Optimal | HW2 | Matt Boler

$X(t) \rightarrow R_x(\tau) = \sigma^2 e^{-\beta|\tau|} = E[x(t)\,x(t+\tau)]$

$y(t) = a\,x(t) + b$, $R_y(\tau) = ?$

a) $E[y(t)\,y(t+\tau)] = E[(ax(t)+b)(ax(t+\tau)+b)]$

$= E[a^2 x(t)\,x(t+\tau) + abx(t) + abx(t+\tau) + b^2]$

$= a^2 E[x(t)\,x(t+\tau)] + ab\,E[x(t)] + \cancel{ab\,E[x(t+\tau)]}$

$+ ab E[x(t+\tau)] + b^2$

$= a^2\sigma^2 e^{-\beta|\tau|} + 2ab\,E[X] + b^2 = R_y(\tau)$

b) $R_{xy}(\tau) = E[x(t)\,y(t+\tau)] = E[x(t)(ax(t+\tau)+b)]$

$= E[ax(t)\,x(t+\tau) + bx(t)]$

$= a\sigma^2 e^{-\beta|\tau|} + b\,E[X]$