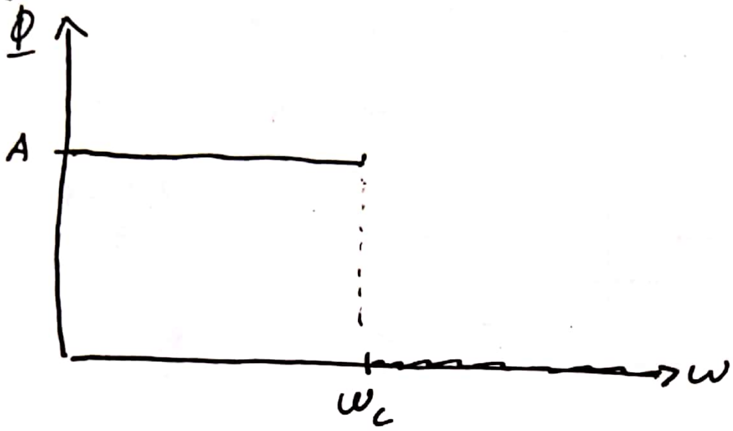
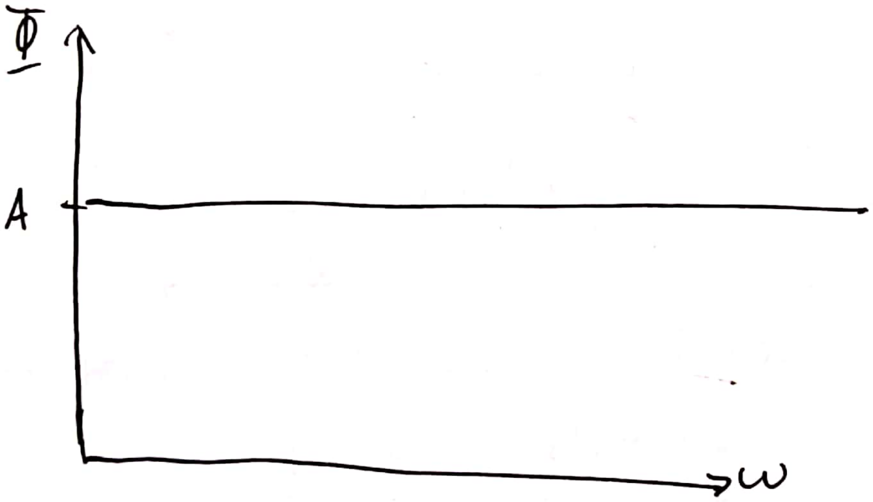
5: Noise PSD Sketches

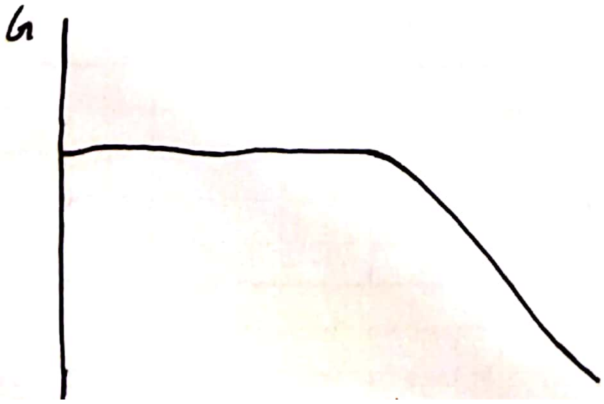
i) $\Phi_1(\omega) = \begin{cases} A; & |\omega| \leq \omega_c \\ 0; & \text{else} \end{cases}$



ii) $\Phi_2(\omega) = A \; \forall \omega$



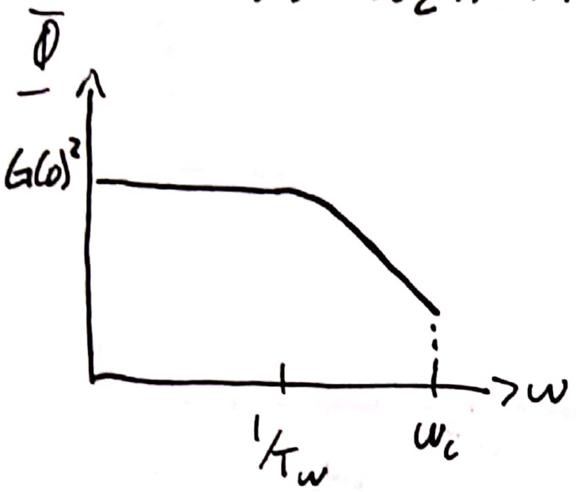
~~G(s)~~ $G(\omega)$



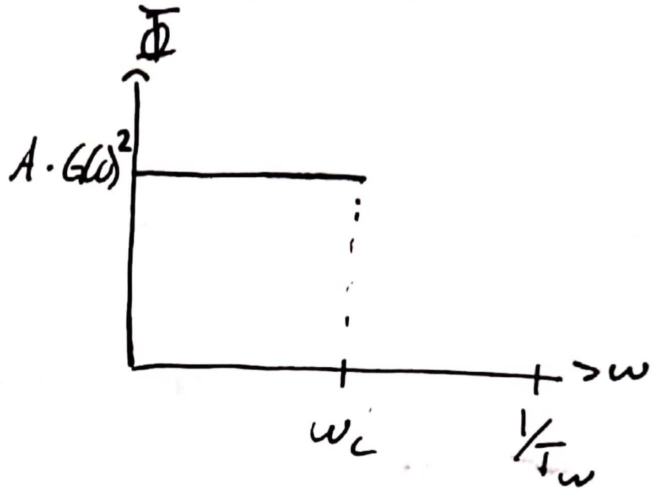
5: $S_y(\omega)$ Sketches

i) $S_y(\omega) = |G(\omega)|^2 S_1(\omega)$

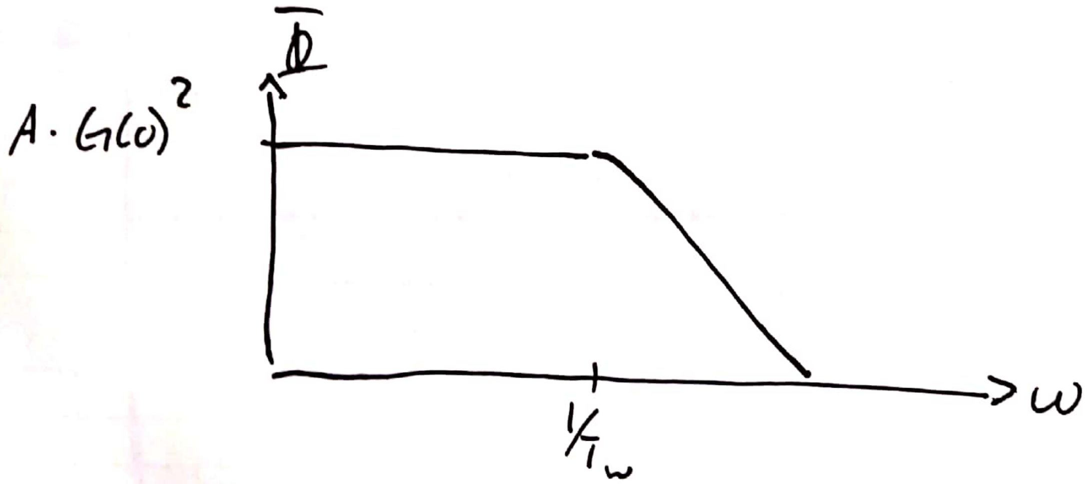
2 cases: $\omega_c > 1/\tau_w$ and $\omega_c < 1/\tau_c$



vs



ii) $S_y(\omega) = |G(\omega)|^2 S_2(\omega)$



5: PSD functions

i) $\Phi_Y = S_1(\omega)\,|G(\omega)|^2$

$$= \begin{cases} A \cdot |G(\omega)|^2 & ; \ |\omega| < \omega_c \\ 0 & ; \ \text{else} \end{cases}$$

ii) $\Phi_Y = S_2(\omega)\,|G(\omega)|^2$

$$= A \cdot |G(\omega)|^2 \quad \forall \omega$$

5: $E[y^2] = ?$

y zero mean $\rightarrow E[y^2] = \sigma_Y^2$

$G(s) = \cancel{\frac{1}{T_w w}} \quad \frac{1}{T_w s + 1} \rightarrow$ difference eq^tn

eig $= 1/T_w \rightarrow \lambda_d = \exp(-\Delta t / T_w)$

$G(z) = \frac{1-\lambda_d}{z-\lambda_d} \rightarrow y(z)[z-\lambda_d] = (1-\lambda_d)u(z)$

$\hookrightarrow y(k) - \lambda_d y(k-1) = (1-\lambda_d)u(k-1)$

$y(k) = \lambda_d y(k-1) + (1-\lambda_d)u(k-1)$

$$E[y^2] = E[y(k)^2]$$

$$= E[\{\lambda y(k-1) + (1-\lambda)\omega(k-1)\}\{\lambda y(k-1) + (1-\lambda)\omega(k-1)\}]$$

$$= E[\lambda^2 y^2(k-1) + (1-\lambda)^2 \omega^2(k-1) + 2\lambda y(k-1)(1-\lambda)\omega(k-1)]$$

$$= E[\lambda^2 y^2(k-1)] + E[(1-\lambda)^2 \omega^2(k-1)] + E[2\lambda y(k-1)(1-\lambda)\omega(k-1)]$$

$$= \lambda^2 E[y^2(k-1)] + (1-\lambda)^2 E[\omega^2(k-1)] + \underbrace{2\lambda(1-\gamma) E[y(k-1)\omega(k-1)]}_{0 \text{ b/c } y, \omega \text{ indep}}$$

$$= \lambda^2 \sigma_Y^2 + (1-\lambda)^2 \sigma_\omega^2$$

$\hookrightarrow$ if λ small ~~or~~ and σ_ω^2 very large, ~~the~~ then approximately equivalent to $\sigma_\omega = \text{inf}$.

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