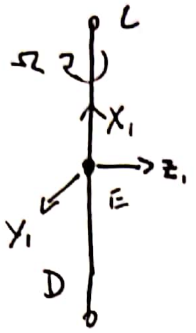
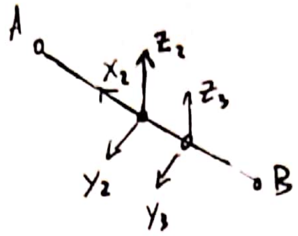
Matt Boler

6.3





$\{F\}_1$: COM of $\overline{CD}$

$\{F\}_2$: COM of $\overline{AB}$

$\{F\}_3$: attachment point on $\overline{AB}$ aligned w/ $\overline{AB}$

I_{CD} : $I_{xx} = 0,\ I_{yy} = \frac{1}{12}mL^3,\ I_{zz} = \frac{1}{12}mL^3 = (I_{CD})^1$

I_{AB} : $I_{xx} = 0,\ I_{yy} = \frac{1}{12}mL^3,\ I_{zz} = \frac{1}{12}mL^3 = (I_{AB})^2$

$(I_{AB})^3$: translate I_{AB} to attachment point (E)

$$X_{o'} = -L/6,\quad Y_{o'} = 0,\quad Z_{o'} = 0$$

$$\rightarrow I_{xx}^3 = I_{xx}^2 + 0$$

$$I_{yy}^3 = I_{yy}^2 + m(L^2/36)$$

$$I_{zz}^3 = I_{zz}^2 + m(L^2/36)$$

$(I_{AB})^1$: rotate into $\{F\}_1$ need $R_{3\to1}$

$$\left.\begin{aligned} X_1 &= X_3\cos\theta + Z_3\sin\theta \\ Y_1 &= Y_3 \end{aligned}\right\} R = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$Z_1 = Z_3\cos\theta - X_1\sin\theta$$

$$(I_{AB})^1 = R(I_{AB})^3R^T\ ,\quad I = (I_{AB})^1 + (I_{CD})^1$$

EOMs:

$\cancel{\Sigma M_{D,x} = \Sigma M_{c,x} = 0}$

$\cancel{\Sigma M_{D,y} = C_z L + \frac{L}{6}\sin\theta mg}$

Cont.

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$$\left.\begin{aligned}\Sigma M_{E,x} &= 0 \\ \Sigma M_{E,y} &= \frac{L}{6} mg\sin\theta - \frac{L}{2} C_z + \frac{L}{2} D_z \\ \Sigma M_{E,z} &= C_y \frac{L}{2} - D_y \frac{L}{2}\end{aligned}\right\} = I \cancel{\begin{Bmatrix}\alpha_x \\ \alpha_y \\ \alpha_z\end{Bmatrix}}^{0} + [\bar{\omega}]_x I \begin{Bmatrix}\omega_x = -\Omega \\ 0 \\ 0\end{Bmatrix} = \bar{0}$$

$0 = C_y \frac{L}{2} - D_y \frac{L}{2} = -D_y \frac{L}{2} - D_y \frac{L}{2} \rightarrow D_y = 0,\ C_y = 0$

$\Sigma F_y = 0 = C_y + D_y \rightarrow \cancel{C_y \neq 0, D_y \neq 0}\ C_y = -D_y$

$0 = \frac{L}{6} mg\sin\theta - \frac{L}{2} C_z + \frac{L}{2} D_z = \frac{L}{6} mg\sin\theta + L D_z = 0$

$\Sigma F_z = 0 = C_z + D_z \rightarrow C_z = -D_z$

$\rightarrow D_z = -\frac{mg}{6}\sin\theta$

$\rightarrow C_z = \frac{mg}{6}\sin\theta$

$\Sigma F_x = 0 = C_x + D_x - 2mg \rightarrow (C_x + D_x) = 2mg$

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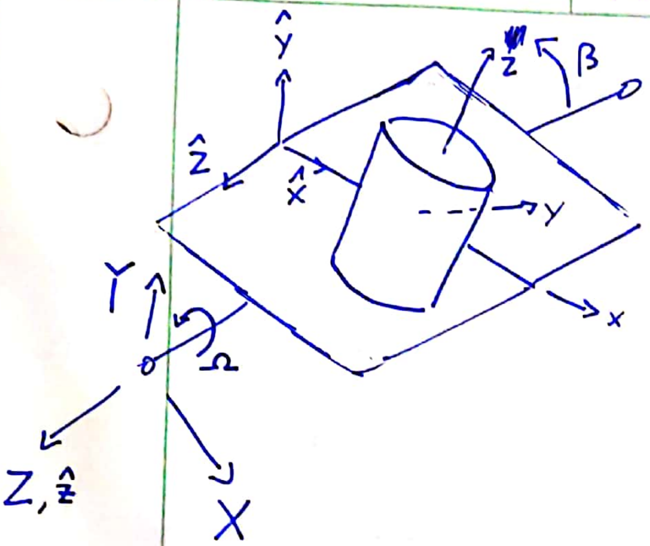

$\bar{\omega} = \cancel{\hat{e}_1 \Omega + \dot{\beta}\hat{e}}$
$= \Omega \bar{e}_1 + \dot{\beta} \bar{e}_2$
$= \Omega \bar{K} + \dot{\beta} \hat{\imath}$

XYZ fixed $\rightarrow \bar{\Omega}_1 = 0$

~~$\hat{x}\hat{y}\hat{z}$ attached to frame~~ :

xyz attached to cylinder: $\bar{\Omega}_2 = \bar{\omega}$

$\bar{\alpha} = \left(\cancel{\Omega \bar{\Omega}_1 \times \bar{K}}_{\to 0} + \dot{\beta} \bar{\Omega}_2 \times \hat{\imath}\right) + \cancel{\dot{\Omega} \bar{K}}_{\to 0} + \ddot{\beta}\hat{\imath}$

$\bar{K} = -\hat{k}\cos\beta - \hat{\jmath}\sin\beta$

$\bar{\alpha} = \ddot{\beta}\hat{\imath} - \Omega\cos(\beta)\hat{\jmath} + \Omega\sin(\beta)\hat{k}$

$I_{xx} = I_{yy} = \frac{1}{12} m(3R^2 + L^2)$

$I_{zz} = \frac{1}{2} mR^2$

$\Sigma M_{GY} = I_{yy}\alpha_y - (I_{zz} - I_{xx})\omega_z\omega_x$

$-\Gamma\sin\beta = I_{yy}(-\cos\beta) - (I_{zz} - I_{xx})\dot{\beta}(-\Omega\cos\beta)$

$\Sigma M_{GX} = I_{xx}\alpha_x - (I_{yy} - I_{zz})\omega_y\omega_z$

$0 = I_{xx}\ddot{\beta} - (I_{yy} - I_{zz})\Omega^2 \sin\beta\cos\beta$

$\Sigma M_{GZ} = I_{zz}\alpha_z - (I_{xx} - I_{yy})\omega_x\omega_y$

$-\Gamma\cos\beta = I_{zz}\Omega\sin\beta - (I_{xx} - I_{yy}) \cdot \dot{\beta}(-\Omega\sin\beta)$

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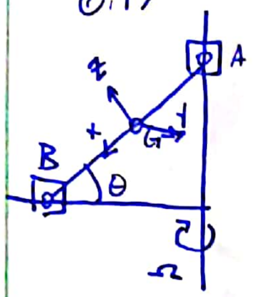

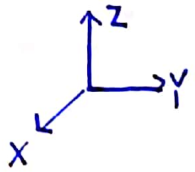

$\bar{r}_{G/B} = -L/2\ \hat{x}$

$\bar{r}_{G/A} = L/2\ \hat{x}$

$I_{xx} \approx 0$

$\bar{\omega} = (-\omega)\bar{K} + \dot{\theta}\hat{y}$

$\bar{K}$: const. dir. $\rightarrow \bar{\Omega}_1 = 0$, $\bar{K} = \hat{z}\cos\theta - \hat{x}\sin\theta$

$\hat{y}$: rotate at $(-\omega)\bar{K} \rightarrow \bar{\Omega}_2 = (-\Omega)\bar{K}$

$\bar{\omega} = (-\Omega)(\hat{z}\cos\theta - \hat{x}\sin\theta) + \dot{\theta}\hat{y}$

$\bar{\alpha} = \ddot{\theta}\hat{y} + \dot{\theta}\left[(-\Omega)(\hat{z}\cos\theta - \hat{x}\sin\theta) \times \hat{y}\right]$

$= \Omega\dot{\theta}\cos\theta\hat{x} + \ddot{\theta}\hat{y} + \Omega\dot{\theta}\sin\theta\hat{z}$

$\bar{a}_G = \dot{V}_B(\hat{x}\cos\theta + \hat{z}\sin\theta) + \bar{\alpha} \times \bar{r}_{G/B} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{G/B})$

$= \dot{V}_A(-\hat{z}\cos\theta + \hat{x}\sin\theta) + \bar{\alpha} \times \bar{r}_{G/A} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{G/A})$

$= \cancel{\dot{V}_B(\hat{x}\cos\theta + \hat{z}\sin\theta)}$

$= \left[\dot{\theta}^2 \frac{L}{2} + \dot{V}_b\cos\theta + \Omega^2\cos(\theta)^2 \frac{L}{2}\right]\hat{x}$

$+ \left[-L\Omega\dot{\theta}\sin(\theta)\right]\hat{y}$

$+ \left[\Omega^2\sin\theta\cos\theta\frac{L}{2} + \ddot{\theta}\frac{L}{2} + \dot{V}_b\sin\theta\right]\hat{z}$

$\Sigma M_{G,y} = F_A\frac{L}{2}\sin\theta - F_B\frac{L}{2}\cos\theta$

$= I_{yy}\alpha_y - I_{zz}\omega_x\omega_z$

$= \frac{1}{12}mL^2(\ddot{\theta} - \Omega^2\sin\theta\cos\theta)$

$\Sigma F_{G,x} = mg\sin\theta + F_A\cos\theta - F_B\sin\theta = \bar{a}_G \cdot \hat{x} \cdot m$

$\Sigma F_{G,z} = mg\cos\theta + F_A\sin\theta + F_B\cos\theta = \bar{a}_G \cdot \hat{z} \cdot m$

[3 eq, 3 unknown]

Scanned with CamScanner

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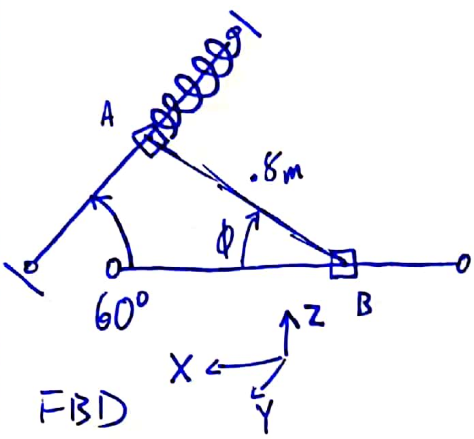
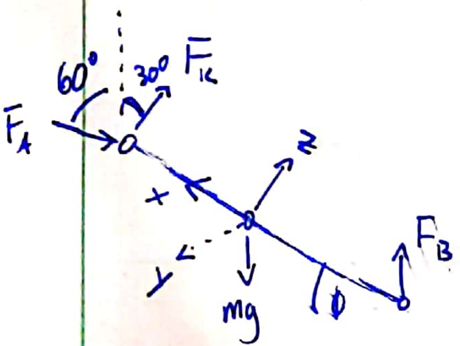
6.49



FBD



F_A moment:

$0 \text{ @ } \phi = 30$

$-F_A \frac{L}{2} \text{ @ } \phi = 120$

$F_A \frac{L}{2} \text{ @ } \phi = -60$

$= -F_A \frac{L}{2} \sin(\phi - 30)$

F_K moment:

$+F_K \frac{L}{2} \text{ @ } \phi = 30$

$0 \text{ @ } \phi = 120$

$= +F_K \frac{L}{2} \cos(\phi - 30)$

$m_{AB} = 40 \text{ kg}$

$K_s = 9 \text{ kN/m}$

Initial Cond:

$\phi = 0$

$\Delta X_k = 0.3\text{m}$

a) $\phi_{max} = ?$

$\bar{\omega} = -\dot{\phi}\hat{y}$

$$\Sigma M_y = -F_B \frac{L}{2}\cos\phi + F_K \frac{L}{2}\cos(\phi - 30) - F_A \frac{L}{2}\sin(\phi - 30)$$