Sampling and Aliasing

This project will give you some hands-on experience with sampling 2-D signals and will demonstrate the effects of aliasing.

You should review your notes on sampling and aliasing before beginning this project. **NOTE: It is essential that you understand aliasing thoroughly and that you do each step accurately. Otherwise, you may do the entire project incorrectly and fail to learn anything from it.**

Exercises

1. Create a 1024x1024 image containing a horizontal white line three pixels wide on a black background. Use `imrotate` with bicubic interpolation to rotate the image 25 degrees. Extract a 512x512 section out of the original image and the rotated image so that the line in the extracted image has ends on the left and right edges rather than cutting across a corner. Compare the two 512x512 images visually, and comment on the similarities or differences, if any. Use `truesize` so that each pixel in the display corresponds to a pixel in the image. (Make sure this is the case for every display in this exercise. See the help entry.) Examine the pixel values closely, and describe how the line at an angle is represented in a discrete array.

2. The Fourier magnitude of an image can be viewed by setting the colormap using `colormap(gray(512))`. You can view the the Fourier magnitude to see the stronger features using `imagesc(abs(fftshift(fft2(IMAGE))))` and then view the log-magnitude to see some of the finer details. To view a Fourier magnitude, use a dynamic-range compression mapping such as `log(abs(fftshift(fft2(IMAGE)))+0.01)`, which can then be viewed as an image using `imagesc`. The small constant is there in case any of the Fourier components are nearly zero and can be varied to change the "floor" of the mapping. Note that the origin is in the middle of the image in this display method. What you are viewing here is one period of the periodic frequency spectrum of the discrete image. To get a sense of what is happening with adjacent periods, you may want to view a 3x3 replication of the displayed image using `repmat(IMAGESC_ARG,[3 3])` substituted for the argument to imagesc in the display command above. That is, the goal is to replicate the spectrum, not the spatial-domain image. This display method may also be helpful in understanding some of the following exercises. When displaying this way, examine the middle period of the 3x3 array, which corresponds to a single frequency spectrum of the image.

   NOTE: When you view the log-magnitude, the darker components have very little significance to the issues in this project. Some lower-magnitude features occur as the result of using an FFT, which assumes that the image is periodic. Such features may appear as a strange texture background over the image. Those features will be considered artifacts for this project and not features of interest. You may want to try increasing the small constant to reduce the visibility of the background artifacts (try 1 or 10).

   (a) View the rotated and unrotated 512x512 images.

   (b) Explain using comparisons with simple Fourier transform pairs and properties why the strongest features in the Fourier magnitudes appear as they do in the two images. (Consider the 1-D Fourier transform of a constant, an impulse, and a pulse; the Fourier transform property of separable signals; and the Fourier transform property of a rotated image.)

   (c) Explain why the differences occur.

3. Consider the 512x512 images generated in Step 1 to represent continuous resolution.
   (a) Create new 256x256 images that consist of every other sample of each of the 512x512 images. Taking every other sample will represent the effect of sampling a continuous image. View the images, and explain what, if anything, has changed. Examine the pixel values, and describe what you find.
   (b) View the Fourier magnitudes and explain in terms of aliasing. Remember to display the replicated spectral plots as described in #2. Adjust the size of the image plot so that it is half the size of the plot in #2, which will make the frequency spectra correspond to the same size for comparison purposes. What happens to the strong features of the Fourier magnitude of the rotated line?

4. Nonideal sampling can be simulated by averaging each 2x2 block in a "continuous" image to create a new image whose pixels are the block averages of the input image and whose size is one-half the original in each dimension. (This averaging approximates integrating over a continuous pixel region.)
   (a) Write a **general** m-file `function` that will implement this operation on an input image of any size and return the downsampled output image. Please include your m-file in your report.
   (b) Compare the 256×256 rotated-line image resulting from this process to the 256×256 rotated-line image resulting from the previous step, and explain the differences in terms of our in-class analysis of nonideal sampling.
   (c) Look at the periodically replicated Fourier log-magnitudes and explain the differences compared to the previous step. Remember to display the replicated spectral plots as described in #2.

5. Consider sampling a sinusoid.
   (a) Create a 512x512 image $f(m,n) = \cos(0.65\pi m + 0.2\pi n)$. Create a new image from every other sample of this image.
   (b) View the two images and their Fourier magnitudes (clearer than log-magnitudes in this case). Explain the differences. Remember to display the replicated spectral plots as described in #2. In the second image, adjust the size of the image plot so that it is half the size of the plot of the first spectrum, which will make the frequency spectra correspond to the same size for comparison purposes.
   (c) What apparent frequencies are found in the downsampled image? (You should be able to calculate specific numerical frequencies in radians/sample. Use the original equation for $f(m,n)$ to determine this rather than the Fourier plot. The plot should correspond to what you find mathematically.)

Write a **brief** memo that summarizes your findings. (You may use two pages if absolutely necessary, but one page is best. Images can be small.) You will have to be selective in your discussion to meet the space requirements, but answer all questions at least briefly. (That means you may need to leave some things out that you did in the exercise.) Include at least one image or image comparison and one code snippet. If the project specifically asks that an image or code snippet be included, then you must include that. The memo should be written so that it makes sense without reference to the project instructions.

Submit a PDF of your project memo on Canvas by class time on the due date.

NOTE: All out-of-class work is to be done **independently** and should represent your work alone. Sharing of programming tips and discussing general concepts is ok. Collaborating on experiments or code-writing is not. **Any** such collaboration on these assignments will be considered an act of dishonesty and will be treated accordingly. Memos may be checked using Turnitin (TM) for excessive similarity to one another and to documents available online.

For further help:

- Matlab Primer
- Matlab Documentation