Math 450H, Fall 2022; Final Project II:

Monte-Carlo method for solving Laplace's equation

Use Monte Carlo method to solve Laplace's equation in a rectangular domain. The domain is defined by $0 \leq x \leq a$, $0 \leq y \leq b$, and the boundary condition is $u(x,0) = u(0,y) = u(a,y) = 0$, $u(x,b) = 12\ V$. Use $a = 7$ inches, $b = 9$ inches. Perform as many random walks as you consider to be enough for the results to be reasonably accurate (one way of deciding how many random walks are needed it to check that the results don't change too much if more random walks are performed).

Find and plot equipotential lines in $1\ V$ intervals. Compare your results with the theoretical result. Discuss any differences. Explain in your own words why random walkers satisfy Laplace's equation. Your report should contain answers to the questions, and the plot of equipotential lines.

Note: Coordinate with your colleagues working on related projects V and VI regarding similarities and differences between your results. Discuss in particular the properties of the solution close to the corners (where the potential jumps from 0 to 12 V).

Note that the goal of your presentation is to explain the results in a clear form so that your colleagues could understand them. Therefore, the presentation should be clear, coordinated and concise without extensive technical details - the details could go into the report. The code should be uploaded to canvas or emailed to the instructor.

Literature:
Farlow "Partial differential equations for scientists and engineers, Lesson 43";
Books of your choice.