A matrix multiplication note

19 October 2012

Consider two matrices F and G. Claim: The matrix product FG can be interpreted as:

- a linear combination of the columns of F, or
- a linear combination of the rows of G.

These may seem to be a mysterious claims, but a 2×2 example might help clear things. Consider the example matrices:

$$F = \left[\begin{array}{c|c} 1 & 2 \\ 3 & 4 \end{array}\right], \quad G = \left[\begin{array}{cc} 8 & 7 \\ \hline 6 & 5 \end{array}\right].$$

Note that matrix F has been partitioned into 2 columns, and matrix G partitioned into 2 rows. Their product FG can be represented as:

$$\begin{aligned} FG &= \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 8 & 7 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} \begin{bmatrix} 6 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 7 \\ 24 & 21 \end{bmatrix} + \begin{bmatrix} 12 & 10 \\ 24 & 20 \end{bmatrix} \\ &= \begin{bmatrix} 20 & 17 \\ 48 & 41 \end{bmatrix} \end{aligned}$$

In the first line, each sum term is the product of a column from F and a row from G. Therefore, the product FG can be interpreted as a linear combination of columns (from F), or a linear combination of rows (from G). The 2nd and 3rd lines can be verified by hand.