http://www.busbar.com/resources/formulas/

$$°F := 1 \cdot \Delta°F$$

Properties:

$$T_{amb} := 40 \cdot °C = 72\ °F$$

$$A_{wire}(AWG) := \pi \cdot \left(\frac{5 \cdot 92^{\left(\frac{36 - AWG}{39}\right)} \cdot mil}{2} \right)^2$$

$$l_{max} := 3010\ ft$$

Properties of Copper at 20°C:

$$TC_{cu} := \frac{0.393\%}{°C}$$

$$\rho_{cu} := 0.68 \cdot 10^{-6} \cdot \Omega \cdot in$$

DC Resistance Formulas:

$$R_{DC.20C}(l, area) := \rho_{cu} \cdot \frac{l}{area}$$

Temperature Adjusted Resistance Formulas:

$$R_T(R_{20C}, T_{cu}) := R_{20C} \cdot (1 + TC_{cu} \cdot (T_{cu} - 20 \cdot °C))$$

$$R_T(1, 95 \cdot °C) = 1.295 \qquad R_T(1, 125 \cdot °C) = 1.413$$

$$R_{DC.T}(l, area, T_{cu}) := \rho_{cu} \cdot \frac{l}{area} \cdot (1 + TC_{cu} \cdot (T_{cu} - 20 \cdot °C))$$

Robot Cable Resistance Measurements:

$$R_{DC.T}(1000\ ft, A_{wire}(20.31), 22 \cdot °C) = 11.015\ \Omega$$ calibrate

$$R_{DC.T}(l_{max}, A_{wire}(20.31), T_{amb}) = 35.481\ \Omega$$

$$R_{DC.T}(1000\ ft, A_{wire}(14.71), 22 \cdot °C) = 3.006\ \Omega$$ calibrate

$$R_{DC.T}(l_{max}, A_{wire}(14.71), T_{amb}) = 9.684\ \Omega$$

Robot Cable Resistance Formula:

$$R_{wire.max} := R_{DC.T}(l_{max}, A_{wire}(20.31), T_{amb}) + R_{DC.T}(l_{max}, A_{wire}(14.71), T_{amb}) = 45.164\ \Omega$$

Robot Droop and Truck Voltage derivations from Ohms Law and P=I*V:

$$V_{robot} := 150\ V \qquad Pin_{robot} := 80\ W$$

$$I_{line} := \frac{Pin_{robot}}{V_{robot}} = 533.333\ mA$$

$$\frac{150\ V - 130\ V}{45\ \Omega} \cdot 130\ V = 57.778\ W$$

$$I_{line} \cdot R_{wire.max} = 24.088\ V$$

$$\frac{165\ V - 130\ V}{45\ \Omega} \cdot 130\ V = 101.111\ W$$

$$V_{droop} := \frac{Pin_{robot}}{V_{robot}} \cdot R_{wire.max} = 24.088\ V \qquad V_{truck} := V_{robot} + \frac{Pin_{robot}}{V_{robot}} \cdot R_{wire.max} = 174.088\ V$$

Output Power measurements:

Circuit Constants:

$V_{out} := 24\ V$

$\eta_{eff} := 0.7$

Right LED lamp Current: $I_{LED.r} := 1.1\ A$ $\quad Pd_{LED.r} := \dfrac{I_{LED.r} \cdot V_{out}}{\eta_{eff}}$

Left LED lamp Current: $I_{LED.l} := 1\ A$ $\quad Pd_{LED.l} := \dfrac{I_{LED.l} \cdot V_{out}}{\eta_{eff}}$

Input Voltage Measurements:

No-load input voltage: $Vt_0 := 154.2708\ V$

Single LED lamp input voltage: $Vt_1 := 143.4613\ V$

Dual LED lamps input voltage: $Vt_2 := 129.015\ V$

Circuit Simulation Values for checks:

$V_{truck} := 155\ V$

$Pd_{bias} := 2.5\ W$

$R_{line} := 45\ \Omega$

Three ways to calculate Truck Voltage:

$$V_{truck} = Vt_0 + \frac{Pd_{bias}}{Vt_0} \cdot R_{line} \qquad [1]$$

$$V_{truck} = Vt_1 + \frac{Pd_{LED.l} + Pd_{bias}}{Vt_1} \cdot R_{line} \qquad Pd_{LED.l} = \frac{I_{LED.l} \cdot V_{out}}{\eta_{eff}} \qquad [2]$$

$$V_{truck} = Vt_2 + \frac{Pd_{LED.l} + Pd_{LED.r} + Pd_{bias}}{Vt_2} \cdot R_{line} \qquad Pd_{LED.r} = \frac{I_{LED.r} \cdot V_{out}}{\eta_{eff}} \qquad [3]$$

Solve for Pdbias and Rline by setting [1] = [2]:

$$Vt_0 + \frac{Pd_{bias}}{Vt_0} \cdot R_{Line} = Vt_1 + \frac{Pd_{LED.l} + Pd_{bias}}{Vt_1} \cdot R_{line}$$

$$Vt_0 - Vt_1 = \left(\frac{Pd_{LED.l}}{Vt_1} + \frac{Pd_{bias}}{Vt_1} - \frac{Pd_{bias}}{Vt_0} \right) \cdot R_{line}$$

$$\frac{Vt_0 - Vt_1}{R_{line}} - \frac{Pd_{LED.l}}{Vt_1} = \left(\frac{1}{Vt_1} - \frac{1}{Vt_0} \right) Pd_{bias} = \frac{Vt_0 - Vt_1}{Vt_0 \cdot Vt_1} \cdot Pd_{bias}$$

$$Pd_{bias} = \left(\frac{Vt_0 - Vt_1}{R_{line}} - \frac{Pd_{LED.l}}{Vt_1} \right) \cdot \frac{Vt_0 \cdot Vt_1}{Vt_0 - Vt_1} \qquad [4]$$

$Pd_{bias} = 2.5\ W$

check: $\left(\dfrac{Vt_0 - Vt_1}{R_{line}} - \dfrac{Pd_{LED.l}}{Vt_1} \right) \cdot \dfrac{Vt_0 \cdot Vt_1}{Vt_0 - Vt_1} = 2.502\ W$

$$R_{line} = \frac{Vt_0 \cdot Vt_1}{Pd_{LED.l} \cdot \frac{Vt_0}{Vt_0 - Vt_1} + Pd_{bias}}$$ [5]

check: $$\frac{Vt_0 \cdot Vt_1}{Pd_{LED.l} \cdot \frac{Vt_0}{Vt_0 - Vt_1} + Pd_{bias}} = 45\ \Omega$$

$R_{line} = 45\ \Omega$

Solve for Pdbias by setting [1] = [3]:

$$Vt_0 + \frac{Pd_{bias}}{Vt_0} \cdot R_{line} = Vt_2 + \frac{Pd_{LED.l} + Pd_{LED.r} + Pd_{bias}}{Vt_2} \cdot R_{line}$$

$$Vt_0 - Vt_2 = \left(\frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2} + \frac{Pd_{bias}}{Vt_2} - \frac{Pd_{bias}}{Vt_0}\right) \cdot R_{line}$$

$$\frac{Vt_0 - Vt_2}{R_{line}} = \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2} + \frac{Pd_{bias}}{Vt_2} - \frac{Pd_{bias}}{Vt_0} = \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2} + \left(\frac{1}{Vt_2} - \frac{1}{Vt_0}\right) \cdot Pd_{bias}$$

$$\left(\frac{1}{Vt_2} - \frac{1}{Vt_0}\right) \cdot Pd_{bias} = \frac{Vt_0 - Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2} = \frac{Vt_0 - Vt_2}{Vt_0 \cdot Vt_2} \cdot Pd_{bias}$$

$$Pd_{bias} = \left(\frac{Vt_0 - Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2}\right) \cdot \frac{Vt_0 \cdot Vt_2}{Vt_0 - Vt_2}$$ [6]

check: $$\left(\frac{Vt_0 - Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2}\right) \cdot \frac{Vt_0 \cdot Vt_2}{Vt_0 - Vt_2} = 2.495\ W$$

$Pd_{bias} = 2.5\ W$

Solve for Pdbias by setting [2] = [3]

$$Vt_1 + \frac{Pd_{LED.l} + Pd_{bias}}{Vt_1} \cdot R_{line} = Vt_2 + \frac{Pd_{LED.l} + Pd_{LED.r} + Pd_{bias}}{Vt_2} \cdot R_{line}$$

$$\frac{Vt_1 - Vt_2}{R_{line}} = \left(\frac{Pd_{LED.l} + Pd_{LED.r} + Pd_{bias}}{Vt_2} - \frac{Pd_{LED.l} + Pd_{bias}}{Vt_1}\right)$$

$$\frac{Vt_1 - Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2} + \frac{Pd_{LED.l}}{Vt_1} = \left(\frac{1}{Vt_2} - \frac{1}{Vt_1}\right) \cdot Pd_{bias}$$

$$Pd_{bias} = \frac{\frac{Vt_1 - Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2} + \frac{Pd_{LED.l}}{Vt_1}}{\frac{1}{Vt_2} - \frac{1}{Vt_1}} = \frac{\frac{Vt_1 - Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2} + \frac{Pd_{LED.l}}{Vt_1}}{\frac{Vt_1 - Vt_2}{Vt_1 \cdot Vt_2}}$$

$$Pd_{bias} = \frac{Vt_1 \cdot Vt_2}{R_{line}} - \left(Pd_{LED.l} + Pd_{LED.r}\right) \cdot \frac{Vt_1}{Vt_1 - Vt_2} + Pd_{LED.l} \cdot \frac{Vt_2}{Vt_1 - Vt_2} \qquad [7]$$

check: $\frac{Vt_1 \cdot Vt_2}{R_{line}} - \left(Pd_{LED.l} + Pd_{LED.r}\right) \cdot \frac{Vt_1}{Vt_1 - Vt_2} + Pd_{LED.l} \cdot \frac{Vt_2}{Vt_1 - Vt_2} = 2.49\ W$

$Pd_{bias} = 2.5\ W$

Substitute [4] for Pdbias into [6] and solve for Rline:

$$\left(\frac{Vt_0 - Vt_1}{R_{line}} - \frac{Pd_{LED.l}}{Vt_1}\right) \cdot \frac{Vt_0 \cdot Vt_1}{Vt_0 - Vt_1} = \left(\frac{Vt_0 - Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2}\right) \cdot \frac{Vt_0 \cdot Vt_2}{Vt_0 - Vt_2}$$

$$\frac{Vt_1}{R_{line}} - \frac{Pd_{LED.l}}{Vt_0 - Vt_1} = \frac{Vt_2}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_0 - Vt_2}$$

$$Vt_1 - \frac{Pd_{LED.l}}{Vt_0 - Vt_1} \cdot R_{line} = Vt_2 - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_0 - Vt_2} \cdot R_{line}$$

$$R_{line} = \frac{Vt_2 - Vt_1}{\frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_0 - Vt_2} - \frac{Pd_{LED.l}}{Vt_0 - Vt_1}} \qquad [8]$$

check: $\frac{Vt_2 - Vt_1}{\frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_0 - Vt_2} - \frac{Pd_{LED.l}}{Vt_0 - Vt_1}} = 45.006\ \Omega$

$R_{line} = 45\ \Omega$

Test with new simulation measurements:

$$Vt_0 := 148.90872\ V$$

$$Vt_1 := 144.43907\ V$$

$$Vt_2 := 140.03065\ V$$

$$I_{LED.l} := 575\ mA$$

$$I_{LED.r} := 532\ mA$$

introduce errors for comparison:

$$Vt_0' := Vt_0 + 258\ \mu V$$

$$Vt_1' := Vt_1 + 350\ \mu V$$

$$Vt_2' := Vt_2 + 1\ V$$

$$I_{LED.l}' := I_{LED.l} + 0.5\ mA$$

$$I_{LED.r}' := I_{LED.r} - 0.5\ mA$$

Use equation [8] to calculate Rline and store as a constant (therefore, Vt0 only needs to be measured once):

$$Pd_{LED.l} := \frac{I_{LED.l} \cdot V_{out}}{\eta_{eff}} = 19.714\ W \qquad Pd_{LED.l}' := \frac{I_{LED.l}' \cdot V_{out}}{\eta_{eff}} = 19.731\ W$$

$$Pd_{LED.r} := \frac{I_{LED.r} \cdot V_{out}}{\eta_{eff}} = 18.24\ W \qquad Pd_{LED.r}' := \frac{I_{LED.r}' \cdot V_{out}}{\eta_{eff}} = 18.223\ W$$

$$R_{line} := \frac{Vt_2 - Vt_1}{\dfrac{Pd_{LED.l} + Pd_{LED.r}}{Vt_0 - Vt_2} - \dfrac{Pd_{LED.l}}{Vt_0 - Vt_1}} = 32.501\ \Omega$$

$$R_{line}' := \frac{Vt_2' - Vt_1'}{\dfrac{Pd_{LED.l}' + Pd_{LED.r}'}{Vt_0' - Vt_2'} - \dfrac{Pd_{LED.l}'}{Vt_0' - Vt_1'}} = -8.46\ \Omega$$

Use equation [7] to calculate Pdbias:

$$Pd_{bias} := \frac{Vt_1 \cdot Vt_2}{R_{line}} - \left(Pd_{LED.l} + Pd_{LED.r}\right) \cdot \frac{Vt_1}{Vt_1 - Vt_2} + Pd_{LED.l} \cdot \frac{Vt_2}{Vt_1 - Vt_2} = 4.981\ W$$

$$Pd_{bias}' := \frac{Vt_1' \cdot Vt_2'}{R_{line}'} - \left(Pd_{LED.l}' + Pd_{LED.r}'\right) \cdot \frac{Vt_1'}{Vt_1' - Vt_2'} + Pd_{LED.l}' \cdot \frac{Vt_2'}{Vt_1' - Vt_2'} = -3.2 \cdot 10^{3}\ W$$

$$\left(\frac{Vt_0 - Vt_2'}{R_{line}} - \frac{Pd_{LED.l} + Pd_{LED.r}}{Vt_2'}\right) \cdot \frac{Vt_0 \cdot Vt_2'}{Vt_0 - Vt_2'} = -71.244\ W$$

Use equation [3] to calculate Vtruck:

$$V_{truck} := Vt_2 + \frac{Pd_{LED.l} + Pd_{LED.r} + Pd_{bias}}{Vt_2} \cdot R_{line} = 149.996\ V$$

$$V_{truck}' := Vt_2' + \frac{Pd_{LED.l}' + Pd_{LED.r}' + Pd_{bias}'}{Vt_2'} \cdot R_{line}' = 330.696\ V$$

$$V_{truck}'' := Vt_2' + \frac{Pd_{LED.l} + Pd_{LED.r} + Pd_{bias}}{Vt_2'} \cdot R_{line} = 150.925\ V$$

$V_{truck} - V_{truck}'' = -929.341\ mV$ $V_{hyst} := 5\ V$ Use 5V hysteresis

Strategy:

During boot up, measure Vt0 by averaging 100 samples
Turn on the left LED light and wait 30ms min.
Measure Vt1 and I_LED.left over 100 samples averaging
Turn the right LED light on (both LED's on) and wait 50ms
Measure Vt2 and I_LED.rght over 100 samples averaging
Use equation 8 to calculate Rline and store as a constant
Use equation 7 to calculate external bias power Pdbias and store as a semi-constant variable.
Use equation 3 to measure the truck voltage Vtruck
Keep a rolling 100 sample over 100ms average of Vt2 (operating input voltage)

Periodically check Vt2 (every 150ms?) and recalculate the truck voltage Vtruck.
If the truck voltage increases to 160V, check the Pdbias value by turning off one LED (try to alternate), waiting 50ms, measuring Vt1, turning LED back on, and then use equation 7 to recalculate the Pdbias. Turn the motor on to raise the lift if there is "minimal" change in Pdbias.
If the truck voltage decreases from above 160V to below 155V, stop raising the lift (turn motor off).
If the truck voltage decreases to 150V, check the Pdbias value by turning off one LED, waiting 50ms, measureing Vt1, turning LED back on, and then use equation 7 to recalculate the Pdbias. turn the motor on to lower the lift if there is "minimal" change in Pdbias.
If the truck voltage increases from below 150V to 155V, stop lowering the lift (turn motor off).

Keep a 100 sample average of the input voltage and update the LED display value every 500 ms (so maximum 5ms sampling rate, preferably 1ms or 2ms sampling rate).