MATH 5670-6670 Fall 2019

Exam 4 Outline. Wednesday, December 4.

Like Exam 3: There will be 5 problems of equal worth.

Allowed

- A calculator
- A self-made one page (**not one sheet**) help sheet

  (suggested: whatever is forgettable, mainly pmf's and pdf's, means, variances, and mgf's (if available) of "named" distributions, both discrete and continuous).

  I suggest (it's not obligatory) to write down pdf's in the plain "mother" form, without the scale and (especially) shift parameters.

$$X \sim \boldsymbol{f(x)} \quad \mapsto \quad Y = sX + \mu \sim \frac{\mathbf{1}}{\boldsymbol{s}} \boldsymbol{f}\left(\frac{\boldsymbol{x-\mu}}{\boldsymbol{s}}\right).$$

  Also, it will be worth to know that some distributions arise through transformations or interpretations of basic random variables or processes, e.g.

  - plain β-Weibull $W = V^{1/\beta}$, where $V \sim \exp(\theta = 1)$;
  - Gamma $\Gamma(\alpha, \theta)$ is the α^{th} signal in a Poisson process of intensity $\lambda = \dfrac{1}{\theta}$;
  - Beta(α, β) is the distribution of the ratio $\dfrac{S_\alpha}{S_{\alpha+\beta}}$;
  - $\chi^2_r \sim \Gamma\left(\alpha = \dfrac{r}{2}, \theta = 2\right)$ is the distribution of $Z_1^2 + \cdots Z_r^2$, where Z_i are iid N(0,1);
- Normal tables or normal calculator. An online resource via cell phone is also allowed.

Coverage

- Chapter 6
- Chapter 7 (visited many times during the entire semester), selected parts
- Consult slides, scans or notes, and the homework file '`class_log_4.pdf`'

Topics

Each problem may have "atomic" parts (a)-(b)-(c)...
However, although they will belong to the same topic, they won't depend on each other.
(In the past, when, e.g., (a) was messed up, then automatically the next ones were doomed).

- **A general bivariate joint pdf**. Two problems.

  A "c-problem", marginals, conditional density, conditional mean, conditional variance (or general moment).

  The problems may involve computation of double integrals.

  Pay attention to the support. Graph it.

  Marginal and conditional densities will depend strongly on it.

  Additional from Chap. 6: 15, 17, 19-23, 41-42, 44

- **Transformations** of a pair (or more) of independent X, Y (or $X_1, \ldots, X_n$). Two problems.

  (like in the file '`BP_16_conditioning_exercises` ')

- **Bi- and multivariate normal**. Conditioning (like in the homework_4 file)

Additional Chap.7 exercises worth to attend:

Problems: 4, 10, 16, 26 (When $Y \geq 0$, then $\mathsf{E}\, Y = \int_0^\infty \mathsf{P}(Y > y)\, dy$) 7.38, 39, 40, 45, 48, 50, 51

TE: 7.2, previously 5.31, 7.1 (see 26 above), 10 (Show that $\mathsf{E}\,[X_i/S_n] = 1/n$, compare 29), 12, 19, 23, 41, 54

Named distributions required to know:

old discrete, of course	continuous uniform
exponential	Weibull
Beta	Gamma
normal	χ^2
	Cauchy