MATH 5670-6670 Fall 2019

Since we will not follow the textbook linearly we will record our progress class by class.
This is the third of four logs (corresponding to four exams).

10/11-14 -16

Chapter 5: all problems and TEs are for grabs

uniform: 5.10-5.14 and then 37-41

now: 5.31 but for general c.d.f. (which is easier!) using TE 5.2 (then a bit of Calculus 1)

TE 5.5. Also, generalize it (easier!): show that for $X \geq 0$ and differentiable function $g(x)$ such that $g(0) = 0$

$$\mathsf{E}\, g(X) = \int_0^\infty g'(t)\, \mathsf{P}(X > t)\, dt.$$

Use the same simple method shown in the class (10/16).

Change of schedule

Exam 3 will be on Wednesday, Oct 30 and Quiz 3 on Friday, Oct 25

Change of Layout

Instead of $8 \times 1 + 3 \times 2 + 2 \times 3$ will have the equal topic-wise split

(general + uniform + exponential + transformations + Gamma + normal + (or so))

10/18

Chapter 5: transforms today

39-42

TE: 15, 19, 24, 25, 29

10/21

Chapter 5: Gamma

BP_10_Poisson_process_Gamma.pdf

The **Gamma function**

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}\, e^{-x}\, dx,\; \alpha > 0.$$

It extends the discrete factorial due to the recursion:

$$\Gamma(\alpha+1) = \alpha\,\Gamma(\alpha), \qquad \Gamma(n+1) = n!.$$

- The Gamma scale-free pdf of X yields the pdf of the scaled $Y = \theta X$:

$$f(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha)}\, e^{-x} \quad \mapsto \quad f_Y(y) = \frac{y^{\alpha-1}}{\theta^\alpha\,\Gamma(\alpha)}\, e^{-y/\theta}.$$

- For an integer shape parameter $\alpha = n$, it's the distribution of the n^{th} signal in a Poisson process and is also called **Erlang's**. The relation between the intensity λ and scale θ is

$$\lambda = \frac{1}{\theta} \quad \text{or} \quad \theta = \frac{1}{\lambda}.$$

  One may consider a fractional "*α^{th} signal*" as well (it has a physical meaning).

- "**Smart integration**":

  Find the p^{th} moment of an exponential random variable X with mean θ.

  Originally, it would require a tedious computation:

$$\mathsf{E}\, X^p = \int_0^\infty x^p\, \frac{1}{\theta}\, e^{-x/\theta}\, dx.$$

  Now, write $X = \theta V$, where $V \sim \exp(1)$. Consequently,

$$\mathsf{E}\, X^p = \mathsf{E}\,(\theta V)^p = \theta^p \int_0^\infty v^p\, e^{-v}\, dx = \theta^p\, \Gamma(p+1), \quad \text{done.}$$

  The moments of a Gamma RV are equally easy to compute.

Chap. 5: Pbs. 8 (done in the class + modifications - higher powers), TE 1, 21

Clearly, Gamma distribution is hardly a favorite distribution of Prof. Ross.
However, it's common in all kinds of applications.
In addition, as it borrows heavily from Calculus, it returns a lot to Calculus.

10/23

Chapter 5 and Section 8.3: normal and CLT

BP_11_normal.pdf

Chap. 5: 15-30, TE 9-12

Sect. 8.3: 6-19, TE 7-8