Restoration

Monday, January 6, 2020 10:47 AM

- reduction or elimination of degradations in an image

Ex: - camera out of focus
- camera or object in motion
- noisy acquisition system
- geometric warp due to lens distortion

Applications:
- forensics (blurry picture of license plate of getaway car)
- military reconnaissance
- Hubble space telescope (spherical aberration)
- consumer photos

Image observation model

$$v(x,y) = g[w(x,y)] + n(x,y)$$

$g[\,]$ is a pointwise nonlinearity

$n(x,y)$ is noise (possibly image-dependent)

$$w(x,y) = \iint h(x,y;x',y')\,u(x',y')\,dx'dy'$$

(blurring operation)

$u(x,y)$ is the original image

For shift-invariant system, point-spread function (PSF) is constant over image.

PSF $\equiv$ impulse response

$$h(x,y;x',y') = h(x-x', y-y'; 0, 0) \triangleq h(x-x', y-y')$$

Then $w(x,y) = h(x,y) * u(x,y)$

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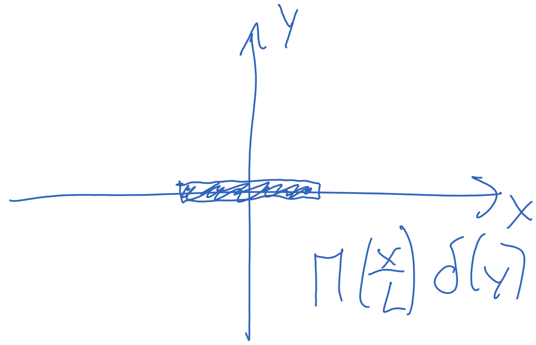
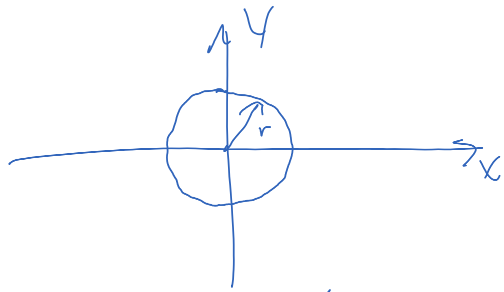
blur models

horizontal uniform motion



$\Pi\left(\frac{x}{L}\right)\delta(y) \longleftrightarrow \mathrm{sinc}(L\omega_x)$

Out of focus lens



$$\begin{cases} \frac{1}{\pi r^2}, & x^2+y^2 \le r^2 \\ 0, & \text{otherwise} \end{cases}$$

$$\xleftrightarrow{\mathcal{F}} \frac{J_1(2\pi f r)}{\pi f r}$$

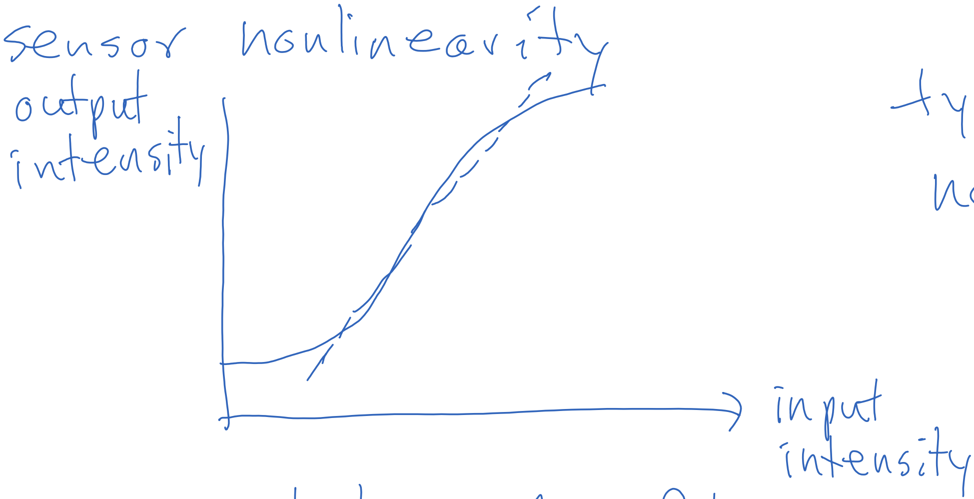
"jinc"

$f = \sqrt{f_x^2 + f_y^2}$

J_1 is Bessel function

of first kind of order 1

* sensor nonlinearity



typical sensor nonlinearity

- photographic film
- some CMOS sensors
- either include nonlinear model in restoration or assume image acquired in linear region

* noise

$$\eta(x,y) = \sqrt{w(x,y)}\,\eta_1(x,y) + \eta_2(x,y)$$

$\sqrt{w(x,y)}\,\eta_1(x,y)$ is Poisson-distributed — for example, random photon emissions

$\eta_2(x,y)$ is Gaussian distributed — electronic (thermal) noise

* can also model quantization, although this is actually a uniform distribution

From this point, we assume:

- shift-invariant blur
- linear sensor
- image-independent noise

Inverse filter

$$v(x,y) = h(x,y) * u(x,y) + \eta(x,y)$$

$$v(m,n) = h(m,n) * u(m,n) + \eta(m,n)$$

$$V(\omega_m,\omega_n) = H(\omega_m,\omega_n)\,U(\omega_m,\omega_n) + N(\omega_m,\omega_n)$$

(stack pixels into tall vectors) $\quad v = H u + n$

inverse filter: $\quad \hat{U}(\omega_m,\omega_n) = \dfrac{1}{H(\omega_m,\omega_n)} V(\omega_m,\omega_n)$

$$= U(\omega_m,\omega_n) + \frac{N(\omega_m,\omega_n)}{H(\omega_m,\omega_n)}$$

two problems:

- noise amplification
- may not be invertible

$\Rightarrow$ pseudoinverse

$$\frac{1}{H(\omega_m,\omega_n)}, \quad |H(\omega_m,\omega_n)| > \varepsilon$$

$$H^{-}(\omega_m, \omega_n) = \begin{cases} \\ 0 & , \text{otherwise} \\ \end{cases}$$

Wiener filter

Minimize $E\{[u(m,n) - \hat{u}(m,n)]^2\}$

for choice of restoration filter $g(m,n)$

Where $\hat{u}(m,n) = g(m,n) * v(m,n)$

- assumes we have a statistical description of $u(m,n)$ + noise

$$r_{uv}(m,n) = E[u(m',n')\, v(m'-m, n'-n)]$$

$$r_{uv}(m,n) = g(m,n) * r_{vv}(m,n)$$

crosscorrelation of u + v

autocorrelation of v

$\Rightarrow$ In freq. domain,

$$S_{uv}(\omega_m, \omega_n) = G(\omega_m, \omega_n)\, S_{vv}(\omega_m, \omega_n)$$

$$G(\omega_m, \omega_n) = S_{uv}(\omega_m, \omega_n)\, S_{vv}^{-1}(\omega_m, \omega_n)$$

If $v(m,n) = h(m,n) * u(m,n) + n(m,n)$

then $S_{vv}(\omega_m, \omega_n) = |H(\omega_m, \omega_n)|^2 S_{uu}(\omega_m, \omega_n)$
$+ S_{nn}(\omega_m, \omega_n)$