

# ELEC7500 Project 5

Matthew Boler

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## 1 Modeling

Our task is to analyze and design a control system for the inverted pendulum. The variables and parameters are given in Table 1.

Symbol	Description	Value
$m$	Your body mass	80kg
$l$	Length of pendulum	0.5m
$M$	Mass of cart	45kg
$H$	Cart rolling friction coefficient	$4.5 \frac{Ns}{m}$
$g$	Gravitational constant	$9.8 \frac{m}{s^2}$
$\theta$	Pendulum angle	varies
$x$	Cart position	varies
$F$	Force exerted on cart	varies

Table 1: System variables and parameters

The dynamics of the system are given by Equations 1 and 2.

$$\ddot{x} \cos(\theta) + l\ddot{\theta} - g \sin(\theta) = 0 \quad (1)$$

$$(M + m)\ddot{x} + ml\ddot{\theta} \cos(\theta) - ml\dot{\theta}^2 \sin(\theta) = F - H\dot{x} \quad (2)$$

We define the system state to be

$$\mathbf{z} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

with the system input  $u = F$ . We rewrite the dynamic equations in state-variable form

$$\dot{\mathbf{z}} = f(\mathbf{z}, u)$$

- 2    Equilibrium State**
- 3    Linearization**
- 4    State Stability**
- 5    Transfer Function**
- 6    Other Properties**
- 7    State Feedback**
- 8    State Estimation**
- 9    Computer Simulations**
  - 9.1   Linear Model with State Feedback**
  - 9.2   Linear Model with Estimated State Feedback**
  - 9.3   Nonlinear Model with Linear State Feedback**
  - 9.4   Nonlinear Model with Estimated State Feedback**
- 10   Revisiting the Separation Principle**