ELEC7500 Project 5

Matthew Boler

January 7, 2020

1 Modeling

Our task is to analyze and design a control system for the inverted pendulum. The variables and parameters are given in Table 1.

Symbol	Description	Value
m	Your body mass	80kg
l	Length of pendulum	0.5m
M	Mass of cart	45kg
H	Cart rolling friction coefficient	$4.5\frac{Ns}{m}$
g	Gravitational constant	$9.8\frac{m}{s^2}$
θ	Pendulum angle	varies
x	Cart position	varies
F	Force exerted on cart	varies

Table 1: System variables and parameters

The dynamics of the system are given by Equations 1 and 2.

$$\ddot{x}\cos(\theta) + l\ddot{\theta} - g\sin(\theta) = 0 \tag{1}$$

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos(\theta) - ml\dot{\theta}^2\sin(\theta) = F - H\dot{x} \tag{2}$$

We define the system state to be

$$\mathbf{z} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

with the system input $u = F$. We rewrite the dynamic equations in state-variable form

$$\dot{\mathbf{z}} = f(\mathbf{z}, u)$$