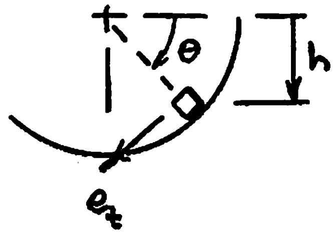
Spring 2020

Homework #2

Exercise 2.1



Given $v^2 = 2gh$

Find $\bar{v}(s)$ & $\bar{a}(s)$

Solution: $\bar{v} = v\bar{e}_t$, $\bar{a} = \dot{v}\bar{e}_t + \frac{v^2}{\rho}\bar{e}_n$

but $\theta = \frac{s}{R} \Rightarrow h = R\sin\theta = R\sin\frac{s}{R}$

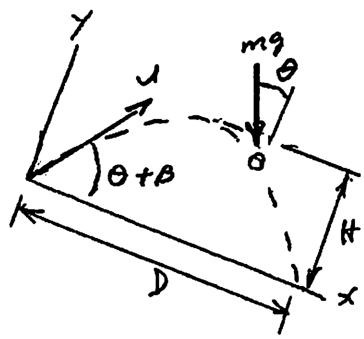
so $v^2 = 2gR\sin\frac{s}{R}$

Differentiate: $2v\dot{v} = 2gR\frac{\dot{s}}{R}\cos\frac{s}{R} = 2gv\cos\frac{s}{R} \Rightarrow \dot{v} = g\cos\frac{s}{R}$

Then $\bar{v} = (2gR\sin\frac{s}{R})^{1/2}\,\bar{e}_t$

$\bar{a} = g\cos\frac{s}{R}\,\bar{e}_t + 2g\sin\frac{s}{R}\,\bar{e}_n$

Exercise 2.18



Given $v_0 = u$, β, and θ

Find D, H, & v at $y = H$

Solution: $\Sigma \bar{F} \cdot \bar{i} = mg \sin\theta = m\ddot{x}$

$\Sigma \bar{F} \cdot \bar{j} = -mg\cos\theta = m\ddot{y}$

At $t = 0$: $x = y = 0$, $\dot{x} = u\cos(\beta+\theta)$, and

$\dot{y} = u\sin(\beta+\theta)$

Integrate eqs of motion and satisfy initial conditions:

$$\dot{x} = (g\sin\theta)t + \dot{x}(0) = gt\sin\theta + u\cos(\beta+\theta)$$

$$x = \tfrac{1}{2}gt^2\sin\theta + ut\cos(\beta+\theta)$$

$$\dot{y} = -(g\cos\theta)t + \dot{y}(0) = -gt\cos\theta + u\sin(\theta+\beta)$$

$$y = -\tfrac{1}{2}gt^2\cos\theta + ut\sin(\theta+\beta)$$

To find range D, set $y = 0 \Rightarrow t_D = \dfrac{2u\sin(\theta+\beta)}{\cos\theta}$

$$D = x(t_D) = t_D\left[\tfrac{1}{2}gt_D\sin\theta + u\cos(\beta+\theta)\right]$$

$$= \frac{2u\sin(\theta+\beta)\cos\beta}{g(\cos\theta)^2}$$

To find height H, set $\dot{y} = 0 \Rightarrow t_H = \dfrac{u\sin(\theta+\beta)}{g\cos\theta} = \tfrac{1}{2}t_D$

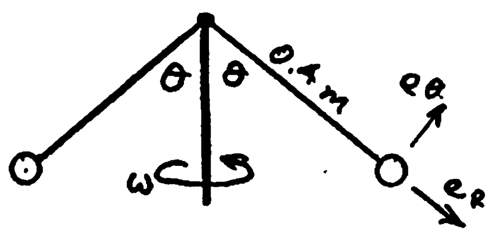
$$H = y(t_H) = t_H\left[-\tfrac{1}{2}g\,t_H\cos\theta + u\sin(\theta+\beta)\right]$$

$$= \frac{u^2\sin(\theta+\beta)^2}{2g\cos\theta}$$

At $t = t_H$, $\bar{v} = \dot{x}\,\bar{i} = \left[-gt_H\cos\theta + u\sin(\theta+\beta)\right]\bar{i}$

$$= \frac{u\cos\beta}{\cos\theta}\,\bar{i}$$

Exercise 2.30



Given $\omega = 1800(\frac{2\pi}{60})$ rad/s, $\dot{\omega} = 0$,
$\theta = \frac{\pi}{3}\sin(120t)$
Find: $\bar{v}$ & $\bar{a}$ vs. t, then evaluate at $\theta = 0$ and $\theta = \pi/3$.
Solution: Use spherical coords with θ as the polar angle

$r = 0.4$, $\dot{r} = \ddot{r} = 0$, $\dot{\theta} = 40\pi\cos(120t)$, $\ddot{\theta} = -4800\pi\sin(120t)$

Correspondingly, $\bar{v} = r\dot{\theta}\,\bar{e}_\theta + r\omega\sin\theta\,\bar{e}_\psi$ (ψ is the azimuth)

$$\bar{a} = [-r\dot{\theta}^2 - r\omega^2(\sin\theta)^2]\,\bar{e}_r + [r\ddot{\theta} - r\omega^2(\sin\theta)(\cos\theta)]\,\bar{e}_\theta$$
$$+ 2r\dot{\theta}\omega\cos\theta\,\bar{e}_\psi$$

$\theta = 0 \Rightarrow \sin(120t) = 0 \Rightarrow t = 0, \frac{\pi}{120}$. (Periodic functions)

$t = 0 \Rightarrow \bar{v} = 50.27\,\bar{e}_\theta$ m/s, $\bar{a} = -6317\,\bar{e}_r + 18950\,\bar{e}_\psi$ m/s^2 ◅

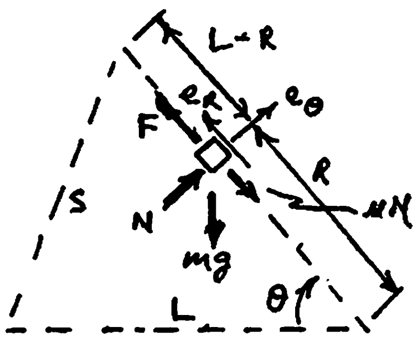
$t = \frac{\pi}{120} \Rightarrow \bar{v} = -50.27\,\bar{e}_\theta$ m/s, $\bar{a} = -6317\,\bar{e}_r - 18950\,\bar{e}_\psi$ m/s^2 ◅

$\theta = \frac{\pi}{3} \Rightarrow \sin(120t) = 1 \Rightarrow t = \frac{\pi}{240}$, $t = \frac{\pi}{80}$

$t = \frac{\pi}{240} \Rightarrow \bar{v} = 65.30\,\bar{e}_\psi$ m/s, $\bar{a} = -10659\,\bar{e}_r - 12186\,\bar{e}_\theta$ m/s^2 ◅

$t = \frac{\pi}{80} \Rightarrow \bar{v} = -65.30\,\bar{e}_\psi$ m/s, $\bar{a} = -10659\,\bar{e}_r + 12186\,\bar{e}_\theta$ m/s^2 ◅

Exercise 2.34



Given $L=0.3m$, $l = s+(L-R)=0.3m$, $m = 0.5\,kg$, $\dot\theta = 5\,rad/s$, $\ddot\theta=0$ @ $\theta = 53.1301^\circ$, $\mu = 0.4$

Find F & N

Solution; $\dot\theta > 0 \Rightarrow \dot R > 0 \Rightarrow \bar F$ inward

$s = 2L\sin\frac{\theta}{2}$, $\dot s = L\dot\theta\cos\frac{\theta}{2}$

$\ddot S = -\frac{1}{2}L\dot\theta^2\sin\frac{\theta}{2}$

From cable length $s+(L-R)=0.3 \Rightarrow R = S+L-0.3 \Rightarrow \dot R = \dot s$, $\ddot R = \ddot S$

At $\theta = 53.13^\circ$; $s=0.2683$, $\dot s = 1.3416$, $\ddot s = -1.6771$, $R=0.2683$

Then $\bar a = (\ddot R - R\dot\theta^2)\bar e_R + 2\dot R\dot\theta\,\bar e_\theta = -8.385\,\bar e_R + 2.683\,\bar e_\theta\ m/s^2$

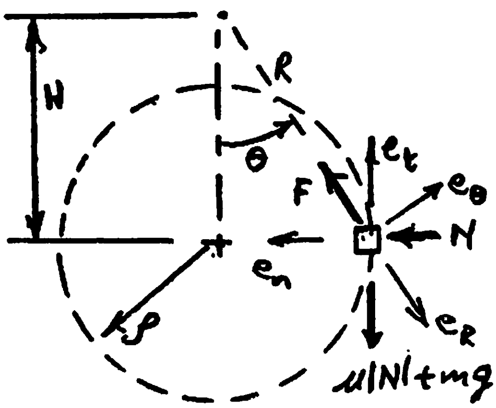
$\Sigma F_R = F - \mu N - mg\sin\theta = ma_R$

$\Sigma F_\theta = N - mg\cos\theta = ma_\theta$

Thus $N = m(a_\theta + g\cos\theta) = 4.284\,N$ $\triangleleft$

$F = m(a_R + g\sin\theta) + \mu N = 1.444\,N$ $\triangleleft$

Exercise 2.51



Given $\rho = 0.8\,m$, $H = 1.2\,m$, $\dot{R} = -25\,m/s$, $\ddot{R} = 0$, $m = 0.2\,kg$

Find (a) v & $\dot{v}$ at the given position, (b) F corresponding to $\mu = 0.4$.

Solution; $\theta = \tan^{-1}\left(\frac{\rho}{H}\right) = 32.01^\circ$

$\bar{e}_t = -\cos\theta\,\bar{e}_R + \sin\theta\,\bar{e}_\theta$

$\bar{e}_n = -\sin\theta\,\bar{e}_R - \cos\theta\,\bar{e}_\theta$

$\bar{v} = v\bar{e}_t$, $\bar{v}\cdot\bar{e}_R = \dot{R}$, $\bar{v}\cdot\bar{e}_\theta = R\dot{\theta}$. Also $R = (\rho^2 + H^2)^{1/2} = 1.4422\,m$

$\dot{R} = -v\cos\theta$ & $v\sin\theta = R\dot{\theta} \Rightarrow v = -\dfrac{\dot{R}}{\cos\theta} = 30.05\,m/s$ ◁

and $\dot{\theta} = \dfrac{v}{R}\sin\theta = 11.556\,rad/s$

Then $\bar{a} = -R\dot{\theta}^2\bar{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\bar{e}_\theta = \dot{v}\bar{e}_t + \dfrac{v^2}{\rho}\bar{e}_n$

so $\dot{v}\bar{e}_t\cdot\bar{e}_R + \dfrac{v^2}{\rho}\bar{e}_n\cdot\bar{e}_R = -R\dot{\theta}^2$

$\dot{v} = \dfrac{1}{\cos\theta}\left(R^2\dot{\theta} - \dfrac{v^2}{\rho}\sin\theta\right) = -520.8\,m/s^2$ ◁

Eqs. of motion:

$\Sigma F_t = F\cos\theta - \mu N - mg = m\dot{v}$

$\Sigma F_n = F\sin\theta + N = m\dfrac{v^2}{\rho}$

Solve $F(\cos\theta + \mu\sin\theta) = m\left(g + \dot{v} + \mu\dfrac{v^2}{\rho}\right) \Rightarrow F = 40.08\,N$ ◁