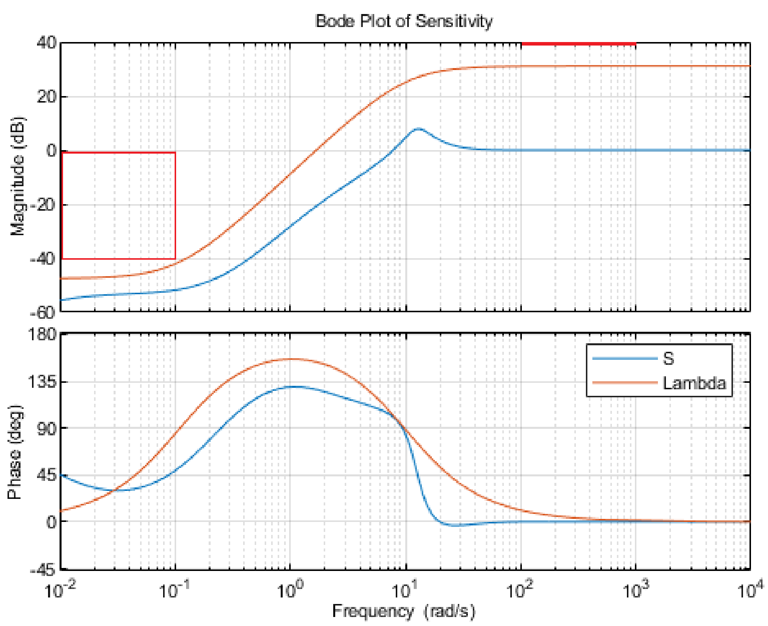


Figure 6: Stability Margins and Frequency Response

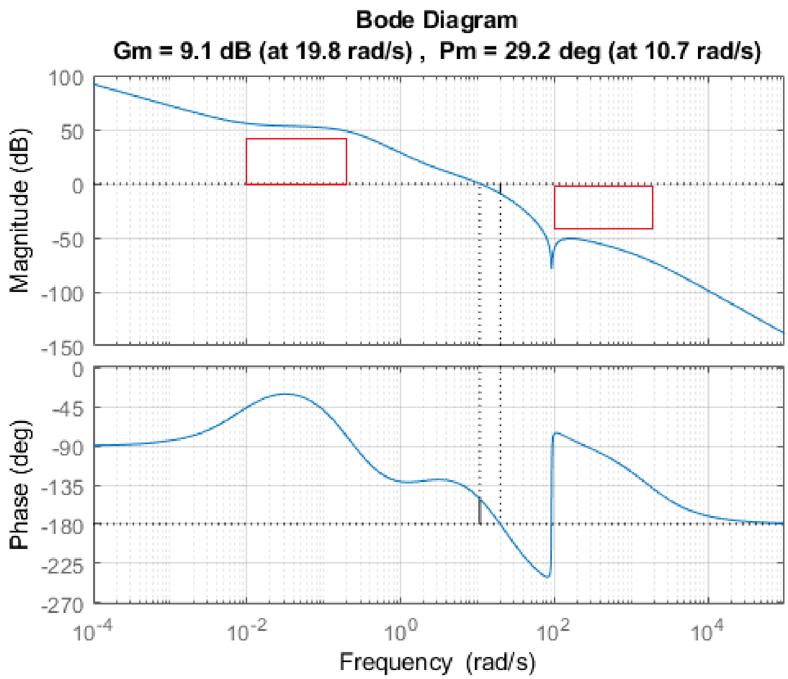
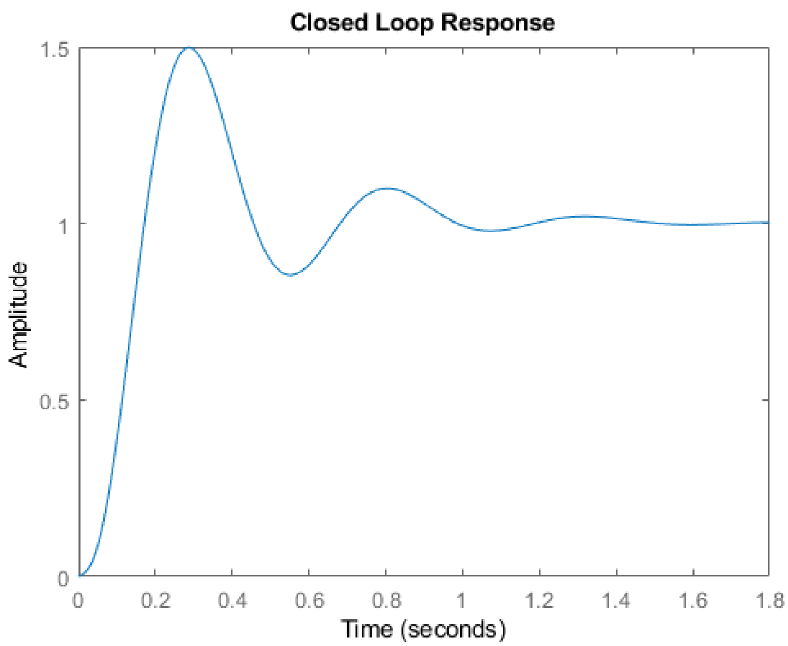


Figure 7: Closed Loop Response



```
%{
Define bode plot settings
%}
opts = bodeoptions;
opts.Grid = 'on';
opts.Xlim = [10e-3, 10e3];
%opts.Ylim = [-80, 80];

%{
Design a control system using frequency shaping for the following
 system:
%}
s = tf('s');
G = 10 / (s+1)^2;

%{
Performance requirements:
    0 steady-state error to a unit step
    -40dB attenuation in [0.01:0.1] rad/s
    -40dB attenuation in [100:1000] rad/s
    10 rad/s bandwidth
    30 deg phase margin
%}

%{
Notes on W(s):
    Performance guarantee is given by
    |W*S| < 1 for all freq

%}

% Designing W:
%{
zeta_z = .707;
wn_z = 0.1;
numerator_w = (s^2 + 2*zeta_z * wn_z*s + wn_z);

zeta_p = .707;
wn_p = 1000;
denominator_w = (s^2 + 2*zeta_p*wn_p*s + wn_p);
gain_w = 100;
LAMBDA = gain_w * (numerator_w) / denominator_w;
W = LAMBDA^-1;
%}
load('W_current.mat')
LAMBDA
W = LAMBDA^-1

load('K_current.mat');
K = K_des

L = G*K;
```

```
S = 1/(1 + L);
T = L / (1 + L);

T
stepinfo(T)

%{
margin(T)

figure(1);
f = bodeplot(LAMBDA, opts);
title('Bode Plot of \Lambda');

figure(2);
a = bodeplot(S, LAMBDA, opts);
legend('S', 'Lambda');
title('Bode Plot of Sensitivity');

figure(3);
h = bodeplot(L, opts);
title('Bode Plot of G(s) * K(s)');

figure(4)
g = bodeplot(S*W, opts);
title('Bode Plot of S(s) * W(s)');

figure(5)
b = bodeplot(G, opts);
title('Bode Plot of G(s)');

figure(6)
step(T);
title("Closed Loop Response");
%}


LAMBDA =

  35.948 (s+0.1049) (s+0.1117)
  ----------------------------
           (s+9.99)^2

Name: C
Continuous-time zero/pole/gain model.


W =

   0.027818 (s+9.99)^2
  ---------------------
  (s+0.1049) (s+0.1117)

Continuous-time zero/pole/gain model.
```

```
K =

  115.74 (s+1)^2 (s+1.825) (s+0.008682) (s^2 + 1.433s + 8583)
  -----------------------------------------------------------
              s (s+1613) (s+21.4)^2 (s+0.2313)^2

Name: C
Continuous-time zero/pole/gain model.


T =



  1157.4 s (s+1)^4 (s+1.825) (s+0.2313)^2 (s+21.4)^2 (s+1613) (s
+0.008682)


                                                    (s^2 + 1.433s +
 8583)



 -------------------------------------------------------------------------


  s (s+1)^4 (s+2.021) (s+0.2313)^2 (s+21.4)^2 (s+35.36) (s+0.008665)



                                 (s+1613) (s+1613) (s^2 + 6.611s +
 157.6)



Continuous-time zero/pole/gain model.


ans =

  struct with fields:

        RiseTime: 0.1060
    SettlingTime: 1.3636
     SettlingMin: 0.8541
     SettlingMax: 1.4986
       Overshoot: 50.1013
      Undershoot: 0
            Peak: 1.4986
        PeakTime: 0.2865
```

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