### Exercise1

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## (1) Explanatory Analysis

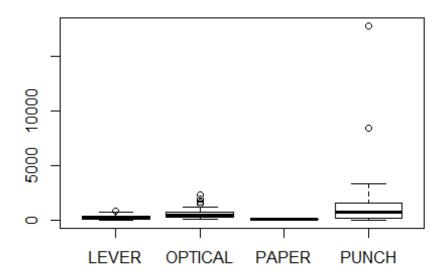
# Does using certain kinds of voting equipment lead to higher rates of undercount?

Read in data and create new column for the amount of undercounted votes

```
votes = read.csv('georgia2000.csv', header = TRUE)
summary(votes)
##
                      ballots
         county
                                        votes
                                                         equip
  APPLING: 1
                   Min.
                        :
                             881
                                    Min.
                                           :
                                               832
                                                     LEVER
##
                                                            :74
##
   ATKINSON: 1
                   1st Qu.:
                             3694
                                    1st Qu.:
                                              3506
                                                     OPTICAL:66
    BACON
                   Median : 6712
                                    Median :
                                              6299
                                                     PAPER : 2
                          : 16927
##
    BAKER
               1
                   Mean
                                    Mean
                                           : 16331
                                                     PUNCH: 17
                   3rd Qu.: 12251
##
    BALDWIN: 1
                                    3rd Qu.: 11846
                          :280975
##
    BANKS
                   Max.
                                    Max.
                                           :263211
    (Other) :153
##
                                         atlanta
##
         poor
                         urban
                                                            perAA
##
   Min.
           :0.0000
                     Min.
                            :0.0000
                                      Min.
                                             :0.00000
                                                        Min.
                                                                :0.0000
##
   1st Qu.:0.0000
                     1st Qu.:0.0000
                                      1st Qu.:0.00000
                                                        1st Qu.:0.1115
##
   Median :0.0000
                     Median :0.0000
                                      Median :0.00000
                                                        Median :0.2330
##
   Mean
           :0.4528
                     Mean
                            :0.2642
                                             :0.09434
                                                        Mean
                                                                :0.2430
                                      Mean
    3rd Ou.:1.0000
                     3rd Ou.:1.0000
                                      3rd Ou.:0.00000
                                                        3rd Ou.:0.3480
##
## Max.
           :1.0000
                            :1.0000
                                      Max.
                                             :1.00000
                                                        Max.
                                                                :0.7650
##
##
                          bush
         gore
   Min.
               249
                     Min.
                                271
##
##
    1st Qu.:
             1386
                     1st Qu.:
                               1804
   Median : 2326
                     Median :
##
                               3597
##
   Mean
             7020
                     Mean
                               8929
##
    3rd Qu.: 4430
                     3rd Qu.:
                               7468
           :154509
##
   Max.
                     Max.
                            :140494
##
votes$undercount = votes$ballots - votes$votes
summary(votes$undercount)
##
      Min. 1st Qu. Median
                              Mean 3rd Qu.
                                              Max.
##
             152.5
                     296.0
                             595.5
                                     523.5 17760.0
```

Boxplot Undercount against equipment. Lever and Paper are extremely accurate, while optical and punch are less reliable

boxplot(undercount~equip, data = votes)



```
lm.fit = glm(undercount~equip, data = votes)
summary(lm.fit)
##
## Call:
## glm(formula = undercount ~ equip, data = votes)
##
## Deviance Residuals:
                      Median
##
       Min
                 1Q
                                   3Q
                                           Max
## -2260.5
             -246.8
                      -110.9
                                116.1 15501.5
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
                   229.9
## (Intercept)
                              171.9
                                      1.338
                                               0.183
## equipOPTICAL
                   362.3
                              250.3
                                      1.447
                                               0.150
## equipPAPER
                  -173.4
                             1059.5 -0.164
                                               0.870
                                      5.111 9.32e-07 ***
## equipPUNCH
                  2032.5
                              397.7
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 2186205)
##
```

```
## Null deviance: 396571504 on 158 degrees of freedom
## Residual deviance: 338861726 on 155 degrees of freedom
## AIC: 2778.2
##
## Number of Fisher Scoring iterations: 2
```

# If so, should we worry that this effect has a disparate impact on poor and minority communities?

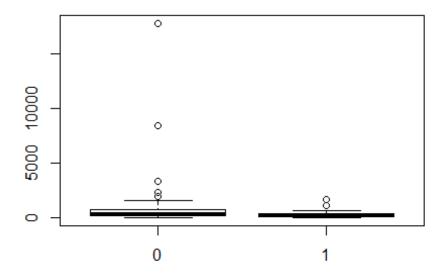
Xtab and Boxplot show that the poor have on average less undercounting and the large outliers pertain to the rich.

```
x1 = xtabs(~undercount + poor, data = votes)
x1
##
              poor
## undercount 0 1
##
        0
         2
               0 1
##
##
         20
               1 0
##
         23
               1 0
         27
               0 1
##
##
         35
               1 0
         41
##
               0 1
         49
##
               0 1
##
         52
               0 1
##
         56
               0 1
##
         57
               0 1
               0 1
##
         64
         65
               2 2
##
##
         67
               1 0
         70
##
               0 1
##
         78
               0 2
##
         80
               0 1
##
         81
               0 1
         85
               1 0
##
##
         88
               0 2
##
         102
               1 0
##
         105
               1 0
##
         107
               0 1
##
         110
               1 0
##
         111
               0 1
##
         113
               0 1
##
         118
               1 0
##
         119
               0 1
               1 1
##
         123
##
         124
               1 1
##
         136
               1 0
##
         147
               0 1
##
         158
               0 1
```

```
##
               0 1
         159
##
         166
                0 1
##
         167
               0 1
##
         169
               0 1
##
         170
               1 0
##
         171
               0 1
##
         176
               1 0
##
         178
               0 1
##
               0 1
         181
##
         193
                1 0
##
                2 0
         195
##
         197
               0 1
##
         201
               1 0
##
               0 1
         203
##
               0 2
         205
##
               0 1
         213
##
         216
               1 0
##
         217
                1 0
##
         222
                1 0
##
         235
               0 1
##
         236
               0 1
##
         240
               1 0
##
               1 0
         241
##
         243
               1 0
##
               0 1
         244
##
         246
                2 0
##
         248
               0 1
##
         250
               0 1
##
         257
               1 0
##
         261
               1 0
##
               1 0
         265
##
         269
               0 1
##
               1 0
         272
##
               0 1
         282
##
         295
               1 0
##
         296
               1 1
##
         297
               1 0
##
         301
               1 0
##
         304
               0 1
##
         307
               0 1
##
               1 0
         317
##
         330
               1 0
##
               0 1
         336
##
         338
               0 1
##
         344
               0 1
##
         345
                1 0
##
               1 0
         346
##
               0 1
         348
##
         352
               0 1
##
         353
               0 2
```

```
##
               1 1
         362
##
         364
               0 1
##
         370
               1 0
##
         383
               0 1
##
         393
               1 0
##
         397
               0 1
##
         404
               1 0
##
         407
               1 0
##
         413
               1 0
##
         415
               0 1
##
         420
               1 0
##
         421
               1 0
##
         442
               1 0
##
         452
               0 1
##
               0 1
         457
##
               0 1
         469
##
         475
               0 1
##
         479
               1 0
##
               0 1
         492
##
         493
               0 1
##
         509
               1 0
##
         518
               0 1
##
         529
               0 1
##
               1 0
         552
##
         589
               1 0
##
         615
               1 0
##
         618
               0 1
##
         655
               0 1
##
         659
               1 0
##
         675
               1 0
##
         686
               1 0
##
         687
               1 0
##
         694
               1 0
##
         702
               1 0
##
         719
               1 0
##
         739
               1 0
               1 0
##
         759
##
         767
               1 0
##
               1 0
         782
##
         786
               1 0
##
         879
               1 0
##
         955
               1 0
##
         980
               1 0
##
         984
               1 0
##
         995
               1 0
##
         999
               1 0
##
               0 1
         1080
##
         1097
               1 0
##
         1110
               1 0
##
         1151 1 0
```

```
##
        1193 1 0
##
        1323 1 0
##
        1499 1 0
##
        1598 1 0
##
        1688 0 1
##
        1907
             1 0
##
        1911
             1 0
##
        2299
             1 0
##
        2308 1 0
##
        3366 1 0
##
        8372 1 0
##
        17764 1 0
boxplot(undercount~poor, data = votes)
```

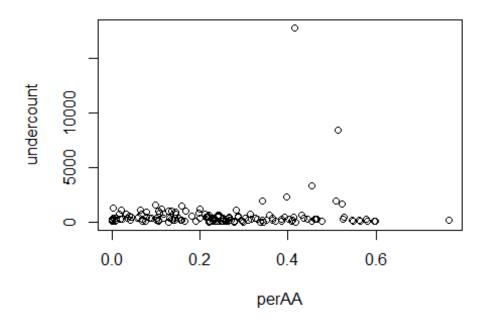


We can also see that the rich use the undercounted equipment much more frequently than the poor people.

```
x2 = xtabs(~equip + poor, data = votes)
p1 = prop.table(x2, margin =1)
p1
##
            poor
## equip
                     0
             0.3918919 0.6081081
##
     LEVER
     OPTICAL 0.7272727 0.2727273
##
##
     PAPER
             0.0000000 1.0000000
##
     PUNCH 0.5882353 0.4117647
```

plotting Undercount against Percent AA shows little correlation, but a few large ouliers can be seen in higher AA populations.

```
plot(undercount~perAA, data = votes)
```



```
x = glm(undercount~perAA, data = votes)
summary(x)
##
## Call:
## glm(formula = undercount ~ perAA, data = votes)
##
## Deviance Residuals:
                      Median
##
       Min
                 1Q
                                    3Q
                                            Max
## -1046.2
             -482.8
                      -221.3
                                  21.9 16963.1
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                  307.0
                                      1.365
                                               0.174
## (Intercept)
                              225.0
## perAA
                 1187.2
                              769.5
                                      1.543
                                               0.125
##
## (Dispersion parameter for gaussian family taken to be 2488210)
##
       Null deviance: 396571504 on 158 degrees of freedom
##
## Residual deviance: 390648953
                                  on 157
                                          degrees of freedom
## AIC: 2796.8
```

```
##
## Number of Fisher Scoring iterations: 2
```

#### Conclusion

In conclusion it is not the poor population that is discriminated against but rather the rich population. When looking at the AA population there is very little correlation to undercounting other than a few outliers.

## (2)Bootstrapping

### **Analyze the 5 ETFs**

Import the ETFs and view the first 5 rows

```
library(mosaic)
## Loading required package: car
## Loading required package: dplyr
##
## Attaching package: 'dplyr'
##
## The following objects are masked from 'package:stats':
##
##
       filter, lag
##
## The following objects are masked from 'package:base':
##
       intersect, setdiff, setequal, union
##
##
## Loading required package: lattice
## Loading required package: ggplot2
## Loading required package: mosaicData
##
## Attaching package: 'mosaic'
##
## The following objects are masked from 'package:dplyr':
##
       count, do, tally
##
##
## The following object is masked from 'package:car':
##
       logit
##
##
## The following objects are masked from 'package:stats':
##
##
       binom.test, cor, cov, D, fivenum, IQR, median, prop.test,
##
       quantile, sd, t.test, var
##
```

```
## The following objects are masked from 'package:base':
##
##
       max, mean, min, prod, range, sample, sum
library(fImport)
## Loading required package: timeDate
## Loading required package: timeSeries
library(foreach)
#Import stocks
stocks = c("SPY", "TLT", "LQD", "EEM", "VNQ")
prices = yahooSeries(stocks, from='2011-01-01', to='2015-08-05')
# The first few rows
head(prices)
## GMT
##
              SPY.Open SPY.High SPY.Low SPY.Close SPY.Volume SPY.Adj.Close
## 2011-01-03
                126.71
                          127.60 125.70
                                             127.05
                                                     138725200
                                                                     115.9587
## 2011-01-04
                127.33
                          127.37
                                  126.19
                                             126.98
                                                     137409700
                                                                     115.8948
## 2011-01-05
                126.58
                          127.72 126.46
                                             127.64
                                                     133975300
                                                                     116.4972
## 2011-01-06
                127.69
                          127.83 127.01
                                             127.39
                                                     122519000
                                                                     116.2690
## 2011-01-07
                127.56
                          127.77
                                  126.15
                                             127.14
                                                     156034600
                                                                     116.0409
## 2011-01-10
                126.58
                          127.16 126.20
                                             126.98
                                                     122401700
                                                                     115.8948
##
              TLT.Open TLT.High TLT.Low TLT.Close TLT.Volume TLT.Adj.Close
## 2011-01-03
                 93.20
                           94.31
                                   92.95
                                              93.41
                                                      13799400
                                                                     81.38658
                 93.41
                           93.77
                                   92.91
                                              93.52
## 2011-01-04
                                                      10466900
                                                                     81.48242
## 2011-01-05
                 92.49
                           92.69
                                   91.19
                                              91.46
                                                                     79.68757
                                                      17568000
                 91.54
                                              91.86
## 2011-01-06
                           92.12
                                   91.16
                                                       9317500
                                                                     80.03609
## 2011-01-07
                 91.51
                           92.63
                                   91.03
                                              92.35
                                                      12694600
                                                                     80.46302
## 2011-01-10
                 92.58
                           92.99
                                   92.18
                                              92.85
                                                       8295200
                                                                     80.89866
##
              LQD.Open LQD.High LQD.Low LQD.Close LQD.Volume LQD.Adj.Close
                108.22
                          108.89
                                  108.06
                                             108.86
                                                       2447900
                                                                     91.64534
## 2011-01-03
                108.98
                          109.19
                                  108.75
                                             109.00
                                                        962500
                                                                     91.76320
## 2011-01-04
                          108.58
## 2011-01-05
                108.51
                                 108.00
                                             108.20
                                                       1209200
                                                                     91.08971
                                             108.36
                108.32
                          108.64
## 2011-01-06
                                  108.20
                                                       1089700
                                                                     91.22441
## 2011-01-07
                108.26
                          109.06
                                  108.20
                                             108.91
                                                       1118600
                                                                     91.68744
## 2011-01-10
                108.78
                          109.20
                                  108.78
                                             109.10
                                                        909200
                                                                     91.84739
##
              EEM.Open EEM.High EEM.Low EEM.Close EEM.Volume EEM.Adj.Close
## 2011-01-03
                 48.03
                           48.31
                                   48.03
                                              48.10
                                                      40116200
                                                                     43.98749
## 2011-01-04
                 48.26
                           48.32
                                   47.75
                                              48.32
                                                      45176300
                                                                     44.18868
## 2011-01-05
                 47.90
                           48.30
                                   47.88
                                              48.20
                                                      47527100
                                                                     44.07894
## 2011-01-06
                 48.01
                           48.04
                                   47.58
                                              47.69
                                                      43977400
                                                                     43.61254
## 2011-01-07
                 47.51
                           47.62
                                   46.93
                                              47.25
                                                      57124900
                                                                     43.21016
## 2011-01-10
                 46.80
                           46.85
                                   46.51
                                              46.76
                                                      62556500
                                                                     42.76206
##
              VNQ.Open VNQ.High VNQ.Low VNQ.Close VNQ.Volume VNQ.Adj.Close
                 55.76
                           56.44
                                              56.40
                                                       1957400
## 2011-01-03
                                   55.73
                                                                     47.62828
## 2011-01-04
                 56.48
                           56.63
                                   55.00
                                              55.32
                                                       2315700
                                                                     46.71624
                 55.15
                           55.63
                                   55.14
                                              55.52
                                                       1512700
## 2011-01-05
                                                                     46.88514
```

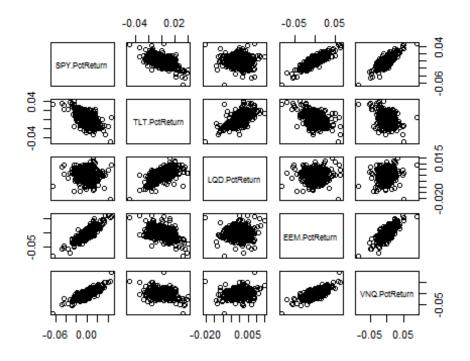
```
## 2011-01-06
                 55.67
                           55.68
                                   54.97
                                              55.00
                                                       1819300
                                                                     46.44601
                                                       2001200
## 2011-01-07
                 55.29
                           55.45
                                   54.50
                                              55.02
                                                                     46.46290
## 2011-01-10
                 54.80
                           55.14
                                   54.42
                                              55.01
                                                                     46.45446
                                                       1668400
```

Creat a function to calculate daily returns of each ETF and then compute the returns

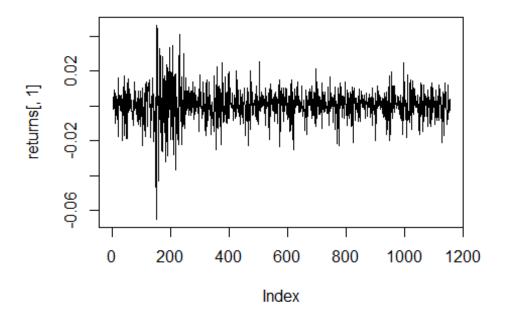
```
YahooPricesToReturns = function(series) {
    mycols = grep('Adj.Close', colnames(series))
    closingprice = series[,mycols]
    N = nrow(closingprice)
    percentreturn = as.data.frame(closingprice[2:N,]) /
as.data.frame(closingprice[1:(N-1),]) - 1
    mynames = strsplit(colnames(percentreturn), '.', fixed=TRUE)
    mynames = lapply(mynames, function(x) return(paste0(x[1], ".PctReturn")))
    colnames(percentreturn) = mynames
    as.matrix(na.omit(percentreturn))
}
```

Plot the Returns

```
pairs(returns)
```



```
plot(returns[,1], type='l')
```



Calculate the betas of each stock determined against the market to see which investments are riskier

```
lm_TLT = lm(returns[,2]~returns[,1])
lm_LQD = lm(returns[,3]~returns[,1])
lm_EEM = lm(returns[,4]~returns[,1])
lm_VNQ = lm(returns[,5]~returns[,1])
coef(lm_TLT); coef(lm_LQD); coef(lm_EEM); coef(lm_VNQ)
##
     (Intercept) returns[, 1]
##
    0.0007091476 -0.5626797102
##
     (Intercept) returns[, 1]
    0.0002286759 -0.0428450593
##
##
     (Intercept)
                  returns[, 1]
## -0.0007506257
                  1.2301980660
##
     (Intercept)
                  returns[, 1]
## -0.0000285025
                  0.9399541016
```

Look at the residuals and their correlations

```
residuals = cbind(resid(lm_TLT), resid(lm_LQD), resid(lm_EEM), resid(lm_VNQ))
cor(residuals)
```

```
## [,1] [,2] [,3] [,4]

## [1,] 1.00000000 0.7888184 0.03087361 0.2707303

## [2,] 0.78881844 1.0000000 0.17097169 0.3493197

## [3,] 0.03087361 0.1709717 1.00000000 0.1280538

## [4,] 0.27073031 0.3493197 0.12805378 1.0000000
```

## **Evenly weighted Portfolio:**

Set the seed and simulate performance for the safe portfolio. Here the bonds are evenly weighted

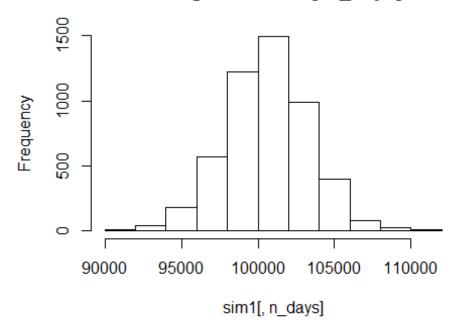
```
n_days=20
set.seed(3000)
sim1 = foreach(i=1:5000, .combine='rbind') %do% {
    totalwealth = 100000
    weights = c(0.2, 0.2, 0.2, 0.2, 0.2)
    holdings = weights * totalwealth
    wealthtracker = rep(0, n_days) # Set up a placeholder to track total
wealth
    for(today in 1:n days) {
        return.today = resample(returns, 1, orig.ids=FALSE)
        holdings = holdings + holdings*return.today
        totalwealth = sum(holdings)
        holdings = weights * totalwealth
        wealthtracker[today] = totalwealth
    }
    wealthtracker
```

Show a selection of Sim1 and plot a histogram of the wealth over 20 days

```
head(sim1)
##
                 [,1]
                           [,2]
                                     [,3]
                                                [,4]
                                                          [,5]
                                                                    [6,]
## result.1 100842.83 100807.89 100843.17 100762.98 101161.61 100701.92
## result.2 100694.09 99254.37 99456.25
                                           98838.68 99004.05
                                                               98161.16
## result.3 100035.54 100094.03 100571.32 100188.06 100506.17 100278.45
## result.4 100507.56 100809.95 100417.39 100931.90 101408.68 101130.46
## result.5 100123.68 100640.27 100986.27 101252.22 100583.03
                                                               99696.41
## result.6 99623.42 99608.27 100005.36 100844.86 100615.73
                                                               99591.93
##
                 [,7]
                           [,8]
                                     [,9]
                                              [,10]
                                                         \lceil ,11 \rceil
                                                                   \lceil ,12 \rceil
## result.1 100961.95 100710.30 100564.89 100618.91 100304.57 100498.93
## result.2 98235.78 98357.28 98027.71
                                          97528.78
                                                     97982.59
                                                               97877.37
## result.3 99826.58 99647.02 99619.48 98939.66 99314.46
                                                              99202.39
## result.4 101489.06 101349.91 101979.57 101991.98 102081.31 102119.02
## result.5 100165.18 100250.51 99830.64 99937.80 99201.50
                                                               99631.72
## result.6 99992.18 100027.65 99887.70 99536.16 99335.44 99109.74
                [,13]
                          [,14]
                                    [,15]
                                              [,16]
                                                         [,17]
                                                                   [,18]
## result.1 100695.81 100127.85 99789.70 100093.82 100470.40 100429.44
## result.2 98457.40 97822.33 97784.23 96980.26 97751.36 98001.20
```

```
## result.3 98964.03
                       98092.31 98548.75
                                           98521.51
                                                     98676.78
                                                               98536.17
## result.4 102905.05 102857.38 102746.25 101569.24 101989.80 102049.98
## result.5 99996.01 100433.81 100623.57 100283.38
                                                     99274.82
                                                               99311.50
            99487.26
                       99459.95
                                 99126.93
                                                     98094.09
## result.6
                                           98768.10
                                                               97428.78
                [,19]
##
                          [,20]
## result.1
            99349.71
                       99498.96
## result.2
            97978.77
                       97370.37
## result.3
            98176.61
                       98209.07
## result.4 102263.94 102222.25
## result.5 100053.44 100646.38
## result.6 96710.97
                       97117.65
hist(sim1[,n_days])
```

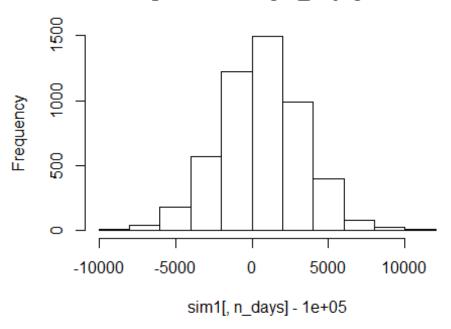
## Histogram of sim1[, n\_days]



plot a histogram showing profit/loss over the 20 days

hist(sim1[,n\_days]- 100000)

## Histogram of sim1[, n\_days] - 1e+05



Calculate 5% value at risk

```
quantile(sim1[,n_days], 0.05) - 100000
## 5%
## -3869.17
```

Show the average profit or loss

```
mean(sim1[,n_days]- 100000)
## [1] 618.9943
```

#### Safe Portfolio

Set the seed and simulate performance for the safe portfolio. Here the bonds have been heavily weigted

```
set.seed(4000)

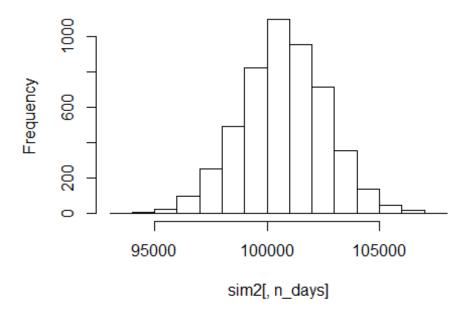
sim2 = foreach(i=1:5000, .combine='rbind') %do% {
    totalwealth = 100000
    weights = c(0.15, 0.3, 0.4, 0.0, 0.15)
    holdings = weights * totalwealth
    wealthtracker = rep(0, n_days) # Set up a placeholder to track total
wealth
    for(today in 1:n_days) {
        return.today = resample(returns, 1, orig.ids=FALSE)
```

```
holdings = holdings + holdings*return.today
    totalwealth = sum(holdings)
    holdings = weights * totalwealth
    wealthtracker[today] = totalwealth
}
wealthtracker
}
```

Show a selection of Sim2 and plot a histogram of the wealth over 20 days

```
head(sim2)
                 [,1]
##
                                                          [,5]
                           [,2]
                                     [,3]
                                               [,4]
                                                                    [,6]
## result.1 100442.76 100730.17 100725.81 101085.10 100708.31 100661.34
## result.2 100737.43 100878.77 101143.97 100827.06 101024.26 100842.55
## result.3 99303.76
                       99040.03 98752.78
                                           98297.78
                                                     98786.38
## result.4 100384.21 100565.02 100190.17 100784.47 101342.15 101563.36
## result.5 100162.49 100117.57
                                 99991.39
                                           99684.05
                                                     99777.25
                                                                99811.77
## result.6 100151.03
                       99963.78 100400.43 100980.61 100552.75 100840.47
##
                                     [,9]
                 [,7]
                           [8,]
                                               [,10]
                                                         [,11]
## result.1 101005.36 100936.26 100638.48 100826.11 100525.77 100435.80
## result.2 100817.24 100824.81 101162.39 101946.97 101304.65 101364.11
## result.3 98363.90
                       98512.89 98543.20
                                           98347.81
                                                     98216.66
## result.4 100923.46 101393.27 101839.20 102330.46 102775.77 103111.03
## result.5 99506.18
                       99807.34 99816.33
                                           99373.98
                                                     99507.36
## result.6 101073.80 101209.46 101326.76 101540.99 101206.85 101230.32
                [,13]
##
                          [,14]
                                    [,15]
                                               [,16]
                                                         [,17]
## result.1 100632.23 100105.05 100382.90 100388.20 100582.88 101301.50
## result.2 102153.95 102478.92 103193.41 102976.79 102800.60 103031.71
## result.3 98155.65
                       98306.50 98279.83
                                           97618.74 97730.57
                                                               97730.30
## result.4 103216.85 103022.95 103296.97 103433.82 103666.35 103692.58
## result.5 99396.68
                       99030.88 98884.85
                                           97979.98 97706.00
## result.6 101936.11 101457.89 101356.30 101675.97 101401.39 101550.43
                          [,20]
##
                [,19]
## result.1 101048.14 100433.33
## result.2 103018.87 103373.32
## result.3 97874.27
                       98030.42
## result.4 103944.47 104100.60
                       97957.72
## result.5 97910.35
## result.6 102043.03 102749.30
hist(sim2[,n_days])
```

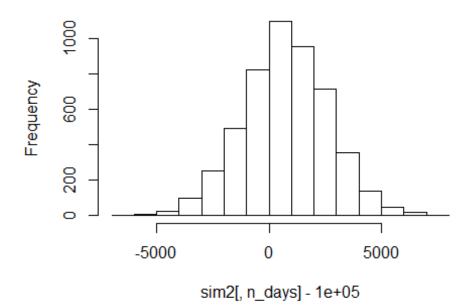
# Histogram of sim2[, n\_days]



plot a histogram showing profit/loss over the 20 days

hist(sim2[,n\_days] - 100000)

# Histogram of sim2[, n\_days] - 1e+05



Calculate 5% value at risk

```
quantile(sim2[,n_days], 0.05) - 100000
## 5%
## -2417.244
```

Show the average profit or loss

```
#Average Profit/Loss
mean(sim2[,n_days]- 100000)
## [1] 734.0009
```

#### **Risky Portfolio**

Set the seed and simulate performance for the risky portfolio. Here the bonds have been excluded

```
set.seed(2000)

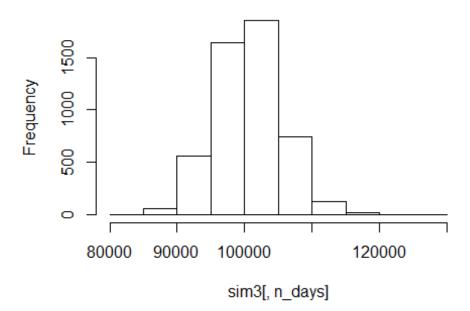
sim3 = foreach(i=1:5000, .combine='rbind') %do% {
    totalwealth = 100000
    weights = c(0.4, 0.0, 0.0, 0.4, 0.2)
    holdings = weights * totalwealth
    wealthtracker = rep(0, n_days) # Set up a placeholder to track total
wealth
    for(today in 1:n_days) {
        return.today = resample(returns, 1, orig.ids=FALSE)
        holdings = holdings + holdings*return.today
        totalwealth = sum(holdings)
        holdings = weights * totalwealth
        wealthtracker[today] = totalwealth
    }
    wealthtracker
}
```

Show a selection of Sim3 and plot a histogram of the wealth over 20 days

```
head(sim3)
##
                 [,1]
                          [,2]
                                             [,4]
                                   [,3]
                                                       [,5]
                                                                 [,6]
## result.1 99880.23 100184.5 100017.2 100458.52 101228.9 100881.73
## result.2 101058.13 101232.2 101793.4 101916.34 102147.7 102571.41
## result.3 99817.85 100037.2 100036.0 101475.47 101232.1 100535.13
## result.4 103068.40 101579.9 102820.5 101010.85 100117.7 99469.64
## result.5 101240.73 101678.6 99453.0 99973.87 97625.2 97651.87
## result.6 100941.02 100487.9 100083.9 99786.64 101138.2 101015.01
                                     [,9]
##
                                              [,10]
                 [,7]
                           [8,]
                                                         \lceil ,11 \rceil
## result.1 100010.42 99775.03 101503.20 101546.61 101648.38 104827.07
## result.2 102658.24 102779.24 101071.49 100270.67 99159.27 101093.51
## result.3 100445.79 99933.25 98876.02 99418.32 99118.70 98061.48
```

```
## result.4 99967.10 101444.45 102918.13 103670.91 102957.16 102449.68
## result.5 97830.44 97608.02 97571.06
                                           97395.53
                                                     98389.70
                                                               97996.39
## result.6 105837.69 104193.51 104422.44 104478.70 103075.06 104287.14
##
                [,13]
                          [,14]
                                    [,15]
                                                         [,17]
                                               [,16]
                                                                   [,18]
## result.1 105488.89 104689.64 102435.51 102386.41 102468.40 103490.39
## result.2 101123.72 101465.81 100815.20 101227.69 101257.94 102329.38
## result.3 96881.90
                       97980.65
                                 98390.16
                                           99041.60
                                                     99709.57 100147.67
## result.4 102199.06 102093.22 102870.53 104506.71 104739.68 104145.47
                       98828.27
                                 97126.38
                                           98523.94
## result.5
             98593.56
                                                     99362.55
## result.6 104206.42 104989.69 103667.83 104575.87 105483.84 105498.81
##
                [,19]
                          [,20]
## result.1 104561.39 105785.17
## result.2 102420.84 101334.10
## result.3 99854.57 101031.83
## result.4 104000.14 104853.38
## result.5 98516.27
                       99574.83
## result.6 106494.24 105646.79
hist(sim3[,n_days])
```

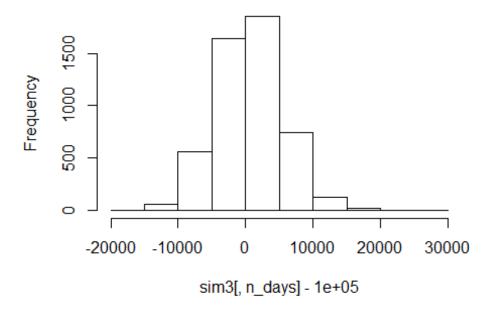
## Histogram of sim3[, n\_days]



plot a histogram showing profit/loss over the 20 days

```
hist(sim3[,n_days] - 100000)
```

## Histogram of sim3[, n\_days] - 1e+05



Calculate 5% value at risk

Show the average profit or loss

```
mean(sim3[,n_days]- 100000)
## [1] 593.1402
```

#### Conclusion

Ultimately the safe portfolio had the best average return and the least amount at stake. The risky portfoli had the worst return and the most money at stake.

## (3) Clustering And PCA

## **Red Vs White Clustering**

Read in the data

```
wine = read.csv('wine.csv')
head(wine)
##
     fixed.acidity volatile.acidity citric.acid residual.sugar chlorides
## 1
               7.4
                                0.70
                                            0.00
                                                             1.9
                                                                     0.076
## 2
               7.8
                                0.88
                                            0.00
                                                             2.6
                                                                     0.098
## 3
               7.8
                                                                     0.092
                                0.76
                                             0.04
                                                             2.3
## 4
              11.2
                                0.28
                                            0.56
                                                             1.9
                                                                     0.075
## 5
               7.4
                                0.70
                                             0.00
                                                             1.9
                                                                     0.076
## 6
               7.4
                                0.66
                                                             1.8
                                                                     0.075
                                             0.00
     free.sulfur.dioxide total.sulfur.dioxide density
                                                          pH sulphates alcohol
## 1
                      11
                                             34 0.9978 3.51
                                                                  0.56
                                                                            9.4
## 2
                       25
                                            67
                                                0.9968 3.20
                                                                  0.68
                                                                            9.8
## 3
                      15
                                                0.9970 3.26
                                                                  0.65
                                                                            9.8
                                            54
## 4
                       17
                                            60
                                                0.9980 3.16
                                                                  0.58
                                                                            9.8
## 5
                       11
                                                                  0.56
                                                                           9.4
                                             34 0.9978 3.51
                                            40 0.9978 3.51
                                                                            9.4
## 6
                       13
                                                                  0.56
##
     quality color
## 1
           5
               red
## 2
           5
               red
           5
## 3
               red
## 4
           6
               red
           5
## 5
               red
           5
## 6
               red
names(wine)
                                                        "citric.acid"
## [1] "fixed.acidity"
                                "volatile.acidity"
## [4] "residual.sugar"
                                "chlorides"
                                                        "free.sulfur.dioxide"
                                                        "pH"
## [7] "total.sulfur.dioxide" "density"
## [10] "sulphates"
                                "alcohol"
                                                        "quality"
## [13] "color"
```

Remove the last 2 columns and scale the data.

```
wine_num = wine [,(1:11)]
wine_scaled = scale(wine_num, center = TRUE, scale = TRUE)
```

cluster the data using 2 centers, in hopes of distinguishing red from white wine.

```
cluster_2 = kmeans(wine_scaled, centers = 2, nstart = 50)
```

Capture the mean and SD of the scaled data.

```
sigma = attr(wine_scaled, "scaled:scale")
mu = attr(wine_scaled, "scaled:center")
```

Unscale the data.

```
cluster_2$center
```

```
fixed.acidity volatile.acidity citric.acid residual.sugar chlorides
## 1
        -0.2804833
                          -0.3953082
                                       0.1143429
                                                       0.1998380 -0.3119753
         0.8286464
                           1.1678795 -0.3378091
## 2
                                                      -0.5903919 0.9216848
##
     free.sulfur.dioxide total.sulfur.dioxide
                                                                   рН
                                                  density
                                     0.4018607 -0.2306934 -0.1920315
## 1
               0.2814861
## 2
                                    -1.1872380 0.6815493 0.5673286
              -0.8316090
##
      sulphates
                    alcohol
## 1 -0.2853595
                 0.02562065
## 2 0.8430523 -0.07569241
cluster_2$center[1,]*sigma + mu
##
          fixed.acidity
                             volatile.acidity
                                                        citric.acid
##
             6.85167903
                                   0.27458385
                                                         0.33524928
##
         residual.sugar
                                    chlorides free.sulfur.dioxide
##
             6.39402555
                                   0.04510424
                                                        35.52152864
## total.sulfur.dioxide
                                      density
                                                                 pН
##
           138.45848785
                                   0.99400486
                                                         3.18762464
##
              sulphates
                                      alcohol
##
             0.48880511
                                  10.52235888
cluster 2$center[2,]*sigma + mu
                                                        citric.acid
##
          fixed.acidity
                             volatile.acidity
##
              8.2895922
                                    0.5319416
                                                          0.2695435
                                    chlorides free.sulfur.dioxide
##
         residual.sugar
##
                                                         15.7647596
              2.6342666
                                    0.0883238
## total.sulfur.dioxide
                                      density
                                                                 рΗ
                                                          3.3097200
##
             48.6396835
                                    0.9967404
##
              sulphates
                                      alcohol
##
              0.6567194
                                   10.4015216
```

See which wines are in each cluster

```
which(cluster_2$cluster == 1)
##
            50 355 495 592 635 650 837 838 1018 1019 1080 1082 1091
      [1]
     [14] 1115 1132 1157 1236 1245 1287 1390 1457 1491 1567 1575 1600 1601
##
##
     [27] 1602 1603 1604 1605 1606 1607 1608 1609 1610 1611 1612 1613 1614
##
     [40] 1615 1616 1617 1618 1619 1620 1621 1622 1623 1624 1625 1626 1627
##
     [53] 1628 1629 1630 1631 1632 1633 1634 1635 1636 1637 1638 1639 1640
##
     [66] 1641 1642 1643 1644 1645 1646 1647 1648 1649 1650 1651 1652 1653
     [79] 1654 1655 1656 1657 1658 1659 1660 1661 1662 1663 1664 1665 1666
##
     [92] 1667 1668 1669 1670 1671 1672 1673 1674 1675 1676 1677 1678 1679
##
    [105] 1680 1681 1682 1683 1684 1685 1686 1687 1688 1689 1690 1691 1692
##
    [118] 1693 1694 1695 1696 1697 1698 1699 1700 1701 1702 1703 1704 1705
##
    [131] 1706 1707 1708 1709 1710 1711 1712 1713 1714 1716 1717 1718 1719
    [144] 1720 1721 1722 1723 1724 1725 1726 1727 1728 1729 1730 1731 1732
##
    [157] 1733 1734 1735 1736 1737 1738 1739 1740 1741 1742 1743 1744 1745
    [170] 1746 1748 1749 1750 1751 1752 1753 1755 1756 1757 1758 1759 1760
##
    [183] 1761 1762 1763 1764 1765 1766 1767 1768 1769 1770 1771 1772 1773
## [196] 1774 1775 1776 1777 1778 1779 1780 1781 1782 1783 1784 1785 1786
```

```
[209] 1787 1788 1789 1790 1791 1792 1793 1794 1795 1796 1797 1798 1799
    [222] 1800 1801 1802 1803 1804 1805 1806 1807 1809 1810 1811 1812 1813
    [235] 1814 1815 1816 1817 1818 1819 1820 1821 1822 1823 1824 1825 1826
##
    [248] 1827 1828 1829 1831 1832 1833 1834 1835 1836 1837 1838 1839 1840
    [261] 1841 1842 1843 1844 1845 1846 1847 1848 1849 1850 1851 1852 1853
    [274] 1854 1855 1856 1857 1858 1859 1860 1861 1862 1863 1864 1865 1866
##
    [287] 1867 1868 1869 1870 1871 1872 1873 1874 1875 1876 1877 1878 1879
    [300] 1880 1881 1882 1883 1884 1885 1886 1887 1888 1889 1890 1891 1892
##
    [313] 1893 1894 1895 1896 1897 1898 1899 1900 1901 1902 1903 1904 1905
    [326] 1906 1907 1908 1909 1910 1911 1912 1913 1914 1915 1916 1917 1918
##
    [339] 1919 1920 1921 1922 1923 1924 1925 1926 1927 1928 1929 1930 1931
    [352] 1932 1933 1934 1935 1936 1937 1938 1939 1940 1941 1942 1943 1944
##
    [365] 1945 1946 1947 1948 1949 1950 1951 1952 1953 1954 1955 1956 1957
##
    [378] 1958 1959 1960 1961 1962 1963 1964 1965 1966 1967 1968 1969 1970
    [391] 1971 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984
##
    [404] 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997
##
    [417] 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010
    [430] 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023
    [443] 2024 2025 2026 2027 2028 2029 2030 2031 2032 2034 2035 2036 2037
##
##
    [456] 2038 2039 2040 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050
##
    [469] 2051 2052 2053 2054 2055 2056 2057 2058 2059 2060 2061 2062 2063
    [482] 2064 2065 2066 2067 2068 2069 2070 2071 2072 2073 2074 2075 2076
##
    [495] 2077 2078 2079 2080 2081 2082 2083 2085 2086 2087 2088 2089 2090
##
    [508] 2091 2092 2093 2094 2095 2096 2097 2098 2099 2100 2101 2102 2103
    [521] 2104 2105 2107 2108 2109 2110 2111 2112 2113 2114 2115 2116 2117
    [534] 2118 2119 2120 2121 2122 2123 2124 2125 2126 2127 2128 2129 2130
    [547] 2131 2132 2133 2134 2135 2136 2137 2138 2139 2140 2141 2142 2143
    [560] 2144 2145 2146 2147 2148 2149 2150 2151 2152 2153 2154 2155 2156
##
##
    [573] 2157 2158 2159 2160 2161 2162 2163 2164 2165 2166 2167 2168 2169
    [586] 2170 2171 2172 2173 2174 2175 2176 2177 2178 2179 2180 2181 2182
    [599] 2183 2184 2185 2186 2187 2188 2189 2190 2191 2192 2193 2194 2195
##
    [612] 2196 2197 2198 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208
##
    [625] 2210 2211 2212 2213 2214 2215 2216 2217 2218 2219 2220 2221 2222
    [638] 2223 2224 2225 2227 2228 2229 2230 2231 2232 2233 2234 2235 2236
    [651] 2237 2238 2239 2240 2241 2242 2243 2244 2245 2246 2247 2248 2249
##
    [664] 2250 2251 2252 2253 2254 2255 2256 2257 2258 2259 2260 2261 2263
    [677] 2264 2265 2266 2267 2268 2269 2270 2271 2272 2273 2274 2275 2276
##
##
    [690] 2277 2278 2279 2280 2281 2282 2283 2284 2285 2286 2288 2289 2290
    [703] 2291 2292 2293 2294 2295 2296 2297 2298 2299 2300 2301 2302 2303
    [716] 2304 2305 2306 2307 2308 2309 2310 2311 2312 2313 2314 2315 2316
##
    [729] 2317 2318 2319 2320 2321 2322 2323 2324 2325 2326 2327 2328 2329
    [742] 2330 2331 2332 2333 2334 2335 2336 2337 2338 2339 2340 2341 2342
##
    [755] 2343 2344 2345 2346 2347 2348 2349 2350 2351 2352 2353 2354 2355
    [768] 2356 2357 2358 2359 2360 2361 2362 2363 2364 2365 2366 2367 2368
##
    [781] 2369 2370 2371 2372 2373 2374 2375 2376 2377 2378 2379 2380 2381
##
    [794] 2382 2383 2384 2385 2386 2387 2388 2389 2390 2391 2392 2393 2394
##
    [807] 2395 2396 2397 2398 2399 2400 2401 2402 2403 2404 2405 2406 2407
   [820] 2408 2409 2410 2411 2412 2413 2414 2415 2416 2417 2418 2419 2420
   [833] 2421 2422 2423 2424 2425 2426 2427 2428 2429 2431 2432 2433 2435
## [846] 2436 2437 2438 2439 2440 2441 2442 2443 2444 2445 2446 2447 2448
```

```
[859] 2449 2450 2451 2452 2453 2454 2455 2456 2457 2458 2459 2460 2461
    [872] 2462 2463 2464 2465 2466 2467 2468 2469 2470 2471 2472 2473 2474
   [885] 2475 2476 2477 2478 2479 2480 2481 2482 2483 2484 2485 2486 2487
##
    [898] 2488 2489 2490 2491 2492 2493 2494 2495 2496 2497 2498 2499 2500
   [911] 2501 2502 2503 2504 2505 2506 2507 2508 2509 2510 2511 2512 2513
    [924] 2514 2515 2516 2517 2518 2519 2520 2521 2522 2523 2524 2525 2526
   [937] 2527 2528 2529 2530 2531 2532 2533 2534 2535 2536 2537 2538 2539
##
  [950] 2540 2541 2542 2543 2544 2545 2546 2547 2549 2550 2551 2552 2553
  [963] 2554 2555 2556 2557 2558 2559 2560 2561 2562 2563 2564 2565 2566
## [976] 2567 2568 2569 2570 2571 2572 2573 2574 2575 2576 2577 2578 2579
## [989] 2580 2581 2582 2583 2584 2585 2586 2587 2588 2589 2590 2591 2592
## [1002] 2593 2594 2595 2596 2597 2598 2599 2600 2601 2602 2603 2604 2605
## [1015] 2606 2607 2608 2609 2610 2611 2612 2613 2614 2615 2616 2617 2618
## [1028] 2619 2620 2621 2622 2623 2624 2625 2626 2627 2628 2629 2630 2631
## [1041] 2632 2633 2635 2638 2639 2641 2642 2643 2644 2645 2646 2647 2648
## [1054] 2649 2650 2651 2652 2653 2654 2655 2656 2657 2658 2659 2660 2661
## [1067] 2662 2663 2664 2665 2666 2667 2668 2669 2670 2671 2672 2673 2674
## [1080] 2675 2676 2677 2678 2679 2680 2681 2682 2683 2684 2685 2686 2687
## [1093] 2688 2689 2690 2691 2692 2693 2694 2695 2696 2697 2698 2699 2700
## [1106] 2701 2702 2703 2704 2705 2706 2707 2708 2709 2710 2711 2712 2713
## [1119] 2715 2716 2717 2718 2719 2720 2721 2722 2723 2724 2725 2726 2727
## [1132] 2728 2729 2730 2731 2732 2733 2734 2735 2736 2737 2738 2739 2740
## [1145] 2741 2742 2743 2744 2745 2746 2747 2748 2749 2750 2751 2753 2754
## [1158] 2755 2756 2757 2758 2759 2760 2761 2762 2763 2764 2765 2766 2767
## [1171] 2768 2769 2770 2771 2772 2773 2774 2775 2776 2777 2778 2779 2780
## [1184] 2781 2782 2783 2784 2785 2786 2787 2788 2789 2790 2791 2792 2793
## [1197] 2794 2795 2796 2797 2798 2799 2800 2801 2802 2803 2804 2805 2806
## [1210] 2807 2808 2809 2810 2811 2812 2813 2814 2815 2816 2818 2819 2820
## [1223] 2821 2822 2823 2824 2825 2826 2827 2828 2829 2830 2831 2832 2833
## [1236] 2834 2835 2836 2837 2838 2839 2840 2841 2842 2843 2844 2845 2846
## [1249] 2847 2848 2849 2850 2851 2852 2853 2854 2855 2856 2857 2858 2859
## [1262] 2860 2861 2862 2863 2864 2865 2866 2867 2868 2869 2870 2871 2873
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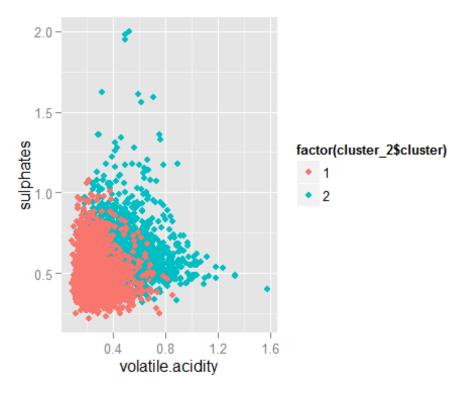
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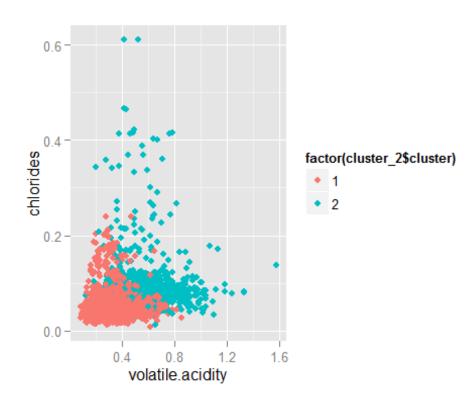
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## [1275] 1294 1295 1296 1297 1298 1299 1300 1301 1302 1303 1304 1305 1306
## [1288] 1307 1308 1309 1310 1311 1312 1313 1314 1315 1316 1317 1318 1319
## [1301] 1320 1321 1322 1323 1324 1325 1326 1327 1328 1329 1330 1331 1332
## [1314] 1333 1334 1335 1336 1337 1338 1339 1340 1341 1342 1343 1344 1345
## [1327] 1346 1347 1348 1349 1350 1351 1352 1353 1354 1355 1356 1357 1358
## [1340] 1359 1360 1361 1362 1363 1364 1365 1366 1367 1368 1369 1370 1371
## [1353] 1372 1373 1374 1375 1376 1377 1378 1379 1380 1381 1382 1383 1384
## [1366] 1385 1386 1387 1388 1389 1391 1392 1393 1394 1395 1396 1397 1398
## [1379] 1399 1400 1401 1402 1403 1404 1405 1406 1407 1408 1409 1410 1411
## [1392] 1412 1413 1414 1415 1416 1417 1418 1419 1420 1421 1422 1423 1424
## [1405] 1425 1426 1427 1428 1429 1430 1431 1432 1433 1434 1435 1436 1437
## [1418] 1438 1439 1440 1441 1442 1443 1444 1445 1446 1447 1448 1449 1450
## [1431] 1451 1452 1453 1454 1455 1456 1458 1459 1460 1461 1462 1463 1464
## [1444] 1465 1466 1467 1468 1469 1470 1471 1472 1473 1474 1475 1476 1477
## [1457] 1478 1479 1480 1481 1482 1483 1484 1485 1486 1487 1488 1489 1490
## [1470] 1492 1493 1494 1495 1496 1497 1498 1499 1500 1501 1502 1503 1504
## [1483] 1505 1506 1507 1508 1509 1510 1511 1512 1513 1514 1515 1516 1517
## [1496] 1518 1519 1520 1521 1522 1523 1524 1525 1526 1527 1528 1529 1530
## [1509] 1531 1532 1533 1534 1535 1536 1537 1538 1539 1540 1541 1542 1543
## [1522] 1544 1545 1546 1547 1548 1549 1550 1551 1552 1553 1554 1555 1556
## [1535] 1557 1558 1559 1560 1561 1562 1563 1564 1565 1566 1568 1569 1570
## [1548] 1571 1572 1573 1574 1576 1577 1578 1579 1580 1581 1582 1583 1584
## [1561] 1585 1586 1587 1588 1589 1590 1591 1592 1593 1594 1595 1596 1597
## [1574] 1598 1599 1715 1747 1754 1808 1830 1972 2033 2084 2106 2209 2226
## [1587] 2262 2287 2430 2434 2548 2634 2636 2637 2640 2714 2752 2817 2872
## [1600] 2986 2994 3096 3177 3308 3383 3435 3456 3465 3526 3532 3551 3624
## [1613] 3626 3692 3754 3762 3786 3949 3973 4022 4024 4075 4090 4189 4194
## [1626] 4242 4268 4381 4449 5171 5262 5448 5501 5572 5639 5813 5916 6073
## [1639] 6249 6250 6392 6415 6445
```

Plots displaying cluster association

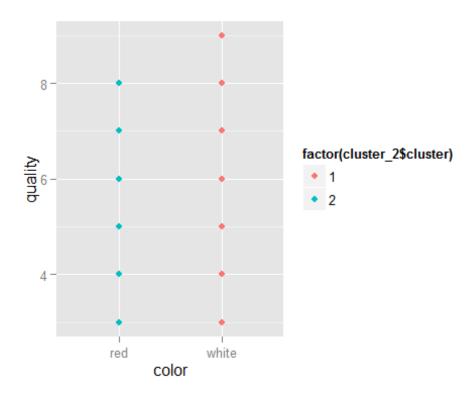
qplot(volatile.acidity, sulphates, data=wine, color=factor(cluster 2\$cluster))



qplot(volatile.acidity, chlorides, data=wine,
color=factor(cluster\_2\$cluster))



qplot(color, quality, data=wine, color=factor(cluster\_2\$cluster))



qplot(wine\$color, cluster\_2\$cluster, data=wine,
color=factor(cluster\_2\$cluster))

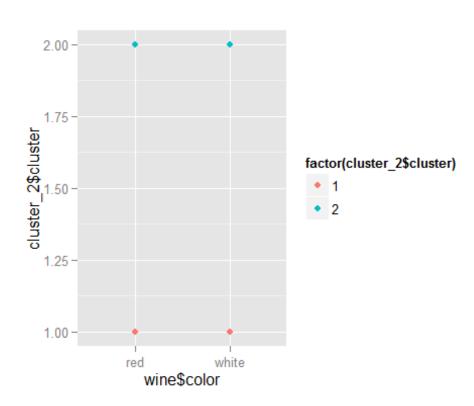


Table displaying that the clusters successfully split red and white wine with minimal error

```
table(wine$color,cluster_2$cluster)

##

##

1 2

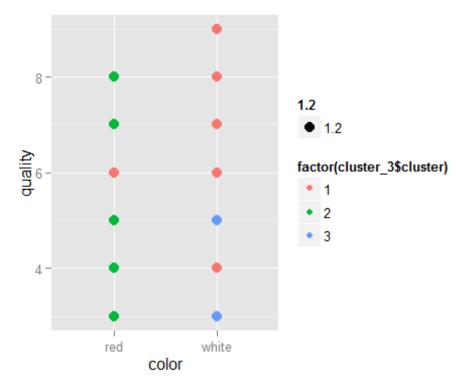
## red 24 1575

## white 4830 68
```

## **Quality Clustering**

Cluster the data into 3 groups

```
cluster_3 = kmeans(wine_scaled, centers = 3, nstart = 50)
table(wine$quality,cluster_3$cluster)
##
##
          1
               2
                     3
     3
          8
              10
                    12
##
     4
##
        100
              68
                    48
     5
##
       638
             696
                   804
##
     6 1371
             622
                   843
##
     7
        740
             182
                   157
##
     8
        148
              15
                    30
     9
               0
                     1
##
          4
qplot(color, quality, data=wine, color=factor(cluster_3$cluster),cex= 1.2)
```



It is dificult to

distinguish the wines by quality.

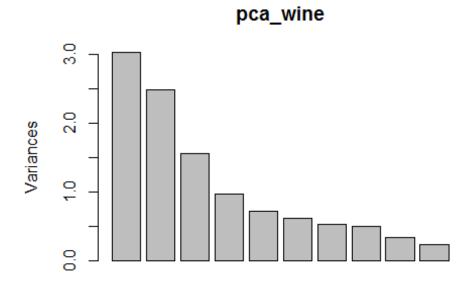
#### **PCA**

Run PCA on the data and look at the result

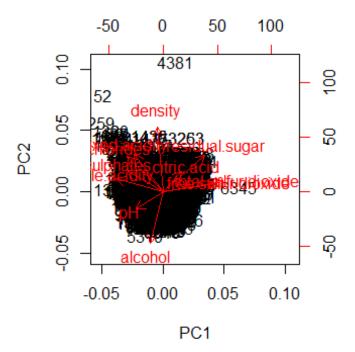
```
pca wine = prcomp(wine num, scale.=TRUE)
pca_wine
## Standard deviations:
   [1] 1.7406518 1.5791852 1.2475364 0.9851660 0.8484544 0.7793021 0.7232971
##
   [8] 0.7081739 0.5805377 0.4771748 0.1811927
##
## Rotation:
##
                              PC1
                                          PC2
                                                      PC3
                                                                 PC4
## fixed.acidity
                       -0.23879890
                                   0.33635454 -0.43430130
                                                          0.16434621
## volatile.acidity
                       -0.38075750
                                   0.11754972 0.30725942
                                                          0.21278489
## citric.acid
                        0.15238844
                                   0.18329940 -0.59056967 -0.26430031
## residual.sugar
                                   0.32991418 0.16468843
                        0.34591993
                                                          0.16744301
## chlorides
                       -0.29011259
                                   0.31525799 0.01667910 -0.24474386
## free.sulfur.dioxide
                        0.43091401
                                   0.07193260 0.13422395 -0.35727894
## total.sulfur.dioxide 0.48741806
                                   ## density
                       -0.04493664
                                   0.58403734 0.17560555
                                                          0.07272496
## pH
                       -0.21868644 -0.15586900 0.45532412 -0.41455110
## sulphates
                                   0.19171577 -0.07004248 -0.64053571
                       -0.29413517
## alcohol
                       -0.10643712 -0.46505769 -0.26110053 -0.10680270
##
                             PC5
                                         PC<sub>6</sub>
                                                     PC7
                                                                 PC8
## fixed.acidity
                       -0.1474804 -0.20455371 -0.28307944
                                                         0.401235645
## volatile.acidity
                        0.1514560 -0.49214307 -0.38915976 -0.087435088
## citric.acid
                                  0.22763380 -0.38128504 -0.293412336
                       -0.1553487
## residual.sugar
                       -0.3533619 -0.23347775 0.21797554 -0.524872935
## chlorides
                        0.6143911
                                  0.16097639 -0.04606816 -0.471516850
## free.sulfur.dioxide
                        0.2235323 -0.34005140 -0.29936325
                                                         0.207807585
## total.sulfur.dioxide
                        0.1581336 -0.15127722 -0.13891032
                                                          0.128621319
## density
                       -0.3065613 0.01874307 -0.04675897
                                                          0.004831136
## pH
                       ## sulphates
                       -0.1365769 -0.29692579
                                              0.52534311
                                                          0.165818022
## alcohol
                       -0.1888920 -0.51837780 -0.10410343 -0.399233887
##
                             PC9
                                         PC10
                                                       PC11
## fixed.acidity
                        0.3440567 -0.281267685 -0.3346792663
## volatile.acidity
                       -0.4969327
                                  0.152176731 -0.0847718098
## citric.acid
                       -0.4026887
                                  0.234463340 0.0011089514
## residual.sugar
                        0.1080032 -0.001372773 -0.4497650778
## chlorides
                        0.2964437 -0.196630217 -0.0434375867
## free.sulfur.dioxide
                        0.3666563 0.480243340 0.0002125351
## total.sulfur.dioxide -0.3206955 -0.713663486
                                               0.0626848131
## density
                        0.1128800 -0.003908289
                                               0.7151620723
## pH
                        0.1278367 -0.141310977 -0.2063605036
## sulphates
                       ## alcohol
                        0.2518903 -0.205053085 0.3357018784
```

See some statistics and plots related to the PCA

```
summary(pca_wine)
## Importance of components:
                             PC1
                                    PC2
                                            PC3
                                                    PC4
                                                            PC5
##
                                                                    PC6
## Standard deviation
                          1.7407 1.5792 1.2475 0.98517 0.84845 0.77930
## Proportion of Variance 0.2754 0.2267 0.1415 0.08823 0.06544 0.05521
## Cumulative Proportion
                          0.2754 0.5021 0.6436 0.73187 0.79732 0.85253
                                                     PC10
##
                              PC7
                                       PC8
                                               PC9
                                                             PC11
## Standard deviation
                          0.72330 0.70817 0.58054 0.4772 0.18119
## Proportion of Variance 0.04756 0.04559 0.03064 0.0207 0.00298
## Cumulative Proportion 0.90009 0.94568 0.97632 0.9970 1.00000
sum((pca_wine$sdev)^2)
## [1] 11
plot(pca_wine)
```

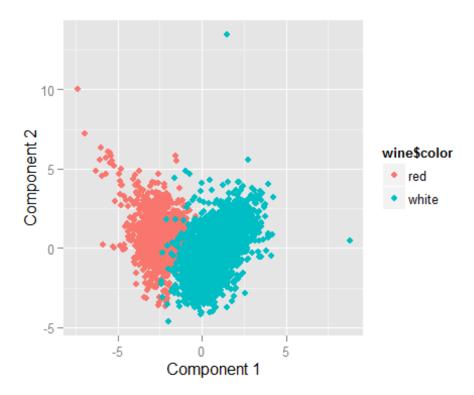


biplot(pca\_wine)



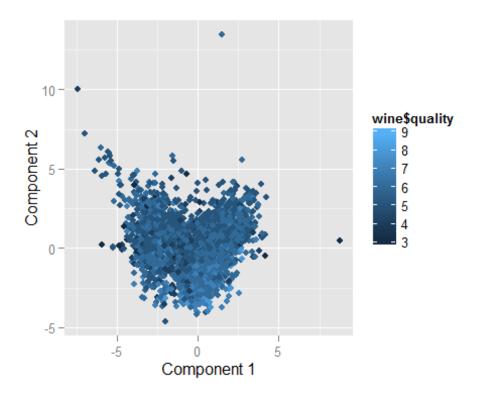
By looking at the score and plotting them, we can again see the distinction from red and white wines

```
scores = pca_wine$x
head(scores)
##
                        PC2
                                  PC3
                                            PC4
                                                        PC5
                                                                   PC6
              PC1
## [1,] -3.205749 0.4164913
                             2.722027 0.7967162 -0.2028619
                                                             0.2273453
## [2,] -3.038817 1.1073769
                             2.046795 0.7701656 1.3225536 -1.6549941
## [3,] -3.071657 0.8788968
                             1.742445 0.8021955
                                                  0.7620531 -0.8483083
## [4,] -1.571141 2.1123820 -2.592717 0.2927620 -0.6046542
                                                             0.7132533
## [5,] -3.205749 0.4164913
                             2.722027 0.7967162 -0.2028619
                                                             0.2273453
                             2.669970 0.6861020 -0.2077957
## [6,] -3.011934 0.3893675
                                                             0.2928529
##
                PC7
                          PC8
                                      PC9
                                                  PC10
                                                              PC11
## [1,] -0.32552850 0.5672348 0.07122341
                                           0.10803795
                                                       0.02745798
## [2,]
        0.05955408 0.5145630 -0.42909559
                                           0.26812827 -0.01547017
        0.16765673 0.4209200 -0.27101087
                                           0.08682522
## [3,]
                                                       0.05414179
## [4,] -0.85115161 0.9295970
                              0.54936730 -0.11665069 -0.10373671
## [5,] -0.32552850 0.5672348
                               0.07122341
                                           0.10803795
                                                       0.02745798
## [6,] -0.28272294 0.6500382
                               0.18849800
                                           0.05506242
                                                       0.06542536
nrow(scores)
## [1] 6497
qplot(scores[,1], scores[,2], color=wine$color, xlab='Component 1',
ylab='Component 2')
```



Again when plotting to see quality, it is dificult to see any distinction.

```
qplot(scores[,1], scores[,2], color=wine$quality, xlab='Component 1',
ylab='Component 2')
```



In conclusion I beleive clustering is more useful in evaluating this data. The plots formed from clustering claerly seperated red and white wines. Additionally by using the table function we were able to see that clustering distinguished between the wines with a high level of precision.

## (4) Market Segmentation

## **K-Means clustering**

Read in the data

```
twitter = read.csv('social marketing.csv')
head(twitter)
##
             X chatter current_events travel photo_sharing uncategorized
## 1 hmjoe4g3k
## 2 clk1m5w8s
                      3
                                      3
                                              2
                                                             1
                                                                            1
## 3 jcsovtak3
                      6
                                      3
                                                                            1
                                              4
                                                             3
                                      5
                                                             2
## 4 3oeb4hiln
                      1
                                              2
                                                                            0
## 5 fd75x1vgk
                      5
                                      2
                                              0
                                                             6
                                                                            1
## 6 h6nvj91yp
                      6
                                      4
                                              2
     tv_film sports_fandom politics food family home_and_garden music news
## 1
                          1
                                    0
                                                 1
                                                                  2
                                                 2
## 2
```

```
## 3
            1
                            0
                                       1
                                             0
                                                     1
                                                                              0
                                                                                    0
## 4
                                                                       0
            0
                            0
                                       2
                                             0
                                                     1
                                                                       0
                                                                              0
                                                                                    0
## 5
                                                     1
## 6
            1
                            1
                                       0
                                             2
                                                                       1
                                                                              1
                                                                                    0
     online_gaming shopping health_nutrition college_uni sports_playing
##
## 1
                   0
                              1
                                                17
                                                               0
                   0
                             0
                                                               0
## 2
                                                 0
                                                                                1
                   0
                             2
                                                 0
                                                               0
                                                                                0
## 3
                   0
                             0
                                                 0
                                                               1
                                                                                0
                   3
                             2
## 5
                                                 0
                                                               4
                                                                                0
                   0
                              5
                                                               0
                                                                                0
## 6
                                                 0
     cooking eco computers business outdoors crafts automotive art religion
##
## 1
            5
                 1
                            1
                                       0
                                                 2
                                                         1
                                                                      0
                                                                          0
                                                                                     1
                                                         2
                                                                          0
## 2
            0
                 0
                            0
                                       1
                                                 0
                                                                      0
                                                                                     0
                                                         2
## 3
            2
                 1
                            0
                                       0
                                                 0
                                                                      0
                                                                          8
                                                                                     0
                                                         3
                                       1
                                                                          2
            0
                 0
                            0
                                                 0
                                                                      0
                                                                                     0
## 4
                                       0
                                                 1
                                                         0
## 5
            1
                 0
                            1
                                                                      0
                                                                          0
                                                                                     0
                            1
                                       1
                                                         0
                                                                      1
                                                                                     0
## 6
            0
                 0
                                                 0
##
     beauty parenting dating school personal_fitness fashion small_business
## 1
           0
                       1
                               1
                                       0
                                                         11
                                                                   0
                       0
                               1
                                       4
                                                          0
                                                                   0
## 2
           0
                                                                                     0
## 3
           1
                       0
                               1
                                       0
                                                          0
                                                                   1
                                                                                     0
## 4
           1
                       0
                               0
                                       0
                                                          0
                                                                    0
                                                                                     0
## 5
           0
                       0
                               0
                                       0
                                                          0
                                                                   0
                                                                                     1
                       0
                               0
                                       0
                                                          0
                                                                   0
## 6
           0
                                                                                     0
##
     spam adult
## 1
         0
## 2
         0
                0
## 3
         0
                0
                0
## 4
         0
## 5
         0
                0
## 6
         0
names(twitter)
    [1]
         "X"
                               "chatter"
                                                     "current_events"
##
         "travel"
                               "photo_sharing"
                                                     "uncategorized"
##
    [4]
    [7] "tv_film"
                               "sports_fandom"
                                                     "politics"
##
## [10] "food"
                               "family"
                                                     "home_and_garden"
## [13] "music"
                               "news"
                                                     "online_gaming"
   [16]
         "shopping"
                               "health_nutrition"
                                                     "college_uni"
##
                                                     "eco"
   [19] "sports_playing"
                               "cooking"
         "computers"
                               "business"
                                                     "outdoors"
   [22]
                               "automotive"
                                                     "art"
## [25] "crafts"
## [28]
         "religion"
                               "beauty"
                                                     "parenting"
                               "school"
                                                     "personal_fitness"
## [31] "dating"
## [34] "fashion"
                               "small_business"
                                                     "spam"
## [37] "adult"
```

Remove the 1st and last 2 columns of the dataset and scale the data.

```
twitter_num = twitter [,(2:35)]
twitter_scaled = scale(twitter_num, center = TRUE, scale = TRUE)
```

cluster the data using 5 centers, in hopes of distinguishing market segments.

```
cluster_twitter = kmeans(twitter_scaled, centers = 5, nstart = 200)
```

Capture the mean and SD of the scaled data.

```
sigma_twitter = attr(twitter_scaled, "scaled:scale")
mu_twitter = attr(twitter_scaled, "scaled:center")
```

Unscale the data.

```
cluster twitter$center
##
          chatter current events
                                      travel photo sharing uncategorized
## 1 -0.086222413
                      0.11809608 -0.09771495
                                                -0.06537001
                                                              -0.07560096
## 2
      0.722175675
                      0.27442218 -0.03596486
                                                               0.45534783
                                                1.03832015
## 3 -0.212834402
                     -0.12948720 -0.23381547
                                                -0.31620751
                                                              -0.16643598
## 4 -0.006995531
                      0.10556173
                                  1.83977588
                                               -0.10723104
                                                              -0.04382044
                     -0.01124003 -0.15545173
## 5 -0.084492666
                                                -0.04798195
                                                               0.14853649
                                  politics
##
          tv_film sports_fandom
                                                  food
                                                             family
                      2.0470620 -0.1982928 1.81765091
                                                         1.49765506
## 1
      0.000746473
## 2
      0.485228283
                     -0.1645516 -0.1310832 -0.15553966
                                                         0.03722871
## 3 -0.149523564
                     -0.2977550 -0.2727615 -0.36584951 -0.27118944
      0.033096281
                      0.1920158
                                2.4282822
                                            0.03297433
## 4
                                                         0.05045762
## 5 -0.104380041
                     -0.1939706 -0.1810003
                                            0.43202406 -0.07556437
##
     home and garden
                                        news online gaming
                           music
                                                                shopping
## 1
           0.1706871
                      0.04803399 -0.06700095
                                                0.02473461 -0.001135387
## 2
           0.2660681 0.57282236 -0.13054926
                                                0.43242891 0.742711633
## 3
          -0.1701953 -0.20187397 -0.24809584
                                               -0.12679472 -0.243105271
## 4
           0.1185947 -0.05033662 1.93533117
                                                -0.07604167 -0.057068483
## 5
           0.1418000 0.02622923 -0.03255397
                                               -0.06324663 -0.007390447
     health nutrition college uni sports playing
                                                      cooking
## 1
           -0.1528754 -0.03268345
                                      0.14438408 -0.08800954
                                                               0.17248722
## 2
           -0.1815311 0.58926171
                                      0.50358625  0.86228797
                                                               0.23181896
## 3
           -0.3295290 -0.15720684
                                     -0.20697001 -0.33998540 -0.23646305
## 4
           -0.2023987 -0.03276814
                                      0.02766946 -0.19240063
                                                               0.09727734
## 5
            2.1505827 -0.15849832
                                      0.02193048
                                                  0.43667699 0.52817655
##
       computers
                   business
                               outdoors
                                             crafts
                                                     automotive
                                                                         art
      0.07670276
                  0.1215802 -0.06801627
                                         0.70338638
                                                      0.17590291
                                                                  0.11089822
## 2 -0.02412705
                  0.3721702 -0.11253202
                                         0.29050312
                                                      0.08501058
                                                                  0.43825776
## 3 -0.25278692 -0.2097674 -0.31512163 -0.25859111 -0.20677952 -0.15790482
## 4
      1.64069987
                  0.3293340
                             0.10575747
                                         0.10738581
                                                     1.07469608 -0.04970671
## 5 -0.07939718
                  0.0374073
                             1.66131375
                                         0.08062137 -0.12643014 -0.01768315
##
        religion
                     beauty
                              parenting
                                             dating
                                                          school
## 1
      2.25268080
                  0.3124905
                             2.12069484 -0.01192335
                                                     1.65716669
                  0.8599830 -0.13690144 0.30780026
## 2 -0.16518697
                                                     0.15654677
## 3 -0.30436878 -0.2904203 -0.31199940 -0.17031941 -0.30572877
## 4 -0.03036794 -0.1505644 0.02142103 0.20654026 -0.04530884
## 5 -0.16501662 -0.1669719 -0.10269878 0.16158299 -0.16709797
```

```
##
     personal fitness
                            fashion small business
## 1
           -0.09784654
                         0.02696846
                                         0.09556896
## 2
           -0.14604173
                        0.96454926
                                         0.44854802
## 3
           -0.34409012 -0.29434889
                                        -0.18012218
## 4
           -0.19706439 -0.15869840
                                         0.22739613
## 5
            2.11143052 -0.07063281
                                        -0.12811374
cluster_twitter$center[1,]*sigma_twitter + mu_twitter
##
             chatter
                       current events
                                                  travel
                                                             photo sharing
##
           4.0944669
                                               1.3616734
                                                                  2.5182186
                             1.6761134
##
                                                                   politics
      uncategorized
                               tv_film
                                           sports_fandom
##
           0.7422402
                             1.0715250
                                               6.0175439
                                                                  1.1875843
                                family
##
                food
                                         home and garden
                                                                      music
##
           4.6248313
                             2.5600540
                                               0.6464238
                                                                  0.7287449
##
                         online_gaming
                                                shopping health nutrition
                news
##
           1.0647773
                             1.2753036
                                               1.3873144
                                                                  1.8798920
##
                       sports_playing
        college uni
                                                  cooking
                                                                        eco
##
           1.4547908
                             0.7800270
                                                                  0.6450742
                                               1.6963563
##
                                                outdoors
                                                                     crafts
           computers
                              business
##
           0.7395412
                             0.5074224
                                               0.7004049
                                                                  1.0904184
##
         automotive
                                    art
                                                religion
                                                                     beauty
##
                             0.9055331
                                               5.4089069
           1.0701754
                                                                  1.1201080
##
          parenting
                                dating
                                                   school personal_fitness
##
          4.1349528
                             0.6896086
                                               2.7368421
                                                                  1.2267206
##
             fashion
                        small business
##
           1.0458839
                             0.3954116
cluster_twitter$center[2,]*sigma_twitter + mu_twitter
##
             chatter
                        current_events
                                                  travel
                                                             photo_sharing
##
           6.9474053
                             1.8744741
                                               1.5028050
                                                                  5.5329593
                                                                   politics
##
      uncategorized
                               tv film
                                           sports fandom
##
          1.2391304
                             1.8751753
                                               1.2384292
                                                                  1.3913043
##
                                family
                                         home and garden
                food
                                                                      music
##
           1.1213184
                             0.9060309
                                               0.7166900
                                                                  1.2692847
##
                        online_gaming
                                                shopping health_nutrition
                news
##
           0.9312763
                             2.3709677
                                               2.7328191
                                                                  1.7510519
##
        college uni
                        sports playing
                                                  cooking
                                                                        eco
##
           3.2566620
                             1.1304348
                                               4.9558205
                                                                  0.6907433
##
                                                outdoors
           computers
                              business
                                                                     crafts
##
           0.6206171
                             0.6809257
                                               0.6465638
                                                                  0.7531557
##
         automotive
                                    art
                                                religion
                                                                     beauty
##
           0.9460028
                             1.4389902
                                               0.7791024
                                                                  1.8471248
##
                                                   school personal fitness
           parenting
                                dating
##
           0.7138850
                             1.2594670
                                               0.9537167
                                                                  1.1107994
##
             fashion
                        small business
##
           2,7601683
                             0.6136045
```

See attributes are emphasized in each cluster to facilitate targeting

```
#Young women. Care about fashion, beauty, cooking, shopping, and photo
sharing
rbind(cluster_twitter$center[1,],cluster_twitter$center[1,]*sigma_twitter +
mu twitter)
##
            chatter current events
                                        travel photo sharing uncategorized
## [1,] -0.08622241
                         0.1180961 -0.09771495
                                                 -0.06537001
                                                               -0.07560096
## [2,]
         4.09446694
                         1.6761134 1.36167341
                                                  2.51821862
                                                                0.74224022
##
            tv film sports fandom
                                    politics
                                                 food
                                                        family
## [1,] 0.000746473
                         2.047062 -0.1982928 1.817651 1.497655
## [2,] 1.071524966
                         6.017544 1.1875843 4.624831 2.560054
                                          news online gaming
##
        home and garden
                             music
                                                                 shopping
## [1,]
              0.1706871 0.04803399 -0.06700095
                                                  0.02473461 -0.001135387
              0.6464238 0.72874494 1.06477733
                                                  1.27530364 1.387314440
## [2,]
        health nutrition college uni sports playing
##
                                                        cooking
## [1,]
              -0.1528754 -0.03268345
                                          0.1443841 -0.08800954 0.1724872
                                          0.7800270 1.69635628 0.6450742
## [2,]
               1.8798920 1.45479082
                                            crafts automotive
##
         computers business
                                outdoors
## [1,] 0.07670276 0.1215802 -0.06801627 0.7033864 0.1759029 0.1108982
## [2,] 0.73954116 0.5074224 0.70040486 1.0904184 1.0701754 0.9055331
                    beauty parenting
                                                   school personal fitness
##
        religion
                                          dating
## [1,] 2.252681 0.3124905 2.120695 -0.01192335 1.657167
                                                               -0.09784654
## [2,] 5.408907 1.1201080 4.134953 0.68960864 2.736842
                                                                1.22672065
           fashion small business
##
## [1,] 0.02696846
                       0.09556896
## [2,] 1.04588394
                       0.39541161
#Your classic young parent (sports, food, family, religion, and parenting)
rbind(cluster_twitter$center[2,],cluster_twitter$center[2,]*sigma_twitter +
mu_twitter)
##
          chatter current events
                                      travel photo sharing uncategorized
                       0.2744222 -0.03596486
                                                  1.038320
## [1,] 0.7221757
                                                               0.4553478
## [2,] 6.9474053
                       1.8744741 1.50280505
                                                  5.532959
                                                               1.2391304
          tv_film sports_fandom
                                  politics
                                                 food
                                                          family
                     -0.1645516 -0.1310832 -0.1555397 0.03722871
## [1,] 0.4852283
## [2,] 1.8751753
                      1.2384292 1.3913043 1.1213184 0.90603086
##
        home and garden
                            music
                                        news online_gaming shopping
              0.2660681 0.5728224 -0.1305493
                                                 0.4324289 0.7427116
## [1,]
                                                 2.3709677 2.7328191
              0.7166900 1.2692847 0.9312763
## [2,]
        health_nutrition college_uni sports_playing cooking
##
## [1,]
                           0.5892617
                                          0.5035862 0.862288 0.2318190
              -0.1815311
               1.7510519
                           3.2566620
                                          1.1304348 4.955820 0.6907433
## [2,]
##
          computers business
                               outdoors
                                            crafts automotive
## [1,] -0.02412705 0.3721702 -0.1125320 0.2905031 0.08501058 0.4382578
       0.62061711 0.6809257 0.6465638 0.7531557 0.94600281 1.4389902
## [2,]
                     beauty parenting
##
          religion
                                          dating
                                                    school personal fitness
## [1,] -0.1651870 0.859983 -0.1369014 0.3078003 0.1565468
                                                                 -0.1460417
## [2,] 0.7791024 1.847125 0.7138850 1.2594670 0.9537167
                                                                  1.1107994
## fashion small business
```

```
## [1,] 0.9645493
                       0.4485480
## [2,] 2.7601683
                       0.6136045
#Average No-desciptive cluster
rbind(cluster_twitter$center[3,],cluster_twitter$center[3,]*sigma_twitter +
mu twitter)
##
           chatter current events
                                     travel photo sharing uncategorized
## [1,] -0.2128344
                                                -0.3162075
                       -0.1294872 -0.2338155
                                                               -0.166436
## [2,]
       3.6476373
                       1.3619573 1.0506117
                                                 1.8330535
                                                                0.657232
##
           tv film sports fandom
                                   politics
                                                  food
                                                           family
                     -0.2977550 -0.2727615 -0.3658495 -0.2711894
## [1,] -0.1495236
       0.8222595
                       0.9505877 0.9618614 0.7479012 0.5567282
## [2,]
##
       home and garden
                             music
                                         news online gaming
## [1,]
             -0.1701953 -0.2018740 -0.2480958
                                                 -0.1267947 -0.2431053
## [2,]
             0.3952986 0.4713361 0.6843368
                                                  0.8680739 0.9496282
##
        health nutrition college uni sports playing
                                                       cooking
                                         -0.2069700 -0.3399854 -0.236463
## [1,]
               -0.329529 -0.1572068
## [2,]
                1.085632
                           1.0940273
                                          0.4372751 0.8320940 0.330295
                               outdoors
                                             crafts automotive
##
         computers
                    business
## [1,] -0.2527869 -0.2097674 -0.3151216 -0.2585911 -0.2067795 -0.1579048
## [2,]
       0.3509235 0.2780043 0.4015351 0.3046294 0.5473735
                                                                0.4674982
##
          religion
                       beauty parenting
                                             dating
                                                        school
## [1,] -0.3043688 -0.2904203 -0.3119994 -0.1703194 -0.3057288
## [2,] 0.5125929 0.3195011 0.4485488 0.4072919 0.4044135
       personal fitness
                            fashion small business
## [1,]
              -0.3440901 -0.2943489
                                        -0.1801222
## [2,]
              0.6344447 0.4583833
                                         0.2249940
#Athletiic/Healthy Cluster (Nutrition, outdoors, and personal fitness)
rbind(cluster twitter$center[4,],cluster twitter$center[4,]*sigma twitter +
mu twitter)
                                      travel photo_sharing uncategorized
##
             chatter current events
                                                 -0.107231
## [1,] -0.006995531
                          0.1055617 1.839776
                                                             -0.04382044
## [2,]
       4.374068554
                          1.6602086 5.789866
                                                  2.403875
                                                              0.77198212
##
          tv_film sports_fandom politics
                                                food
                                                         family
## [1,] 0.03309628
                       0.1920158 2.428282 0.03297433 0.05045762
## [2,] 1.12518629
                       2.0089419 9.149031 1.45603577 0.92101341
##
       home_and_garden
                              music
                                        news online_gaming
                                                              shopping
             0.1185947 -0.05033662 1.935331
                                               -0.07604167 -0.05706848
## [1,]
                                                1.00447094 1.28614009
## [2,]
             0.6080477   0.62742176   5.271237
##
       health_nutrition college_uni sports_playing
                                                       cooking
## [1,]
              -0.2023987 -0.03276814
                                         0.02766946 -0.1924006 0.09727734
              1.6572280 1.45454545
                                         0.66616990 1.3383010 0.58718331
## [2,]
                                        crafts automotive
##
        computers business outdoors
## [1,] 1.640700 0.3293340 0.1057575 0.1073858
                                                  1.074696 -0.04970671
       2.584203 0.6512668 0.9105812 0.6035768
## [2,]
                                                  2.298063 0.64381520
##
                                             dating
           religion
                        beauty parenting
                                                         school
## [1,] -0.03036794 -0.1505644 0.02142103 0.2065403 -0.04530884
## [2,] 1.03725782 0.5052161 0.95380030 1.0789866 0.71385991
```

```
personal fitness
                            fashion small business
## [1,]
              -0.1970644 -0.1586984
                                          0.2273961
                                          0.4769001
## [2,]
               0.9880775 0.7064083
#worldly and current cluster (travel, politics, news, automotive, computers)
rbind(cluster twitter$center[5,],cluster twitter$center[5,]*sigma twitter +
mu twitter)
##
            chatter current events
                                       travel photo sharing uncategorized
## [1,] -0.08449267
                       -0.01124003 -0.1554517
                                                 -0.04798195
                                                                 0.1485365
         4.10057143
                        1.51200000 1.2297143
                                                  2.56571429
                                                                 0.9520000
## [2,]
##
           tv_film sports_fandom
                                   politics
                                                  food
                                                            family
## [1,] -0.1043800
                      -0.1939706 -0.1810003 0.4320241 -0.07556437
## [2,]
         0.8971429
                       1.1748571 1.2400000 2.1645714 0.77828571
        home and garden
                                          news online gaming
##
                             music
                                                                  shopping
## [1,]
              0.1418000 0.02622923 -0.03255397
                                                  -0.06324663 -0.007390447
## [2,]
              0.6251429 0.70628571 1.13714286
                                                   1.03885714
                                                               1.376000000
##
        health nutrition college uni sports playing cooking
                2.150583
                          -0.1584983
                                          0.02193048 0.436677 0.5281765
## [1,]
               12.236571
                           1.0902857
                                          0.66057143 3.496000 0.9188571
## [2,]
                                            crafts automotive
##
          computers
                     business outdoors
## [1,] -0.07939718 0.0374073 1.661314 0.08062137 -0.1264301 -0.01768315
         0.55542857 0.4491429 2.792000 0.58171429 0.6571429 0.69600000
## [2,]
##
          religion
                       beauty parenting
                                            dating
                                                        school
## [1,] -0.1650166 -0.1669719 -0.1026988 0.1615830 -0.1670980
         0.7794286  0.4834286  0.7657143  0.9988571  0.5691429
##
        personal fitness
                             fashion small business
                2.111431 -0.07063281
                                          -0.1281137
## [1,]
                6.540571 0.86742857
                                           0.2571429
## [2,]
```

#### Conclusion

By using K-Menas clustering with 5 centers I was able to successfully group the twitter users into distinct groups. The clusters found were; Young women, Parents, Athletic people, contemporaries, and then a group that was relatively average accross all categories. I think these clusters could provide useful marketing insights to NutrientH20.