MATH 5670-6670 Fall 2019

Quiz 3, take-home, due Wednesday, Oct. 30

Each part is worth 2 points (so there is 1-point bonus).

1. Let X have $\Gamma(\alpha = 3, \theta = 3)$ distribution. .

   Then find $\mathsf{E}\, X^3$ by two methods:

   (a) Using the mgf $M(t) = \mathsf{E}\, e^{tX}$.

   First, derive the formula by yourself and show computations.

   (b) Using the "smart integration" as defined in the class.

2. An emitter shoots an electron into a straight line screen at a random angle. Mark by 0 the point on the screen that is closest to the emitter. Denote by X the random trace of the electron on the screen. Assuming that the distance from the emitter to the screen is s, find the pdf of X.

   Does the mean $\mathsf{E}\, X$ exist? Does $\mathsf{E}\, \sqrt{|X|}$ exist?

3. In the n-space, the distance from a point $\mathbf{x} = (x_1, \ldots, x_n)$ to the origin is given by the formula

$$||\mathbf{x}|| = \sqrt{\sum_{i=1}^{n} x_i^2}.$$

   In the 4-space consider a random point $\mathbf{X} = (X_1, X_2, X_3, X_4)$ whose coordinates are i.i.d. N(0,4) RVs.

   Compute $\mathsf{P}(||\mathbf{X}|| \le 2)$.

   (Hint: *The sum of squares of iid standard N(0,1) RVs has some Gamma distribution which is related to a Poisson process.*)