Agujeros negros de Kerr

RELACIÓN CON AGUJEROS NEGROS ASTROFÍSICOS Y DIFERENCIAS CON AGUJEROS NEGROS DE SCHWARZSCHILD

Matías Borghi

Esquema

- Reseña histórica
- Métrica de Schwarzschild
- Métrica de Kerr
- Un poco de evidencia observacional

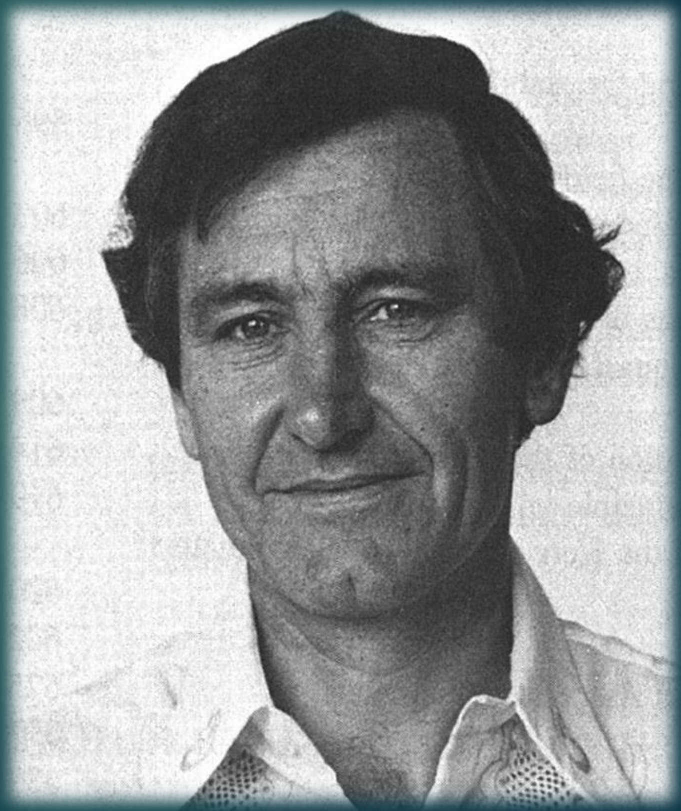
RESEÑA HISTÓRICA (I)

- 1915 – Relatividad General Einstein.
- 1916 – Solución de Schwarzschild.
- 1922 – Ecuaciones de Friedmann.

- Proyecto Manhattan, etc.
- 60's-70's – Época de oro de la astrofísica relativista.
  - 1967 - Descubrimiento del primer púlsar.

RESEÑA HISTÓRICA (II)

- PRINCETON – JOHN ARCHIBALD WHEELER (AGUJEROS NEGROS, AGUJEROS DE GUSANO).
- CAMBRIDGE – DENNIS SCIAMA Y ROGER PENROSE.
- UNI. DE MOSCÚ – YAKOV ZEL'DOVICH.
- UNV. DE HAMBURGO – PASCUAL JORDAN.
- AUSTIN, TEXAS – ALFRED SCHILD Y ROY KERR.



RESEÑA HISTÓRICA (III)

GRAVITATIONAL FIELD OF A SPINNING MASS AS AN EXAMPLE OF ALGEBRAICALLY SPECIAL METRICS

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Goldberg and Sachs[1] have proved that the algebraically special solutions of Einstein's empty-space field equations are characterized by the existence of a geodesic and shear-free ray congruence, k_μ. Among these spaces are the plane-fronted waves and the Robinson-Trautman metrics[2] for which the congruence has nonvanishing divergence, but is hypersurface orthogonal.

In this note we shall present the class of solutions for which the congruence is diverging, and is not necessarily hypersurface orthogonal. The only previously known example of the general case is the Newman, Unti, and Tamburino metrics,[3] which is of Petrov Type D, and possesses a four-dimensional group of isometries.

If we introduce a complex null tetrad (t^* is the complex conjugate of t), with

$$ds^2 = 2tt^* + 2mk,$$

then the coordinate system may be chosen so that

$$t = P(r + i\Delta)d\zeta,$$

$$k = du + 2\,\mathrm{Re}(\Omega d\zeta),$$

$$m = dr - 2\,\mathrm{Re}\{[(r - i\Delta)\dot{\Omega} + iD\Delta]d\zeta\} + \Big\{r\dot{P}/P + \mathrm{Re}[P^{-2}D(D^*\ln P + \dot{\Omega}^*)] + \frac{m_1 r - m_2\Delta}{r^2 + \Delta^2}\Big\}k; \qquad (1)$$

where ζ is a complex coordinate, a dot denotes differentiation with respect to u, and the operator D is defined by

$$D = \partial/\partial\zeta - \Omega\partial/\partial u.$$

P is real, whereas Ω and m (which is defined to be $m_1 + im_2$) are complex. They are all independent of the coordinate r. Δ is defined by

$$\Delta = \mathrm{Im}(P^{-2}D^*\Omega).$$

There are two natural choices that can be made for the coordinate system. Either (A) P can be chosen to be unity, in which case Ω is complex, or (B) Ω can be taken pure imaginary, with P different from unity. In case (A), the field equations are

$$(m - D^*D^*D\Omega) = |\partial_u D\Omega|^2, \qquad (2)$$

$$\mathrm{Im}(m - D^*D^*D\Omega) = 0, \qquad (3)$$

$$D^*m = 3m\dot{\Omega}. \qquad (4)$$

The second coordinate system is probably better, but it gives more complicated field equations.

It will be observed that if m is zero then the field equations are integrable. These spaces correspond to the Type-III and null spaces with

- 1963 – *"Gravitational field of spinning mass as an example of algebraically special metrics".*

- *Brandon Carter investiga el significado de la solución obtenida por Kerr.*

TEOREMA NO-HAIR

- AGUJERO NEGRO : M,J,Q

	J = 0	J ≠ 0
Q = 0	*Schwarzsc hild*	*Kerr*
Q ≠ 0	*Reissner-Nordstrom*	*Kerr-Newmann*

Agujeros negros supermasivos	$10^5 - 10^{10}\ M_\odot$
Agujeros negros intermedios	$10^3 M_\odot$
Agujeros negros estelares	$10\ M_\odot$
Micro agujeros negros	M_{LUNA}

AGUJEROS NEGROS DE SCHWARZSCHILD(I)

- TEOREMA DE BIRKHOFF
- MÉTRICA EN COORDENADAS ESFÉRICAS

$$ds^2 = \left(1 - \frac{2Gm}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2Gm}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

donde $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2.$

AGUJEROS NEGROS DE SCHWARZSCHILD(II)

- SINGULARIDADES

$$r = 0 \text{ y } r_s = 2Gm/c^2$$

- COORDENADAS DE EDDINGTON-FINKELSTEIN

$$ds^2 = \left(1 - \frac{2Gm}{rc^2}\right)(c^2dt^2 - dr_*^2) - r^2 d\Omega^2,$$

$$\text{donde } d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$$

$$\text{y } r_* = r + \frac{2GM}{c^2}\log\left(\frac{r - 2GM/c^2}{2GM/c^2}\right).$$

AGUJEROS NEGROS DE SCHWARZSCHILD(III)

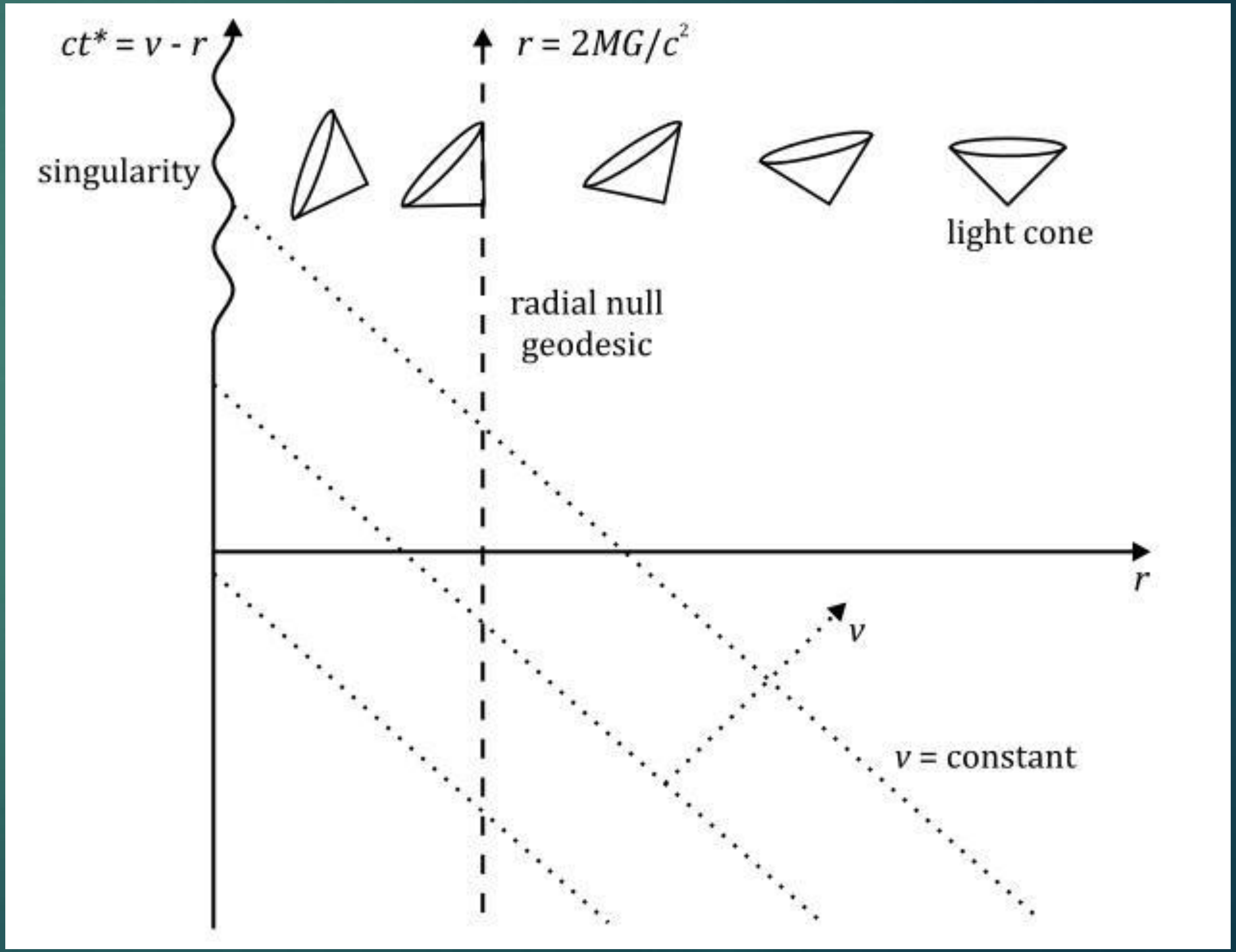
- ¿Es o no una singularidad esencial?
  - ESCALAR DE KRETSCHMANN

$$K = R^{\mu\nu\sigma\rho} R_{\mu\nu\sigma\rho} = \frac{12 r_s^2}{r^6} = \frac{48 G^2 M^2}{c^4 r^6}$$

AGUJEROS NEGROS DE SCHWARZSCHILD(IV)

- HORIZONTE DE EVENTOS

- DIAGRAMA DEL ESPACIO TIEMPO EN COORDENADAS DE EDDINGTON - FINKELSTEIN



AGUJEROS NEGROS DE KERR (I)

- MÉTRICA KERR-SCHILD. COORDENADAS CARTESIANAS

$$ds^2 = c^2 d\hat{t}^2 - dx^2 - dy^2 - dz^2 - \frac{2\mu r^3}{r^4 + a^2 z^2}$$
$$\times \left[c d\hat{t}^2 - \frac{r}{r^2 + a^2}(x dx + y dy) - \frac{a}{r^2 + a^2}(x dy - y dx) - \frac{z}{r} dz \right]^2$$

- MEJOR HACER EL CAMBIO..

$$c d\hat{t} = c dt - 2\mu r/\Delta dr,$$
$$x = (r\cos\phi' + a\sin\phi')\sin\theta,$$
$$y = (r\sin\phi' - a\cos\phi')\sin\theta,$$
$$z = r\cos\theta,$$
$$\text{donde } d\phi' = d\phi - (a/\Delta)dr.$$

AGUJEROS NEGROS DE KERR (II)

- COORDENADAS DE BOYER-LINDQUIST

$$ds^2 = c^2\left(1-\frac{2\mu r}{\rho^2}\right)dt^2+\frac{4\mu acr\sin^2\theta}{\rho^2}dtd\phi-\frac{\rho^2}{\Delta}dr^2-\rho^2 d\theta^2$$

$$-\left(r^2+a^2+\frac{2\mu ra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2$$

donde

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 - 2\mu r + a^2$$

$$\mu = GM/c^2 \text{ y } a = J/Mc$$

AGUJEROS NEGROS DE KERR (III)

- Se puede realizar la sustitución $\Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2\theta$

$$ds^2 = \frac{\Delta - a^2\sin^2\theta}{\rho^2}c^2dt^2 + \frac{4\mu acr\sin^2\theta}{\rho^2}dtd\phi - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \frac{\Sigma^2\sin^2\theta}{\rho^2}d\phi^2$$

$$= \frac{\rho^2\Delta}{\Sigma^2}c^2dt^2 - \frac{\Sigma^2\sin^2\theta}{\rho^2}(d\phi - \omega dt)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2.$$

Con $\omega = 2\mu cra/\Sigma^2$.

AGUJEROS NEGROS DE KERR (IV)

- LÍMITES DE LA MÉTRICA
  - Si $a \to 0 \Leftrightarrow J \longrightarrow 0$

$$\lim_{a\to 0} \Sigma^2 = \lim_{a\to 0} \left[(r^2+a^2)^2 - a^2\Delta \sin^2\theta\right] = r^4,$$

$$\lim_{a\to 0} \rho^2 = \lim_{a\to 0} \left[r^2 + a^2\cos^2\theta\right] = r^2,$$

$$\lim_{a\to 0} \Delta = \lim_{a\to 0} \left[r^2 - 2\mu r + a^2\right] = r^2 - 2\mu r = r^2\left(1-\frac{2\mu}{r}\right).$$

$$\begin{aligned}\lim_{a\to 0} ds^2 &= c^2\left(1-\frac{2\mu}{r}\right)dt^2 - r^2\sin^2\theta d\phi^2 - \frac{dr^2}{\left(1-\frac{2\mu}{r}\right)} - r^2 d\theta^2 \\ &= c^2\left(1-\frac{2\mu}{r}\right)dt^2 - \left(1-\frac{2\mu}{r}\right)^{-1} dr^2 - r^2 d\Omega^2.\end{aligned}$$

AGUJEROS NEGROS DE KERR (V)

- LÍMITES DE LA MÉTRICA
  - Si $\mu \rightarrow 0 \iff M \longrightarrow 0$

$$\lim_{\mu \to 0} ds^2 = c^2 dt^2 - \frac{r^2 + a^2 \cos^2\theta}{r^2 + a^2} dr^2 - (r^2 + a^2 \cos^2\theta) d\theta^2 - (r^2 + a^2) \sin^2\theta d\phi^2$$

$$x = \sqrt{r^2 + a^2} \sin\theta \cos\phi,$$

$$y = \sqrt{r^2 + a^2} \sin\theta \sin\phi,$$

$$z = r \cos\theta,$$

con $r \geq 0$, $0 \leq \theta \leq \pi$ y $0 \leq \phi \leq 2\pi$.

AGUJEROS NEGROS DE KERR (VI)

- SINGULARIDADES

$$\rho = 0 \text{ y } \Delta = 0$$

- ESCALAR DE KRETSCHMANN

$$K = R^{\mu\nu\sigma\rho} R_{\mu\nu\sigma\rho} = \frac{48\mu^2 (r^2 - a^2 \cos^2\theta)\left[(r^2 - a^2\cos^2\theta)^2 - 16 r^2 a^2 \cos^2\theta\right]}{(r^2 + a^2\cos^2\theta)^6}$$

AGUJEROS NEGROS DE KERR (VII)

- A COORDENADAS DE EDDINGTON-FINKELSTEIN

$$ds^2 = \left(1 - \frac{2\mu r}{\rho^2}\right) c^2 dt'^2 - \frac{4\mu r}{\rho^2} cdt' dr - \left(1 + \frac{2\mu r}{\rho^2}\right) dr^2 + \frac{4\mu r a \sin^2\theta}{\rho^2} cdt' d\phi'$$
$$+ 2\frac{(r^2 + a^2) a \sin^2\theta}{\rho^2} dr d\phi' - \rho^2 d\theta^2 - \left[(r^2 + a^2)\sin^2\theta + \frac{2\mu r a^2 \sin^4\theta}{\rho^2}\right] d\phi'^2,$$

donde

$$\begin{aligned} cdt' &= cdt + 2\mu r/\Delta dr \\ d\phi' &= d\phi + a/r dr \end{aligned}$$

AGUJEROS NEGROS DE KERR (VIII)

- SINGULARIDAD ESENCIAL

$$\rho = 0 \Leftrightarrow r^2 + a^2 \cos^2\theta = 0$$

$$\begin{aligned} x^2 + y^2 &= a^2 \\ z &= 0 \end{aligned}$$

$$r = 0 \text{ y } \theta = \frac{\pi}{2}$$

- SINGULARIDADES APARENTES

$$\Delta = 0 \Leftrightarrow r^2 - 2\mu r + a^2 = 0$$

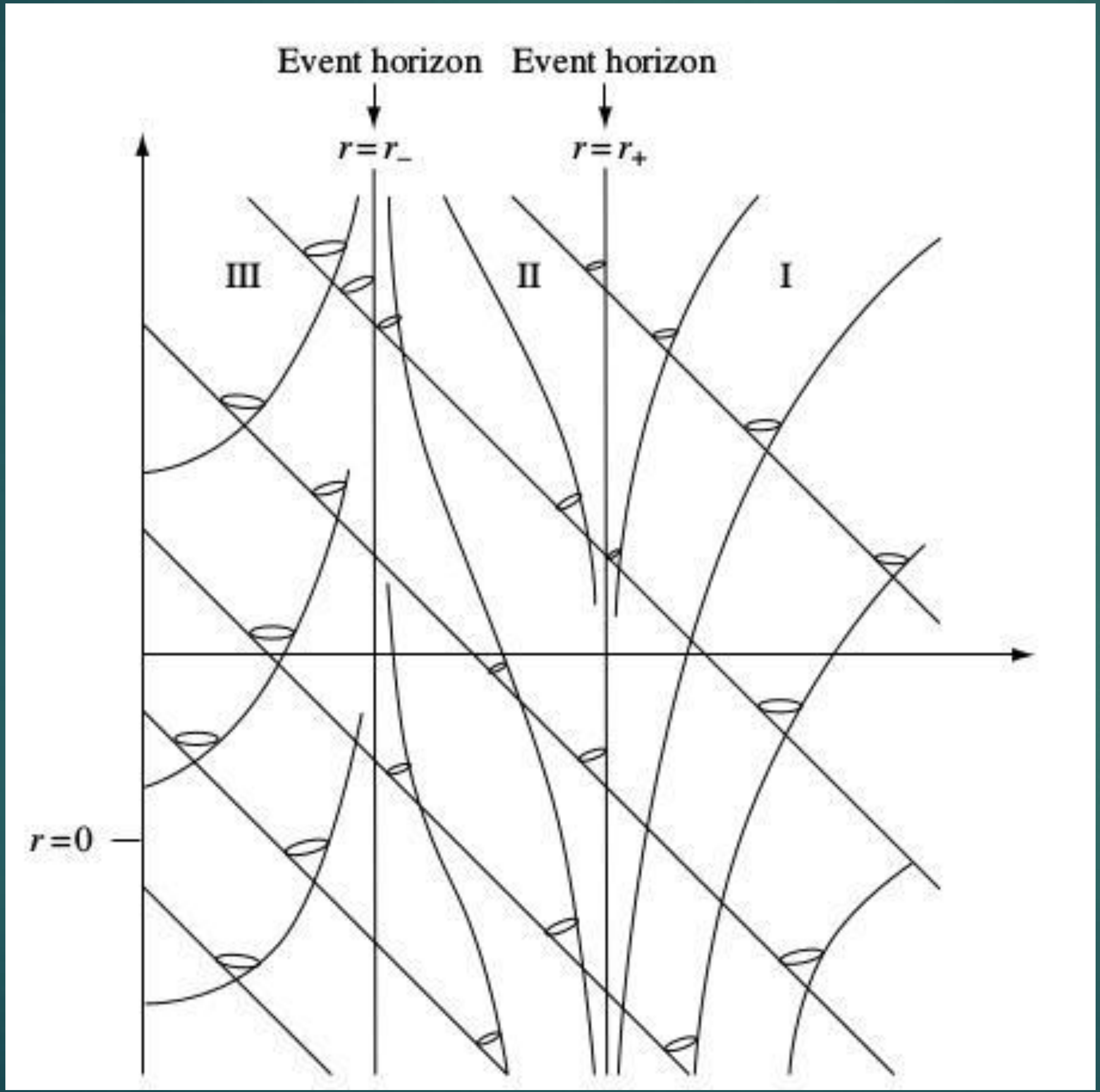
$$r_\pm = \mu \pm \sqrt{\mu^2 - a^2}$$

- HORIZONTE DE EVENTOS

$$g^{rr} = 0 \Leftrightarrow g_{rr} = \infty$$

$$g_{rr} = -\rho^2/\Delta \to \infty \Leftrightarrow \Delta = 0$$

AGUJEROS NEGROS DE KERR (IX)



DIFERENCIAS CON LA MÉTRICA DE SCHWARZSCHILD (I)

- ARRASTRE DEL MARCO DE REFERENCIA (I)

$$\begin{aligned} p_\phi &= cte \\ p_t &= cte \end{aligned}$$

$$\begin{aligned} p^\phi &= g^{\phi\mu}p_\mu = g^{\phi t}p_t + g^{\phi\phi}p_\phi, \\ p^t &= g^{t\mu}p_\mu = g^{tt}p_t + g^{t\phi}p_\phi. \end{aligned}$$

$$p_\phi = 0$$

$$\begin{aligned} p^\phi &= m\frac{d\phi}{d\tau} = g^{\phi t}p_t, \\ p^t &= m\frac{dt}{d\tau} = g^{tt}p_t. \end{aligned}$$

$$\frac{d\phi}{dt} = \frac{g^{\phi t}}{g^{tt}} = \omega(r,\theta) = \frac{2a\mu r\sin^2\theta}{(r^2+a^2)^2 - a^2(r^2-2\mu r+a^2)\sin^2\theta}.$$

DIFERENCIAS CON LA MÉTRICA DE SCHWARZSCHILD (II)

- SUPERFICIES LÍMITE ESTACIONARIAS (I)
  - Fotón con dirección $\pm\phi$

$$g_{tt}dt^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}dtd\phi = 0 \Leftrightarrow dt^2 + g_{\phi\phi}d\phi^2/dt^2 + 2g_{t\phi}d\phi/dt = 0$$

$$\frac{d\phi}{dt} = -\frac{g_{t\phi}}{g_{\phi\phi}} \pm \sqrt{\left(\frac{g_{t\phi}}{g_{\phi\phi}}\right)^2 - \frac{g_{tt}}{g_{\phi\phi}}}.$$

$$g_{tt}(r,\theta) > 0$$

$$g_{tt}(r,\theta) = 0$$

$$g_{tt}(r,\theta) < 0$$

DIFERENCIAS CON LA MÉTRICA DE SCHWARZSCHILD (III)

- SUPERFICIES LÍMITE ESTACIONARIAS (I)

$$g_{tt} = c^2 \left(1 - \frac{2\mu r}{\rho^2}\right) = c^2 \left(\frac{r^2 - 2\mu r + a^2 \cos^2\theta}{\rho^2}\right) = 0$$

$$r_{S\pm} = \mu \pm \sqrt{\mu^2 + a^2 \cos^2\theta}.$$

- Comparando con Schwarzschild..
- Forma de estas superficies a t = cte

$$d\sigma^2 = \rho_S^2 d\theta^2 + \left[\frac{2\mu r_{s\pm}(2\mu r_{s\pm} + 2a^2 \sin^2\theta)}{\rho_{S\pm}^2}\right] \sin^2\theta d\phi^2$$

DIFERENCIAS CON LA MÉTRICA DE SCHWARZSCHILD (IV)

- ERGOSFERA

$g_{tt} < 0$ entre r_+ y r_{S+}

$$v^\mu = (v^t, 0, 0, 0)$$

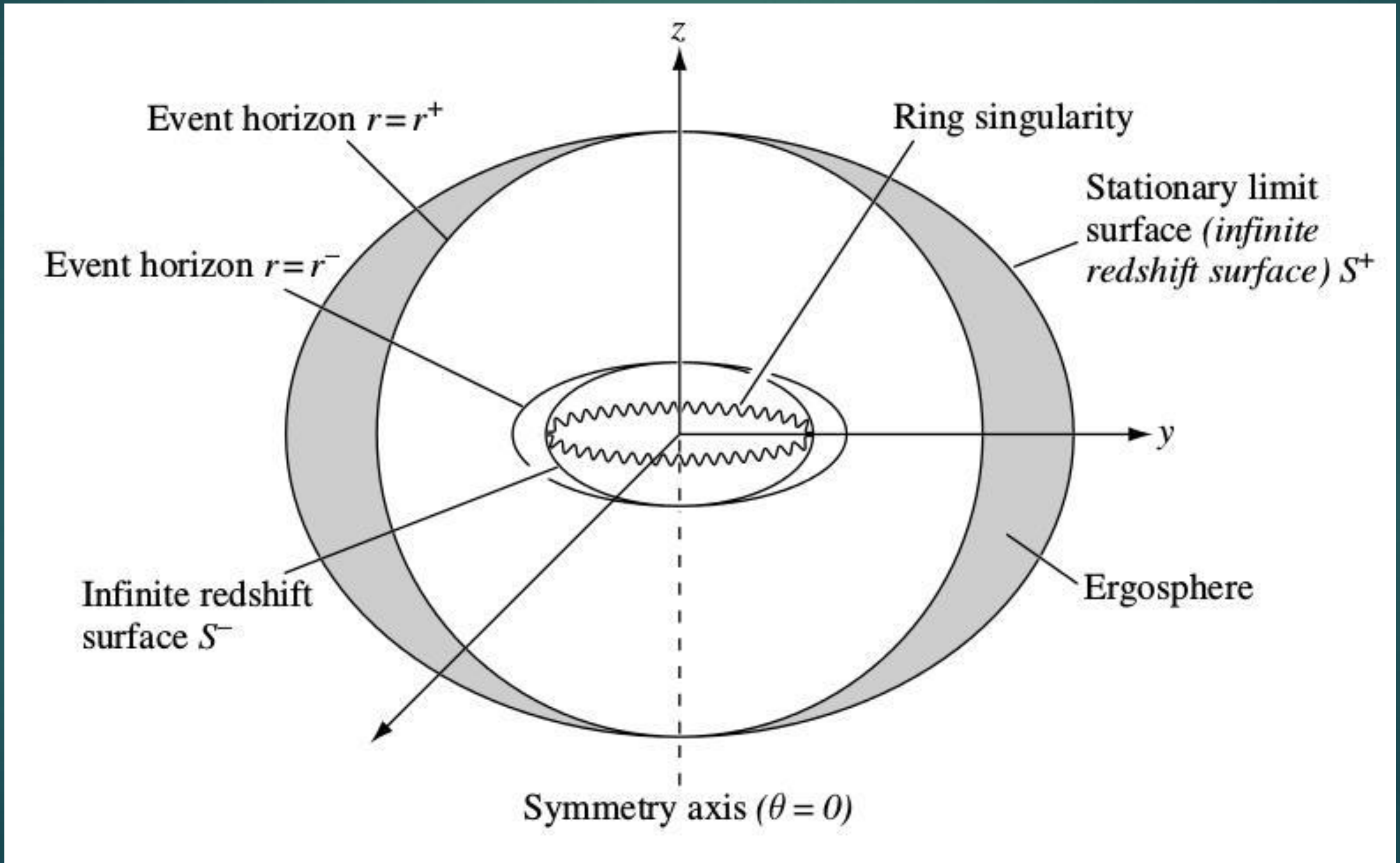
$$v^\mu v_\mu = g_{tt} v^t v^t = g^{tt} (v_t)^2 = c^2$$

$$v^\mu = (v^t, 0, 0, \Omega)$$

$$g_{tt}(v_t)^2 + g_{\phi\phi}(v^\phi)^2 + 2g_{t\phi}v^\phi v^t = c^2$$
$$(u^t)^2(g_{tt}^2 + g_{\phi\phi}\Omega^2 + 2g_{t\phi}\Omega) = c^2.$$

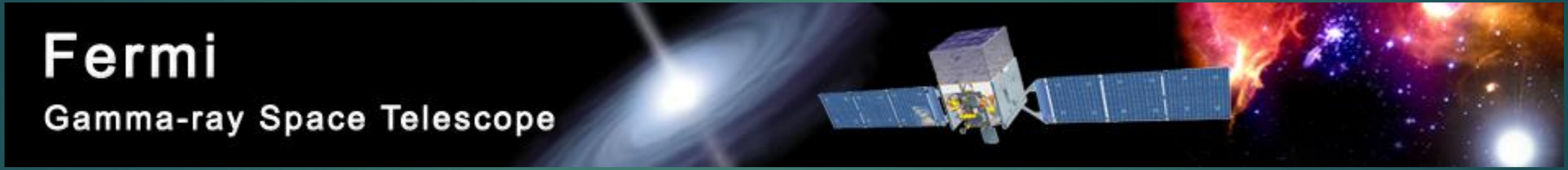
$$\Omega_\pm = -\frac{g_{t\phi}}{g_{\phi\phi}} \pm \sqrt{\left(\frac{g_{t\phi}}{g_{\phi\phi}}\right)^2 - \frac{g_{tt}}{g_{\phi\phi}}} = -\omega \pm \sqrt{\omega^2 - \frac{g_{tt}}{g_{\phi\phi}}}$$

REGIÓN EXTERNA A UN AGUJERO NEGRO DE KERR



AGUJEROS NEGROS ASTROFÍSICOS(I)

- MEDICIÓN DIRECTA
  - RADIACIÓN DE HAWKING

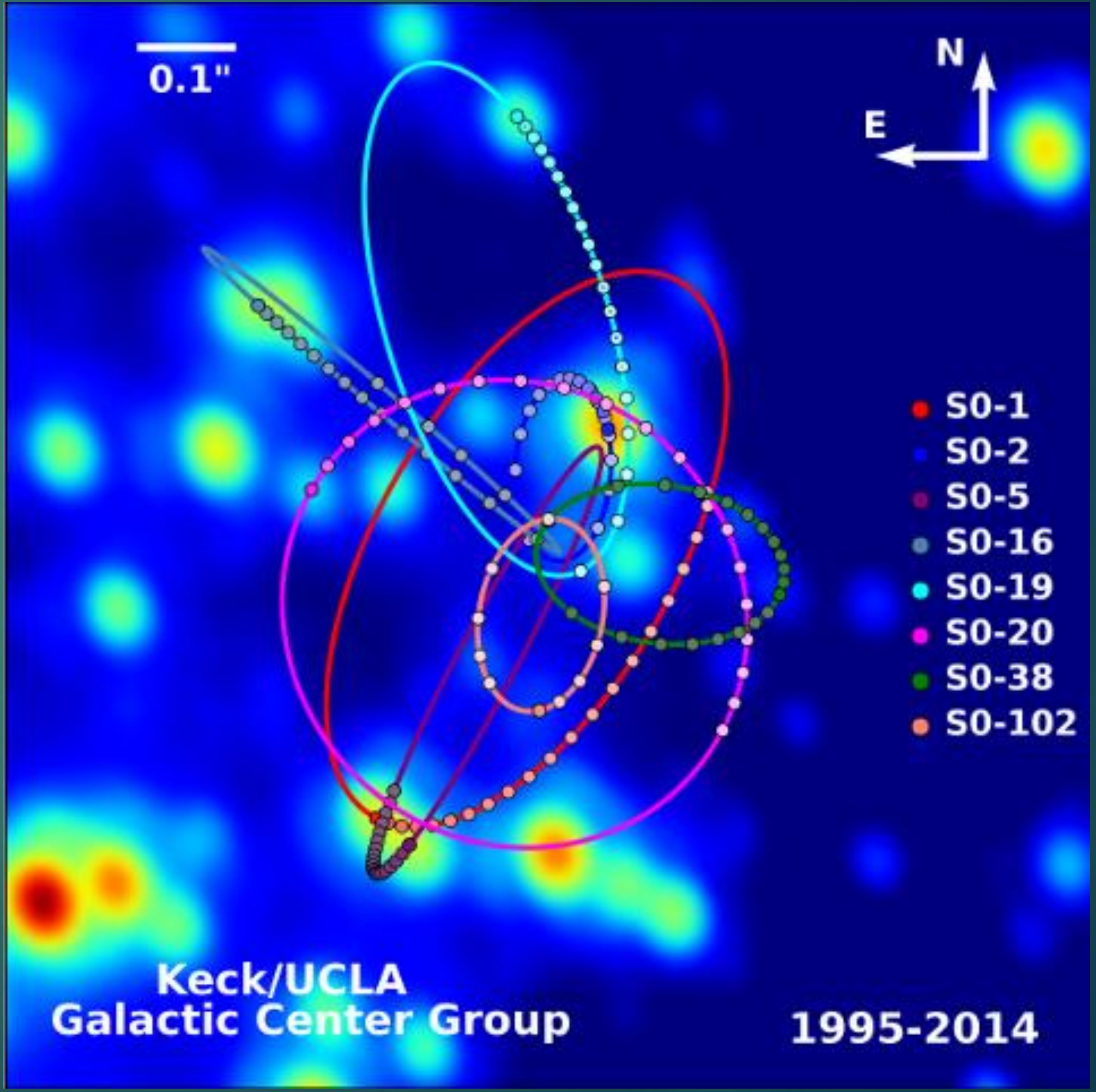


- MEDICIÓN INDIRECTA
  - ONDAS GRAVITACIONALES

AGUJEROS NEGROS ASTROFÍSICOS(II)

- DETERMINACIÓN DE LA MASA DE SAGITARIO A*

$$M = (4{,}4 \pm 0{,}27) \times 10^6 M_\odot$$

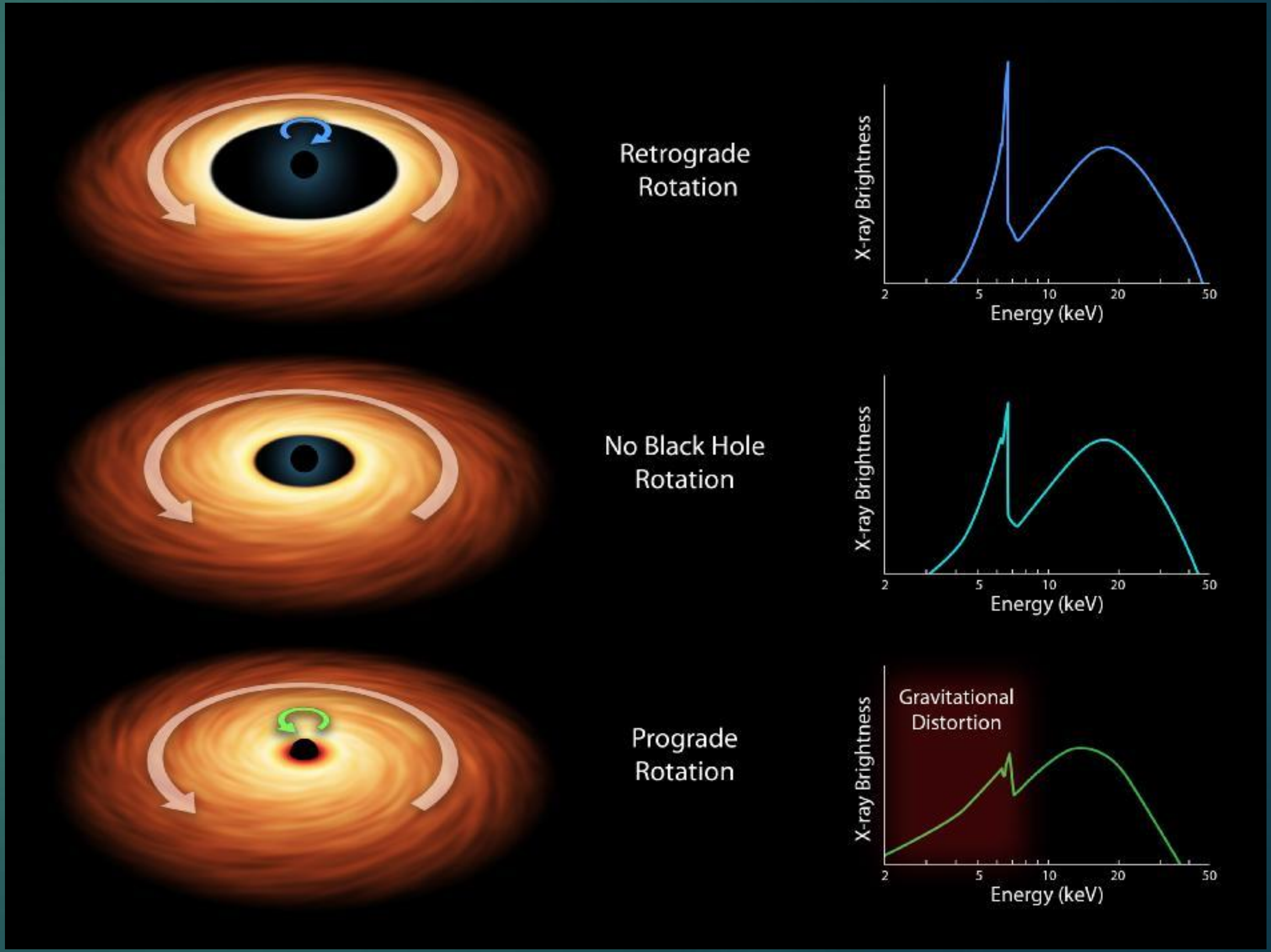


AGUJEROS NEGROS ASTROFÍSICOS (III)

- DETERMINACIÓN DE LA VELOCIDAD ANGULAR DE UN AGUJERO NEGRO (2013)

NuStar - NASA

$\boxed{\omega = 0{,}84c}$



AGUJEROS NEGROS ASTROFÍSICOS(IV)

- OBSERVACIÓN DEL HORIZONTE DE EVENTOS