The material in the following pages is for today's class discussion.

Please review and let me know if you have any questions.

My email is flowegt@auburn.edu
My direct phone line is
334-844-4375

I am available from 8:00-10:00 each morning for any questions or discussion that you would like to have.

Please submit all homework, quizzes, and exams to me via email. If you do not have a scanner available, then please let me know and we will identify an alternative method for you.

First, what will be covered on Exam 2 on Tuesday?

Exam 2 will be held Tuesday, March 24

The exam will cover the content in Chapter 5 and Chapter 6.

The exam will be made available through Canvas. It will be available for you to take starting at 8:00 AM on Tuesday, March 24.

Please return the exam via email or snail mail, by Wednesday, Mach 25, 8:00 AM.

Second, a review of the most important concepts:

2. Angular momentum and inertia properties

2.3 Rate of change of angular momentum

$$\dot{\vec{H}}_A = \frac{\delta \vec{H}_A}{\delta t} + \vec{\omega} \times \vec{H}_A$$

where

$$\frac{\delta \vec{H}_A}{\delta t} = \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z \right)\hat{i} + \left(I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z \right)\hat{j}$$

$$+ \left(I_{zz}\alpha_z - I_{xz}\alpha_z - I_{yz}\alpha_y \right)\hat{k}$$

3. Equation of motion

$$\sum \vec{F} = \frac{d^2}{dt^2}\left(m\vec{r}_{G/A}\right) = m\vec{a}_G \qquad \sum M_A = \frac{\delta \vec{H}_A}{\delta t} + \vec{\omega} \times \vec{H}_A$$

$$\vec{H}_A = \left(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right)\hat{i} + \left(I_{yy}\omega_y - I_{xy}\omega_x - I_{yz}\omega_z\right)\hat{j}$$
$$+\left(I_{zz}\omega_z - I_{xz}\omega_z - I_{yz}\omega_y\right)\hat{k} = \left[I\right]_{xyz}\left\{\omega\right\}_{xyz}$$

$$\frac{\delta \vec{H}_A}{\delta t} = \left(I_{xx}\alpha_x - I_{xy}\alpha_y - I_{xz}\alpha_z\right)\hat{i} + \left(I_{yy}\alpha_y - I_{xy}\alpha_x - I_{yz}\alpha_z\right)\hat{j}$$
$$+\left(I_{zz}\alpha_z - I_{xz}\alpha_z - I_{yz}\alpha_y\right)\hat{k} = \left[I\right]_{xyz}\left\{\alpha\right\}_{xyz}$$

1) point A is center of gravity G, or 2) point A has no acceleration

3. Equation of motion

Matrix form

$$\begin{Bmatrix} \sum F_x \\ \sum F_y \\ \sum F_z \end{Bmatrix} = m \begin{Bmatrix} a_{Gx} \\ a_{Gy} \\ a_{Gz} \end{Bmatrix}$$

$$\begin{Bmatrix} \sum M_{Ax} \\ \sum M_{Ay} \\ \sum M_{Az} \end{Bmatrix} = [I] \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} + \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} [I] \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}$$

Coordinates attached to the body

The special case where *xyz* are principal axes leads to *Euler's equation*:

$$\begin{Bmatrix} \sum M_{Ax} \\ \sum M_{Ay} \\ \sum M_{Az} \end{Bmatrix} = \begin{Bmatrix} I_{xx}\alpha_x - \left(I_{yy} - I_{zz}\right)\omega_y\omega_z \\ I_{yy}\alpha_y - \left(I_{zz} - I_{xx}\right)\omega_x\omega_z \\ I_{zz}\alpha_z - \left(I_{xx} - I_{yy}\right)\omega_x\omega_y \end{Bmatrix}$$

1. Fundamental Principles

1.4 kinetic energy

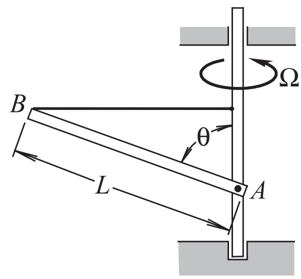
If we select the center of mass as point B, the kinetic energy is simplified as

$$\boxed{\begin{aligned} &T = \frac{1}{2} m \vec{v}_G \bullet \vec{v}_G + \frac{1}{2} \vec{\omega} \bullet \vec{H}_G \text{ for all motions} \\ &T = \frac{1}{2} \vec{\omega} \bullet \vec{H}_O \text{ for pure rotation about point } O \end{aligned}}$$

Third, some example problems from Chapter 6.

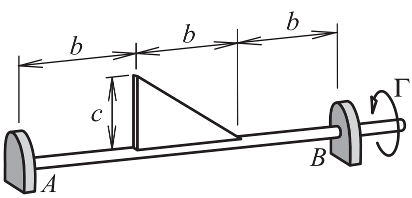
HOMEWORK PROBLEMS

EXERCISE 6.1 The cable holds the angle θ for the bar constant as the vertical shaft rotates at the constant speed Ω. Determine the value of Ω for which the tension in the cable is twice as large as it would be if Ω were zero.



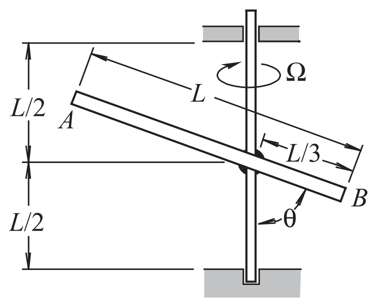
Exercise 6.1

EXERCISE 6.2 Torque Γ causes the horizontal shaft to rotate. The triangular plate's thickness is negligible, as is the mass of the shaft. A torque Γ is applied to the shaft, thereby inducing a time-dependent rotation rate Ω. The system was at rest at $t = 0$. Derive expressions for Ω and the bearing reactions as functions of time. The influence of gravity may be ignored.



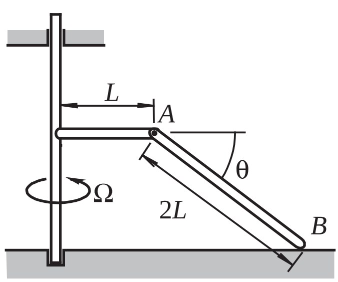
Exercise 6.2

EXERCISE 6.3 The vertical shaft rotates at the constant speed Ω. Determine the forces, including those required to balance the effect of gravity, exerted by the bearings on this shaft. The shaft and the bar have equal mass m.



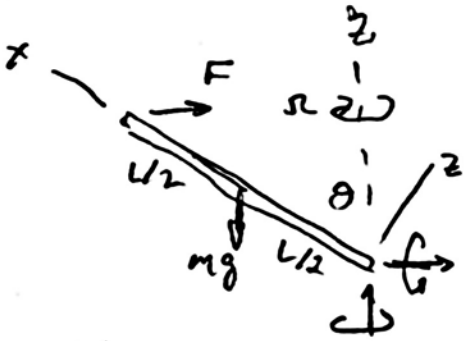
Exercise 6.3

EXERCISE 6.4 Bar AB is pinned to the T-bar and rubs along the ground. The coefficient of friction is μ. A torque acting about the vertical axis imposes a constant rotation rate Ω. Determine the contact forces exerted on bar AB by the ground and the force–couple system exerted on the bar at pin A. (It may be assumed that the friction force acts perpendicular to the vertical plane depicted in the sketch.)



Exercise 6.4

Exercise 6.1



Given constant Ω.

Find Ω for which $F = 2F|_{\Omega=0}$.

Solution: Attach xyz to the bar

$\bar{\omega} = -\Omega\cos\theta\,\bar{i} - \Omega\sin\theta\,\bar{k}, \quad \bar{\alpha} = \bar{0}$

$I_{xx} \approx 0, \; I_{yy} = I_{zz} = \frac{1}{3}mL^2$

$$\Sigma M_{Ay} = -FL\cos\theta + mg\frac{L}{2}\sin\theta = -(I_{zz} - I_{yy})\,\omega_x\omega_z$$
$$= -\frac{1}{3}mL^2\Omega^2\sin\theta\cos\theta$$

When $\Omega = 0$: $FL\cos\theta = mg\frac{L}{2}\sin\theta$

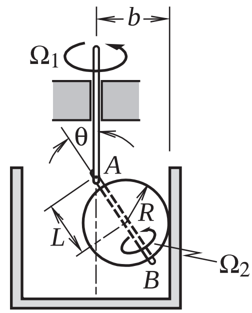
Now set $FL\cos\theta = mgL\sin\theta \Rightarrow$

$$-mg\frac{L}{2}\sin\theta = -\frac{1}{3}mL^2\Omega^2\sin\theta\cos\theta$$

so $\Omega = \left(\frac{3g}{2L\cos\theta}\right)^{1/2}$ because $\sin\theta \neq 0$ ◁

case in which $\beta = 90°$. (b) Determine the forces exerted on the sphere by each surface it contacts if β is an arbitrary acute angle.

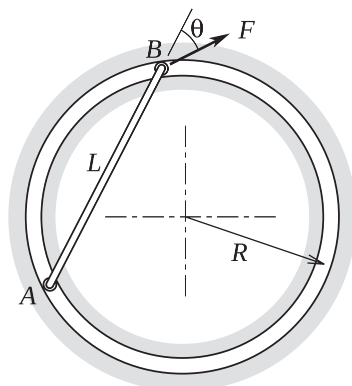
EXERCISE 6.25 The sketch shows the cross section of a stationary cylindrical tank of radius b. A torque, which is not depicted, acting on the vertical shaft maintains the precession rate at a constant angular speed Ω_1 that is very large. The connection between this shaft and shaft AB is an ideal pin, so the angle θ can have any value in the range $0 \le \theta \le \theta_{\max}$, where $L \sin\theta_{\max} + R = b$. The sphere spins freely at angular speed Ω_2 relative to shaft AB. For $t < 0$, a torque applied to shaft AB keeps the sphere in close proximity to the tank's wall, that is, θ is slightly less than $\theta_{\max}$. The spin rate in this condition is $\Omega_2 = 0$. At $t = 0$, the shaft is released, causing the sphere to immediately contact the cylinder. The coefficient of sliding friction between the sphere and the cylinder wall is μ, and all other frictional effects are negligible. Derive the differential equation governing Ω_2 for $t > 0$. The mass of the sphere is m, which is much larger than the mass of shaft AB. *Hint*: The moment exerted on the sphere by shaft AB has a zero component in the direction of the shaft.



Exercise 6.25

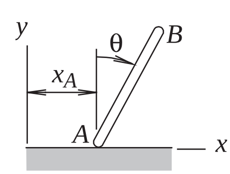
EXERCISE 6.26 Consider an automobile that is following a straight path at speed v. Its wheelbase is L and its center of mass is located at distance b behind the front wheels and distance h above the ground. The coefficient of friction between the tires and the ground is μ. Determine the maximum possible acceleration rate $\dot{v}$ for cases of front-wheel, rear-wheel, and all-wheel drive.

EXERCISE 6.27 Bar AB is attached at both ends to rollers that follow a horizontal circular groove of radius R. Frictional resistance is negligible. At what angle θ relative to the bar should force $\bar{F}$ be applied to maximize the angular acceleration of the bar? What is the corresponding angular acceleration?



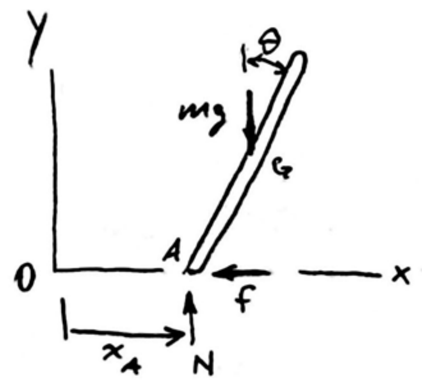
Exercise 6.27

EXERCISE 6.28 The bar of mass m is falling toward the horizontal surface as it slides over the ground. The coefficient of sliding friction is μ. Derive differential equations of motion for the position coordinate x_A of the lower end and the angle of inclination θ.



Exercise 6.28

Exercise 6.28



Find diff eqs for x_A and θ.

Solution:

$$\bar{r}_{G/0} = \left(x_A + \frac{L}{2}\sin\theta\right)\bar{i} + \frac{L}{2}\cos\theta\,\bar{j}$$

$$\bar{a}_G = \ddot{\bar{r}}_{G/0} = \left(\ddot{x}_A + \frac{L}{2}\ddot{\theta}\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta\right) - \left(\frac{L}{2}\ddot{\theta}\sin\theta + \frac{L}{2}\dot{\theta}^2\cos\theta\right)\bar{j}$$

$$\Sigma\bar{M}_G\cdot\bar{k} = -N\left(\frac{L}{2}\sin\theta\right) - f\left(\frac{L}{2}\cos\theta\right) = \frac{1}{12}mL^2(-\ddot{\theta}) \quad (1)$$

$$\Sigma\bar{F}\cdot\bar{i} = -f = m\left(\ddot{x}_A + \frac{L}{2}\ddot{\theta}\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta\right) \quad (2)$$

$$\Sigma\bar{F}\cdot\bar{j} = N - mg = -m\left(\frac{L}{2}\ddot{\theta}\sin\theta + \frac{L}{2}\dot{\theta}^2\cos\theta\right) \quad (3)$$

Eq. (3) gives $N = m\left(g - \frac{L}{2}\ddot{\theta}\sin\theta - \frac{L}{2}\dot{\theta}^2\cos\theta\right)$ (4)

For sliding, $f = \mu N\,\mathrm{sgn}(\dot{x}_A)$, so eq. (2) becomes

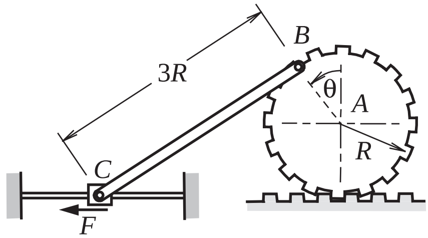
$$\ddot{x}_A + \frac{L}{2}\ddot{\theta}\left[\cos\theta - \mu\sin\theta\,\mathrm{sgn}(\dot{x}_A)\right] - \frac{L}{2}\dot{\theta}^2\left[\sin\theta + \mu\cos\theta\,\mathrm{sgn}(\dot{x}_A)\right] + \mu g\,\mathrm{sgn}(\dot{x}_A) = 0 \quad \triangleleft$$

The second equation is found by substituting N & f into eq. (1). Using eq. (2) for f gives

$$\frac{1}{12}L^2\ddot{\theta} - g\frac{L}{2}\sin\theta + \frac{L^2}{4}\ddot{\theta}(\sin\theta)^2 + \frac{L^2}{4}\dot{\theta}^2\cos\theta\sin\theta + \ddot{x}_A\frac{L}{2}\cos\theta + \frac{L^2}{4}\ddot{\theta}(\cos\theta)^2 - \frac{L^2}{4}\dot{\theta}^2\sin\theta\cos\theta = 0$$

or $\frac{1}{3}L^2\ddot{\theta} + \ddot{x}_A\frac{L}{2}\cos\theta = \frac{1}{2}gL\sin\theta$ $\triangleleft$

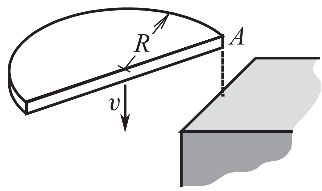
EXERCISE 6.51 Gear A has mass m and centroidal radius of gyration κ. It rolls over the horizontal rack because of the constant horizontal force F acting on collar C. The connecting bar and collar C have negligible mass. The system was at rest at $\theta = 0$. Derive an expression for the speed v of the center of gear A as a function of θ.



Exercise 6.51

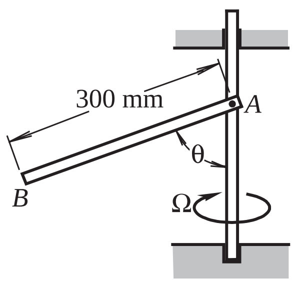
EXERCISE 6.52 Solve Exercise 6.51 for the case in which the mass of rod BC is $m/2$.

EXERCISE 6.53 The semicircular plate is falling at speed v with its plane oriented horizontally. It strikes the ledge at corner A, and the impact is perfectly elastic (that is, the recoil velocity of corner A is v upward). The interval of the collision is Δt. Derive expressions for the velocity of the center of mass and the angular velocity at the instant following the collision. Also, derive an expression for the collision force exerted between the plate and the ledge.



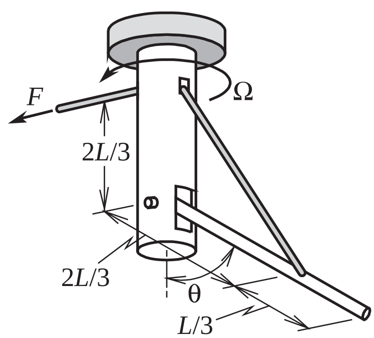
Exercise 6.53

EXERCISE 6.54 Bar AB is pinned to the vertical shaft, which rotates freely. When the bar is inclined at $\theta = 10^\circ$, the rotation rate about the vertical axis is $\Omega = 10$ rad/s, and $\dot{\theta} = 4$ rad/s at that instant. Determine the maximum value of θ in the subsequent motion. The mass of the vertical shaft may be neglected.



Exercise 6.54

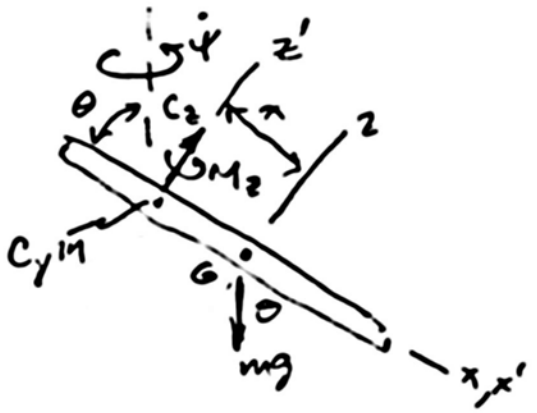
EXERCISE 6.55 The pin allows the angle of inclination θ of the 50-kg bar to change when a constant tensile force of $F = 2$ kN is applied. No torque is applied to the vertical shaft, so the precession rate Ω varies as θ is altered. At the initial position $\theta = 20^\circ$, $\dot{\theta} = 0$, and $\Omega = 5$ rad/s. Determine the values of $\dot{\theta}$ and Ω when $\theta = 90^\circ$.



Exercise 6.55

EXERCISE 6.56 The system in Exercise 6.14 spins freely about the vertical axis because the torque Γ is not present. Initially $\Omega = 4\,(g/L)^{1/2}$ with $\theta = 150^\circ$ and $\dot{\theta} = 0$. The moment of inertia of the T-bar about its rotation axis is $0.5mL^2$, where m is the masses of bar BC. (a) Determine the value of $\dot{\theta}$ when $\theta = 90^\circ$. (b) Determine the minimum value of θ in the ensuing motion.

Exercise 6.15



Given constant $\dot{\psi}$.

Find diff eqs for x & θ, and forces at collar C.

Solution: The collar cannot apply a moment about the x axis, and the shaft cannot exert a moment about the pin in the collar, so the only couple is M_z

$$I_{xx} = 0, I_{yy} = I_{zz} = \frac{1}{12} mL^2$$

$$\bar{\omega} = \dot{\psi}\bar{e}_1 + \dot{\theta}(-\bar{j}), \quad \bar{\alpha} = -\ddot{\theta}\bar{j} - \dot{\theta}(\bar{\omega}\times\bar{j}), \quad \bar{e}_1 = -\cos\theta\,\bar{i} + \sin\theta\,\bar{k}$$

$$\bar{\omega} = -\dot{\psi}\cos\theta\,\bar{i} - \dot{\theta}\bar{j} + \dot{\psi}\sin\theta\,\bar{k}, \quad \bar{\alpha} = -\ddot{\theta}\bar{j} + \dot{\psi}\dot{\theta}(\sin\theta\,\bar{i} + \cos\theta\,\bar{k})$$

Consider $x'y'z'$ attached to the collar $\Rightarrow \bar{\omega}' = \bar{\omega}, \bar{a}_{O'} = \bar{0}$

$$(\bar{v}_G)_{x'y'z'} = \dot{x}\bar{i}, \quad (\bar{a}_G)_{x'y'z'} = \ddot{x}\bar{i}, \quad \bar{r}_{G/O} = x\bar{i}$$

$$\bar{a}_G = \ddot{x}\bar{i} + \bar{\alpha}\times x\bar{i} + \bar{\omega}\times(\bar{\omega}\times x\bar{i}) + 2\bar{\omega}\times\dot{x}\bar{i}$$

$$= [\ddot{x} - x\dot{\theta}^2 - x\dot{\psi}^2(\sin\theta)^2]\bar{i} + 2(\dot{x}\dot{\psi}\sin\theta + x\dot{\psi}\dot{\theta}\cos\theta)\bar{j}$$

$$+ [x\ddot{\theta} + 2\dot{x}\dot{\theta} - x\dot{\psi}^2\sin\theta\cos\theta]\bar{k}$$

Euler's eqs: $\Sigma\bar{M}_G\cdot\bar{i} \equiv 0$

$$\Sigma\bar{M}_G\cdot\bar{j} = xC_z = \frac{1}{12}mL^2(-\ddot{\theta}) - \frac{1}{12}mL^2(-\dot{\psi}\cos\theta)(\dot{\psi}\sin\theta) \qquad (1)$$

$$\Sigma\bar{M}_G\cdot\bar{k} = -xC_y + M_z = \frac{1}{12}mL^2(\dot{\psi}\dot{\theta}\cos\theta) + \frac{1}{12}mL^2(-\dot{\psi}\cos\theta)(\dot{\theta}) \qquad (2)$$

$$\Sigma\bar{F}\cdot\bar{i} = mg\cos\theta = m\bar{a}_G\cdot\bar{i} = m(\ddot{x} - x\dot{\theta}^2 - x\dot{\psi}^2(\sin\theta)^2] \qquad (3)$$

$$\Sigma\bar{F}\cdot\bar{j} = C_y = m\bar{a}_G\cdot\bar{j} = 2m(\dot{x}\dot{\psi}\sin\theta + x\dot{\psi}\dot{\theta}\cos\theta) \qquad (4)$$

$$\Sigma\bar{F}\cdot\bar{k} = C_z - mg\sin\theta = m\bar{a}_G\cdot\bar{k} = m(x\ddot{\theta} + 2\dot{x}\dot{\theta} - x\dot{\psi}^2\sin\theta\cos\theta) \qquad (5)$$

Eq (3) is the d.e. for x.

Exercise 6.15 (cont)

Solve eq (5) for C_z and substitute into eq (1) $\Rightarrow$ d.e. for θ:

$$\left(x^2+\frac{L^2}{12}\right)\left(\ddot{\theta}-\dot{\psi}^2\sin\theta\cos\theta\right)+2x\dot{\theta}\dot{x}+xg\sin\theta=0$$

Solve eq (4) for C_y and substitute into eq. (2):

$$M_z=2m\left(x^2+\frac{L^2}{12}\right)\dot{\psi}\dot{\theta}\cos\theta+2mx\dot{x}\dot{\psi}\sin\theta$$

Finally, Homework Quiz #6 is posted on the Assignments tab in Canvas.

Please download the quiz, complete it, and send your completed quiz to me by 1:00 PM today.