1. Basic Considerations

Outline

- Objectives
- Vector calculations
- Newton’s laws
- Systems of units

1.1 Vectors and complex numbers

- Vectors provide both the graphical and analytical meanings of motion and force.
- Vectors are usually identified by ***boldface*** type to distinguish from scalars.
- A right-hand coordinate system is used for vectors.
- Sign convention for angles: counterclockwise is positive

1.1 Vectors and complex numbers

- Properties

$\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$ (commutative law for addition)

$\boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C}) = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C}$ (associative law for addition)

$m\boldsymbol{A} = \boldsymbol{A}m$ (commutative law for multiplication by a scalar)

$m\,(\boldsymbol{A} + \boldsymbol{B}) = m\boldsymbol{A} + m\boldsymbol{B}$ (distributive law for multiplication by a scalar)

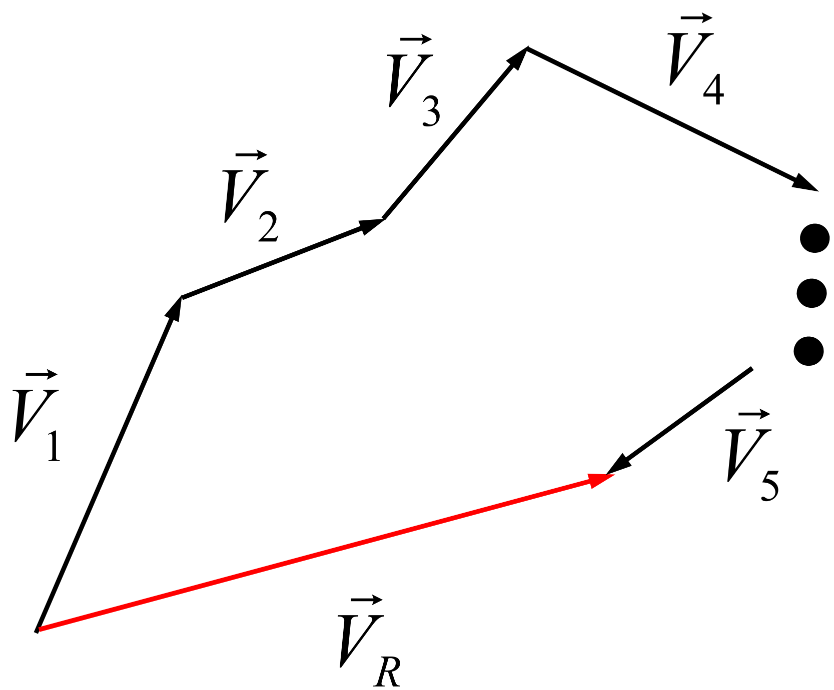
Note: finite rotations about nonparallel axes do not follow the commutative law of addition (we'll see more details in Ch 3.)

$$\theta + \varphi \neq \varphi + \theta$$

1.1 Vectors and complex numbers

- Vector addition

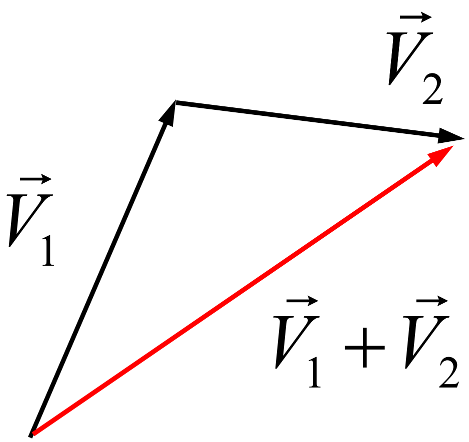
$$\vec{V}_R = \sum_{i=1}^{n} \vec{V}_i = \hat{i}\sum_{i=1}^{n} V_{ix} + \hat{j}\sum_{i=1}^{n} V_{iy} + \hat{k}\sum_{i=1}^{n} V_{iz}$$

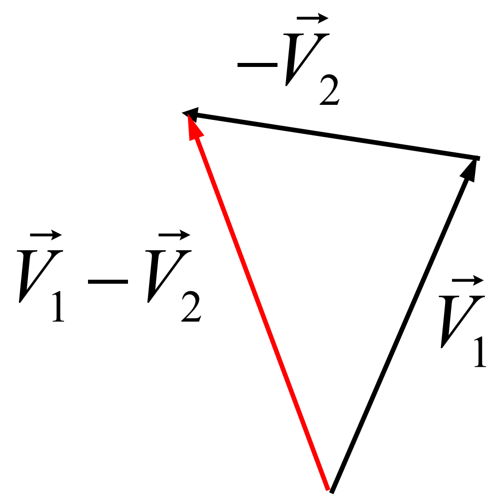


1.1 Vectors and complex numbers

- Vector subtraction (relative motion)

$$\vec{V}_R = \vec{V}_1 - \vec{V}_2$$
$$= \hat{i}\left(V_{1x} - V_{2x}\right) + \hat{j}\left(V_{1y} - V_{2y}\right) + \hat{k}\left(V_{1z} - V_{2z}\right)$$





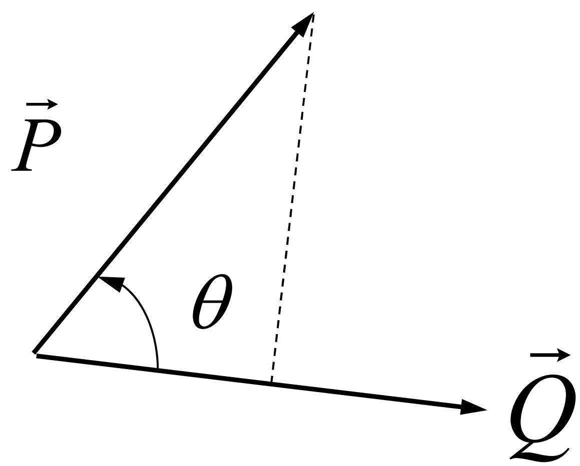
1.1 Vectors and complex numbers

- Dot product (scalar product)

$$\vec{P} \bullet \vec{Q} = \left|\vec{P}\right|\left|\vec{Q}\right| \cos\theta$$

$$\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1$$

$$\hat{i} \bullet \hat{j} = \hat{i} \bullet \hat{k} = \hat{j} \bullet \hat{k} = 0$$



$$\cos\theta = \frac{\vec{P} \bullet \vec{Q}}{\left|\vec{P}\right|\left|\vec{Q}\right|}$$

1.1 Vectors and complex numbers

- Dot product (scalar product)
  - Dot product describes the similarity of two vectors
  - Dot product of two orthogonal vectors (perpendicular to each other) is zero
  - commutative law for dot product

$$\vec{P} \bullet \vec{Q} = \vec{Q} \bullet \vec{P}$$

  - Distributive law for dot product

$$\vec{P} \bullet (\vec{Q} + \vec{R}) = \vec{P} \bullet \vec{Q} + \vec{P} \bullet \vec{R}$$

1.1 Vectors and complex numbers

- Dot product (scalar product)

$$\vec{P} \bullet \vec{Q} = \left(P_x\hat{i} + P_y\hat{j} + P_z\hat{k}\right) \bullet \left(Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k}\right)$$

$$= P_xQ_x + P_yQ_y + P_zQ_z = \begin{bmatrix} P_x & P_y & P_z \end{bmatrix} \begin{bmatrix} Q_x \\ Q_y \\ Q_z \end{bmatrix}$$

$$\vec{P} \bullet \vec{P} = P_x^{\ 2} + P_y^{\ 2} + P_z^{\ 2} = \begin{bmatrix} P_x & P_y & P_z \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

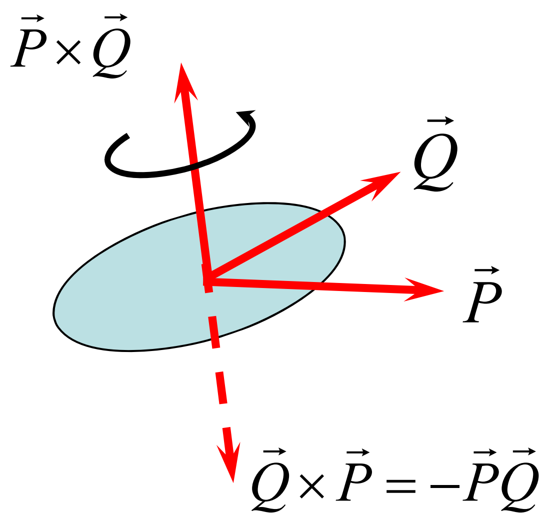
1.1 Vectors and complex numbers

- Cross product (vector product)

The cross product of two vectors is a vector with magnitude

$$\left|\vec{P}\times\vec{Q}\right| = \left|\vec{P}\right|\left|\vec{Q}\right|\sin\theta$$

and a direction specified by the right-hand rule.

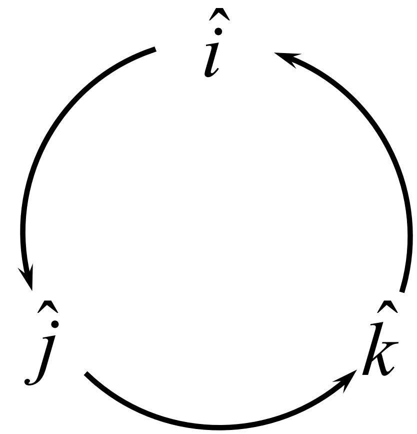


1.1 Vectors and complex numbers

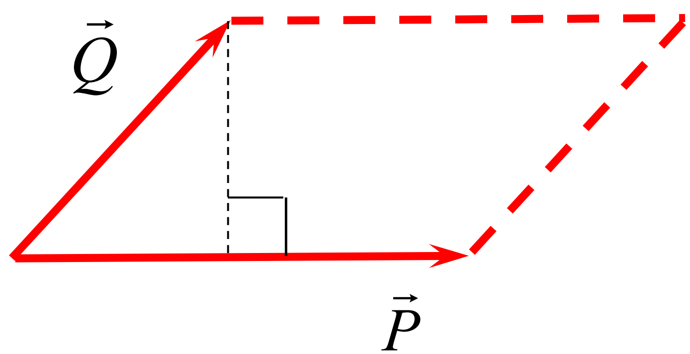
- Cross product (vector product)

$$\hat{i}\times\hat{j}=\hat{k},\ \ \hat{j}\times\hat{k}=\hat{i},\ \ \hat{k}\times\hat{i}=\hat{j}$$

$$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$$



$$\left|\vec{P}\right|\left|\vec{Q}\right|\sin\theta$$



1.1 Vectors and complex numbers

- Cross product (vector product)

$$\vec{P}\times\vec{Q}=\left(P_x\hat{i}+P_y\hat{j}+P_z\hat{k}\right)\times\left(Q_x\hat{i}+Q_y\hat{j}+Q_z\hat{k}\right)$$

$$=\left(P_xQ_z-P_zQ_y\right)\hat{i}+\left(P_zQ_x-P_xQ_z\right)\hat{j}+\left(P_xQ_y-P_yQ_x\right)\hat{k}$$

$$=\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ P_x & P_y & P_z\\ Q_x & Q_y & Q_z\end{bmatrix}$$

1.1 Vectors and complex numbers

- Cross product (vector product)
  - Distributive law for cross product

$$\vec{P}\times(\vec{Q}+\vec{R})=\vec{P}\times\vec{Q}+\vec{P}\times\vec{R}$$

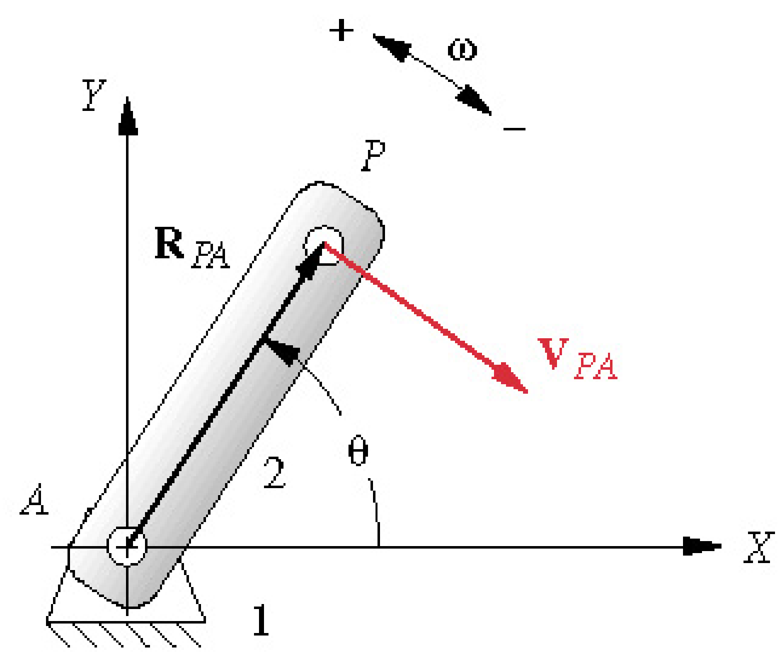
$$\vec{P}\times\left(\vec{Q}\times\vec{R}\right)=\left(\vec{P}\bullet\vec{R}\right)\times\vec{Q}-\left(\vec{P}\bullet\vec{Q}\right)\times\vec{R}$$

$$\left(\vec{P}\times\vec{Q}\right)\times\vec{R}=\left(\vec{P}\bullet\vec{R}\right)\times\vec{Q}-\left(\vec{Q}\bullet\vec{R}\right)\times\vec{P}$$

$$\vec{R}\bullet\left(\vec{P}\times\vec{Q}\right)=\vec{P}\bullet\left(\vec{Q}\times\vec{R}\right)=\vec{Q}\bullet\left(\vec{R}\times\vec{P}\right)$$

1.1 Vectors and complex numbers

- Cross product – example 1

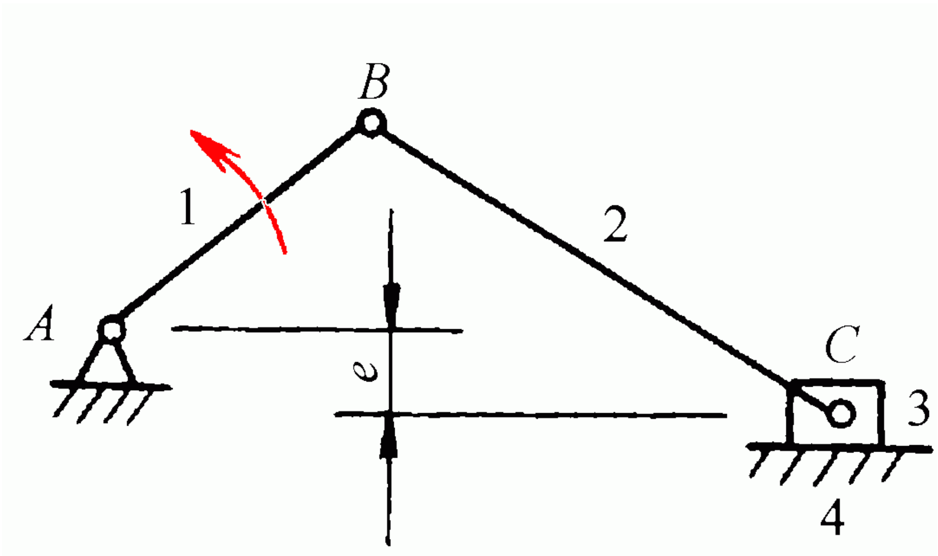


$$\vec{v} = \vec{\omega} \times \vec{r}$$

1.1 Vectors and complex numbers

- Cross product – example 2

$$\vec{M} = \vec{r} \times \vec{F}$$



1.1 Vectors and complex numbers

- Differentiations of a vector

$$\frac{d}{dt}\left(\vec{A}+\vec{B}\right)=\frac{d\vec{A}}{dt}+\frac{d\vec{B}}{dt}$$

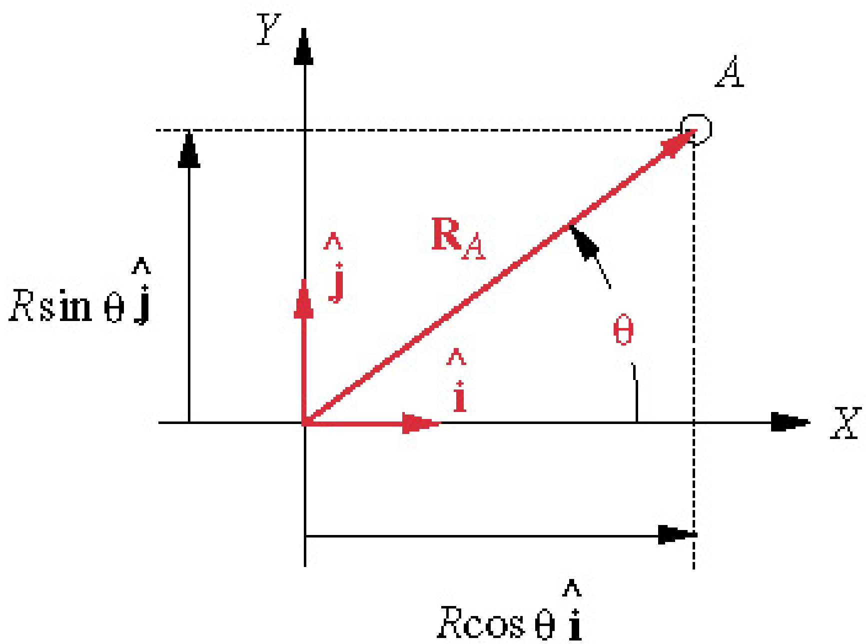
$$\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)=\vec{A}\times\frac{d\vec{B}}{dt}+\frac{d\vec{A}}{dt}\times\vec{B}$$

$$\frac{d}{dt}\left(\vec{A}\bullet\vec{B}\right)=\vec{A}\bullet\frac{d\vec{B}}{dt}+\frac{d\vec{A}}{dt}\bullet\vec{B}$$

$$\frac{d}{dt}\left(m\vec{A}\right)=m\frac{d\vec{A}}{dt}+\frac{dm}{dt}\vec{A}$$

1.1 Vectors and complex numbers

- Complex numbers as vectors



Polar form:

$$|\mathbf{R}_A| \; @ \angle\theta$$

Cartesian form:

$$R\cos\theta\,\hat{\mathbf{i}}\;,\; R\sin\theta\,\hat{\mathbf{j}}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\frac{de^{\pm j\theta}}{d\theta} = \pm j e^{\pm j\theta}$$

1.2 Newton's Laws

1 *A body at rest tends to remain at rest and a body in motion at constant velocity will tend to maintain that velocity unless acted upon by an external force.*

2 *The time rate of change of momentum of a body is equal to the magnitude of the applied force and acts in the direction of the force.*

3 *For every action force there is an equal and opposite reaction force.*

$$A = \pi r^2$$

$$\vec{F} = \frac{d\left(m\vec{v}\right)}{dt} = m\vec{a}$$

$$\vec{T} = \frac{d\left(\vec{H}\right)}{dt} = \dot{\vec{H}}$$

$$\vec{H} = I\vec{\omega}$$

$$\frac{d\vec{H}}{dt} = I\vec{\alpha}$$

Note: The Newton's Second Law is only valid in non-accelerating (or inertial) reference frame.

1.2 Newton's Laws

- Mass - a fundamental concept in physics, roughly corresponding to the intuitive idea of how much matter there is in an object. Mass is a central concept of classical mechanics and related subjects.
- mass and weight are different properties. Mass is an inertial property; that is, the tendency of an object to remain at constant velocity unless acted upon by an external force. Weight is the force created when a mass is acted upon by a gravitational field.

1.2 Newton's Laws

- "First moment of mass"

$$M_x = \int x dm = \int \rho x dA$$

$$M_y = \int y dm = \int \rho y dA$$

$$M_z = \int z dm = \int \rho z dA$$

- The center of gravity (or center of mass)

$$x_c = \frac{M_x}{m} = \frac{\int x \rho dA}{\int \rho dA}$$

$$y_c = \frac{M_y}{m} = \frac{\int y \rho dA}{\int \rho dA}$$

$$x_z = \frac{M_z}{m} = \frac{\int z \rho dA}{\int \rho dA}$$

1.2 Systems of Units

- The International System of Units (SI) is the modern form of the metric system and is generally a system devised around the convenience of the number ten. It is the world's most widely used system of units, both in everyday commerce and in science.
- Download the following document from the International Bureau of Weights and Measures, and read pp. 103-135.

http://www1.bipm.org/utils/common/pdf/si_brochure_8.pdf

1.2 Systems of Units

- SI base units

Name	Symbol	Quantity
metre	**m**	length
kilogram	**kg**	mass
second	**s**	time
ampere	**A**	electric current
kelvin	**K**	thermodynamic temperature
mole	**mol**	amount of substance
candela	**cd**	luminous intensity

- Derived units

1.2 Systems of Units

- US Customary Units
  - US foot – pound - second (fps)
  - US inch – pound - second (ips)

1.2 Systems of Units

http://en.wikipedia.org/wiki/Mass_versus_weight

- Values for gravitational acceleration *g*
  - SI 9.81 m/s^2
  - Ips 386 in/s^2
  - Fps 32.2 ft/s^2
- Conversions
  - 1 lbf = 4.45 N
  - 1 lbm = 1 lbf
  - 1 kg = 2.2 lb
  - 1 blob = 386 lb
- Weight densities
  - Steel 0.28 lbf/in^3
  - Aluminum 0.10 lbf/in^3

1.2 Systems of Units

1 g = 9.81 m/s^2 = 32.2 ft/s^2

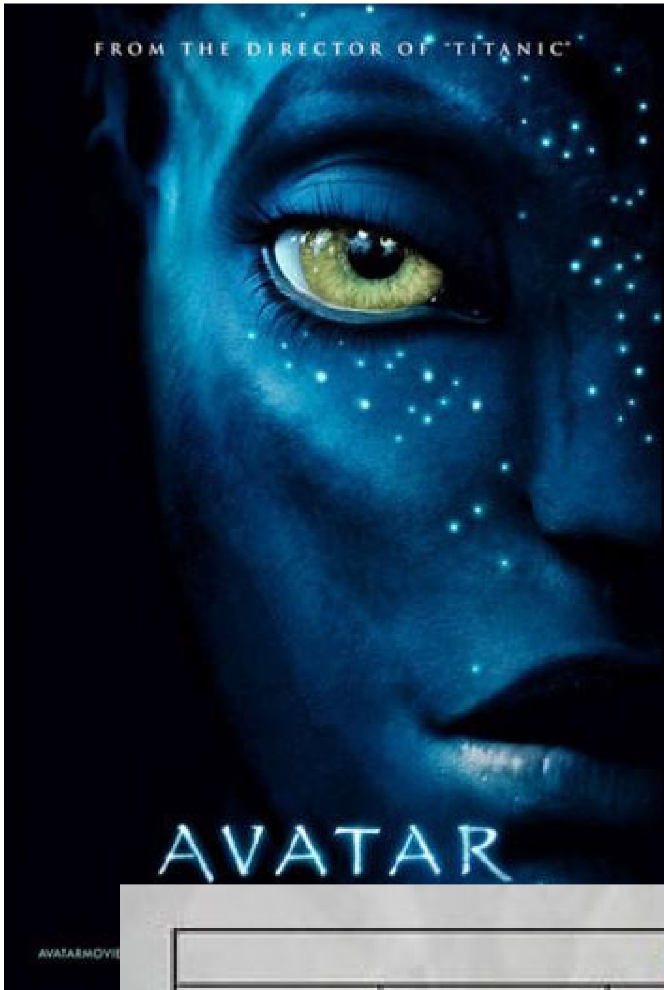
1 HP = 6600 lb · in/s = 745.7 W

1 in = 25.4 mm; 1 m = 39.37 in

1 lbf = 4.4482 N

1 lbm = 0.45359 kg

1 psi = 6894.8 Pa; 1MPa = 145.04 psi



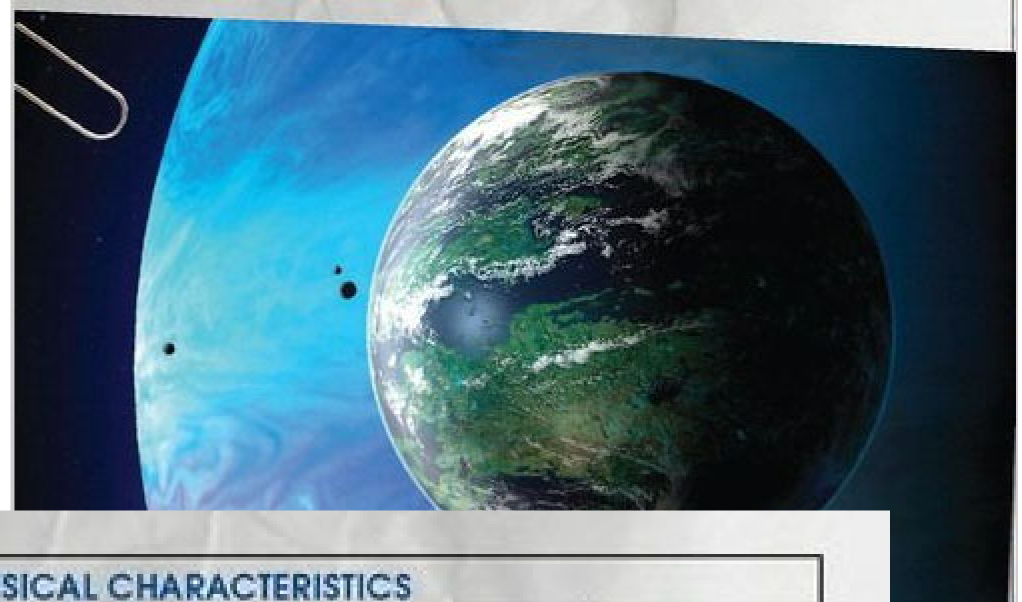
PANDORA

LOCATION: A moon of Polyphemus, a gas giant planet orbiting the star Alpha Centauri A, roughly 4.4 light-years from Earth

ENVIRONMENT: Earth-like, but humans cannot breathe its air without an exopack

INHABITANTS: Na'vi, a highly intelligent humanoid race with a Neolithic society

RESOURCES: The only known source of unobtanium, a superconductor vital to Earth's economy



PHYSICAL CHARACTERISTICS

World	Diameter (kilometers)	Mass	Surface Gravity	Atmospheric Density	Surface Pressure
Earth	1275.27	1	1	1	1
Pandora	11447	0.72	0.8	1.2	1.1

http://browseinside.harpercollins.com/index.aspx?isbn13=9780061896750

- NASA lost the $125 million Mars Climate Orbiter on September 23, 1999
  http://www.cnn.com/TECH/space/9909/30/mars.metric.02/
- The reason is the mission navigation team in California used metric units while Lockheed Martin team used imperial units.
- The thrusters on the spacecraft, which were intended to control its rate of rotation, were controlled by a computer that underestimated the effect of the thrusters by a factor of 4.45. The software was working in pounds force, while the spacecraft expected figures in newtons.
- The spacecraft completed a nearly 10-month journey to Mars and was lost right before the expected entry into Mar's orbit.

Chapter 6 - Introduction
Equations of motion

$$\sum \vec{F} = \frac{d^2}{dt^2}\left(m\vec{r}_{G/A}\right) = m\vec{a}_G \qquad \sum M_A = \frac{\delta \vec{H}_A}{\delta t} + \vec{\omega}\times\vec{H}_A$$

$$\vec{H}_A = \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\hat{i} + \left(I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z\right)\hat{j}$$
$$+\left(I_{zz}\omega_z - I_{xz}\omega_z - I_{yz}\omega_y\right)\hat{k} = \left[I\right]_{xyz}\left\{\omega\right\}_{xyz}$$

$$\frac{\delta \vec{H}_A}{\delta t} = \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z\right)\hat{i} + \left(I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z\right)\hat{j}$$
$$+\left(I_{zz}\alpha_z - I_{xz}\alpha_z - I_{yz}\alpha_y\right)\hat{k} = \left[I\right]_{xyz}\left\{\alpha\right\}_{xyz}$$

1) point A is center of gravity G, or 2) point A has no acceleration

3. Equations of motion

Matrix form

$$\begin{Bmatrix} \sum F_x \\ \sum F_y \\ \sum F_z \end{Bmatrix} = m \begin{Bmatrix} a_{Gx} \\ a_{Gy} \\ a_{Gz} \end{Bmatrix}$$

$$\begin{Bmatrix} \sum M_{Ax} \\ \sum M_{Ay} \\ \sum M_{Az} \end{Bmatrix} = [I] \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} + \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} [I] \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}$$

Coordinates attached to the body

The special case where *xyz* are principal axes leads to *Euler's equation*:

$$\begin{Bmatrix} \sum M_{Ax} \\ \sum M_{Ay} \\ \sum M_{Az} \end{Bmatrix} = \begin{Bmatrix} I_{xx}\alpha_x - \left(I_{yy} - I_{zz}\right)\omega_y\omega_z \\ I_{yy}\alpha_y - \left(I_{zz} - I_{xx}\right)\omega_x\omega_z \\ I_{zz}\alpha_z - \left(I_{xx} - I_{yy}\right)\omega_x\omega_y \end{Bmatrix}$$

Chapter 7 - Introduction

Equations of motion

$$L = T - V \tag{54}$$

then (53) $\Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_j}\right) - \dfrac{\partial L}{\partial q_j} = Q_j^*$ (55)

If there is no non-conservative force, then

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \tag{56}$$

Define the **Lagrangian function** L

Note: set $-\dfrac{\partial L}{\partial q_j} = Q_j^*$ to find the static equilibrium