Multiple View Geometry: Exercise Sheet 2

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Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. Let A be a symmetric matrix, and λ_a, λ_b eigenvalues with eigenvectors v_a and v_b. Prove: if v_a and v_b are not orthogonal, it follows: $\lambda_a = \lambda_b$.

   Hint: What can you say about $\langle Av_a, v_b \rangle$?

2. Let $A \in \mathbb{R}^{n \times n}$ with the orthonormal basis of eigenvectors $v_1, \ldots, v_n$ and eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$. Find all vectors x, that minimize the following term:

   $$\min_{||x||=1} x^T A x$$

   How many solutions exist? How can the term be maximized?

   Hint: Use the expression $x = \sum_{i=1}^{n} \alpha_i v_i$ with coefficients $\alpha_i \in \mathbb{R}$ and compute appropriate coefficients!

3. Let $A \in \mathbb{R}^{m \times n}$. Prove that $\text{kernel}(A) = \text{kernel}(A^\top A)$.

   Hint: Consider a) $x \in \text{kernel}(A) \Rightarrow x \in \text{kernel}(A^\top A)$
   and b) $x \in \text{kernel}(A^\top A) \Rightarrow x \in \text{kernel}(A)$.

Part II: Practical Exercises

This exercise is to be solved **during the tutorial**. Let

$$A_1 = \begin{pmatrix} 2 & 6 & 7 & 8 & 5 \\ 6 & 9 & 6 & 8 & 5 \\ 7 & 6 & 1 & 7 & 5 \\ 8 & 8 & 7 & 12 & 5 \\ 5 & 5 & 5 & 5 & 5 \end{pmatrix} \quad \text{and} \quad A_2 = \begin{pmatrix} 2 & 6 & 7 & 8 & 5 \\ 6 & 9 & 6 & 8 & 5 \\ 7 & 6 & 1 & 7 & 5 \\ 8 & 8 & 7 & 12 & 5 \\ 5 & 5 & 5 & 5 & 0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

(A_1 and A_2 only differ in the bottom-right digit).

1. Do each of the following tasks for both matrices A_1, and A_2. For readability, we omit the index.
   (a) Find out whether the matrix A is invertible.
   (b) Compute the eigenvalue decomposition $A = P\Lambda P^{-1}$ with diagonal matrix Λ. Compute $A - P\Lambda P^{-1}$. What do you observe?
   (c) Compute the Singular Value Decomposition (SVD) $A = U\Sigma V^\top$ with diagonal matrix Σ. Compute $A - U\Sigma V^\top$. What do you observe?
   (d) Compute $\min_x ||Ax - b||^2$ (*Hint: last slide of the first chapter*)