MECH 5970/6970

PROBLEM SET #1

Due in class on Friday, February 6, 2019

Problems:

1. *Coordinate transformations*: Consider a satellite in Earth orbit. The position vector of the satellite in ECEF coordinates ($\vec{p}^{\,e}$) is equal to:

$$\vec{p}^{\,e} = \begin{bmatrix} -4.2198 & -26.428 & 9.4295 \end{bmatrix}^T \times 10^6 \text{ m.} \tag{1}$$

When the satellite is at these position coordinates, its velocity vector in ECEF coordinates is given by:

$$\vec{v}^{\,e} = \begin{bmatrix} 2941.4 & 483.43 & 2667.2 \end{bmatrix}^T \text{ m/s.} \tag{2}$$

Answer the following questions (you may want to write MATLAB scripts to solve some or all of the problems).

(a) What are the satellites's geodetic position coordinates?

(b) Over what major city is the satellite located?

(c) What is the satellites's *speed* in km/s?

(d) Consider a NED coordinate system with its origin located on the reference ellipsoid directly beneath the satellite's current position. What is the satellite's velocity vector (in m/s) expressed in this NED coordinate system?

(e) What are the azimuth and elevation angle of the satellite for an observer located at Quito, Ecuador? The geodetic coordinates for Quito are 0° 8' 28" North, 78° 29' 17" East, and 9228 feet above the reference ellipsoid.

2. (*Navigation on a curved Earth*): How long will it take to fly from Portland, Oregon (45° N and 120° W) to Tashkent, Uzbekistan (45° N and 60° E) in the shortest possible time? Assume an altitude of 10 km, a speed of 800 km/hr and a spherical Earth. Also, describe the trajectory.

3. (*An unwise guidance strategy*): An individual, located at some undisclosed location in Minnesota, wants to launch an Uninhabited Aerial Vehicle (UAV) to International Falls, Minnesota. The individual looks at a map, draws a line from the undisclosed location to International Falls, and determines (correctly) that flying a heading of true north for 1.5 hours, at normal cruising speeds of approximately 100 km/hr will do the job. When the UAV is launched, however, heading sensor errors cause it to fly a true heading of $5°$. Assuming that the UAV has an inexhaustible fuel supply and continues flying a true heading of $5°$ indefinitely, how many hours after the start of the flight will it's again be over it starting point? Why?

4. Do problem 1-5 in *Misra & Enge*. Attached

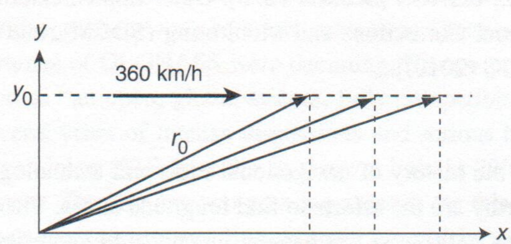
digious amount of calculation by hand. It's all fascinating stuff, and we literally ran through it in a half-dozen pages. The twentieth century brought new technologies of radio and inertial navigation with enormous consequences for war and commerce. To all who would look impatiently at their wristwatches, cell phones, PDAs, or car navigation systems with moving maps for directions to their destinations, we recommend J.E.D. Williams' wonderful monograph for balance and perspective. As for GPS, we'll get plenty of it in the 500-plus pages that follow.

Homework Problems

1-1. Given that 1 minute of latitude is approximately equal to 1 nautical mile (1852 meters), how many significant digits after the decimal must be included for a latitude represented in *degrees* to describe a fix that is accurate to 1 cm? How many significant digits are required after the decimal in the *arc-seconds* field if the latitude is represented in degrees, arc-minutes, and arc-seconds to describe a fix that is accurate to 1 cm? Note: 1 degree = 60 arc-minutes, 1 arc-minute = 60 arc-seconds (you may find arc-minutes referred to as 'minutes' and arc-seconds referred to as 'seconds').

1-2. Suppose you start from location 45° N, 120° W (lat, long) and fly at an altitude of 10 km with ground speed of 885 km/h for eight hours at a constant heading of 45° from true north. Where would you end up? Assume a spherical earth with radius of 6371 km. Two helpful notes: (i) Ground speed in aviation means the speed of an aircraft relative to the surface of the earth (to be distinguished from air speed, which means the speed of an aircraft relative to its surrounding air mass). (ii) It's safe to say that you'll end up pretty far north. Don't worry too much about precision—you get full credit if your answer is within a kilometer of the exact answer.

1-3. An aircraft is carrying a transmitter that is broadcasting a single tone at 100 MHz. The aircraft flies away from you on a straight line at constant altitude with ground speed of 360 km/h. You measure the following Doppler shifts from 100 MHz spaced 0.1 s apart: –33.1679 Hz, –33.1711 Hz, and –33.1743 Hz. Determine the range rates in m/s that correspond to these Doppler measurements. In the figure below, the aircraft altitude is y_0, and its horizontal distance from observer at the three instants of Doppler measurements is shown as x_0, x_1, and x_2, respectively. Set up two linear equations that relate x_1 and x_2 to x_0. Can you set up two non-linear equations that relate x_0 and y_0 to the measurements? For extra credit, solve the equations iteratively.



1-4. A *pseudolite* (short for pseudo-satellite) consists of a generator of a GPS-like signal and a transmitter. Pseudolites are used to augment the GPS signals. Suppose an observer is constrained to be on the line joining two pseudolites PL1 and PL2, which are separated by 1 km. The pseudolite clocks are perfectly synchronized but the observer's clock may have an unknown bias with respect to the pseudolite clocks. Estimate the observer's position and clock bias given that the pseudoranges to PL1 and PL2 are (a) 550 m and 500 m, respectively, and (b) 400 m and 1400 m, respectively.

1-5. You set out from town A and head east to town B 120 km away. Your vehicle has an odometer that is not particularly accurate (it could be off by 1–2 km, or more, after driving 50 km). You carry a decent watch, which is great at keeping time over short intervals, but it has been months since you last reset it. In other words, you can measure time intervals accurately, but do not know exactly what time it is. A short time into your journey, the car breaks down.

(a) According to your odometer, you have traveled 56 km. Estimate your position.

(b) As you push your car to the shoulder of the road, a red bus zooms by heading from A to B. You glance at your watch and notice that it is exactly 21 minutes past the hour. You know that the red buses are prompt, and they leave town A every hour on the hour traveling at exactly 3 km/min. Can you estimate your position without using the odometer information?

(c) Estimate your position and clock bias based on all the information so far. (Hint: Write two equations that relate your position and clock bias to the available information. These equations are sometimes referred to as navigation equations.)

(d) At 25 minutes past the hour by your watch, you observe a blue bus zoom past at 2.5 km/min, going from B to A. Blue buses leave town B every hour on the hour promptly and drive to town A at a constant speed. Estimate your position and clock bias based on all the information so far. (Hint: Recall that your watch can measure time intervals accurately.)

(e) How would your solution be affected if your watch were exactly five minutes faster and all the clocks in town A and town B were running five minutes fast?

(f) Now suppose that your odometer never worked and the only vehicles you see are the identical yellow cabs of carrier L1. These cabs leave town A every minute on the minute and travel at exactly 1 km/min to town B. Can you estimate your position and clock error? Would it help if there were identical green cabs of carrier L2 leaving town B every minute on the minute and traveling at 1 km/min to town A? Explain briefly.