MECH7610: Advanced Dynamics: Homework #1

Due on January 14, 2020

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Problem 1

Determine the number of degrees of freedom of a uniform rod with length L and mass m fixed in the vertical plane by a pin through one end.

Solution

The system can be fully defined by the angle the rod makes with the horizonal, so the system has one degree of freedom.

Problem 2

Write expressions for:

1. Kinetic energy of the system
2. Potential energy of the system

Solution
Part One
The kinetic energy of the system consists of a purely rotational component defined by:

$$T = \frac{1}{2} I_y \omega_y^2$$

The potential energy of the system is limited to the energy due to gravity, equal to:

$$U = mg\frac{L}{2}(1 - \cos(\theta))$$

where z is the height of the center of mass above the center of mass of the earth.

Problem 3

Draw the free body diagram for the system.

Solution

Problem 4

Formulate the equations of motion using Newton's Second Law.

Solution Our equation of motion is:

$$I_y\ddot{\theta} = \Sigma\tau_y = -mg\frac{L}{2}\cos(\theta)$$

Problem 5

Formulate the equations of motion using Lagrange's Equations.

Solution We form our Lagrangian:

$$\begin{aligned} L &= \frac{1}{2} I_y \dot{\theta}^2 - V(\theta) \\ &= \frac{1}{2} I_y \dot{\theta}^2 - mg\frac{L}{2}(1 - \cos\theta) \end{aligned}$$

Taking our Euler-Langrange derivative and setting the sides equal to each other, we get:

$$\begin{aligned} I_y \ddot{\theta} &= -\nabla V \\ &= -mg\frac{L}{2}(1 - \cos(\theta)) \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial \dot{\theta}} &= I_y \dot{\theta} \\ \frac{d}{dt}(\frac{\partial L}{\partial \theta}) &= -mg\frac{L}{2}\sin(\theta) \end{aligned}$$

$$\frac{\partial L}{\partial \dot{\theta}} - \frac{d}{dt}(\frac{\partial L}{\partial \theta}) = I_y \ddot{\theta} + mg\frac{L}{2}\sin(\theta) = 0$$