Assignment 6

Fall 2014
CS595 Web Science
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Contents

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Listings

1 Question 1

1.1 Question

```
We know the result of the Karate Club (Zachary, 1977) split.
Prove or disprove that the result of split could have been predicted
by the weighted graph of social interactions.  How well does the
mathematical model represent reality?

Generously document your answer with all supporting equations, code,
graphs, arguments, etc.
```

1.2 Answer

As was illustrated in the original study of Zachary's karate club, [1], a prediction of the structure of the club if a separation were to occur can be made with a high degree of accuracy using weighted edges based on the perceived "strength" of each relationship it modeled. The prediction method outlined in the original paper was an implementation of the *maximum flow-minimum cut labeling procedure* [2]. The pickled [3] dataset of the existing karate club, with weights for each edge, was obtained from `http://nexus.igraph.org/api/dataset_info?id=1&format=html` and used to create the graph shown in Figure 1.
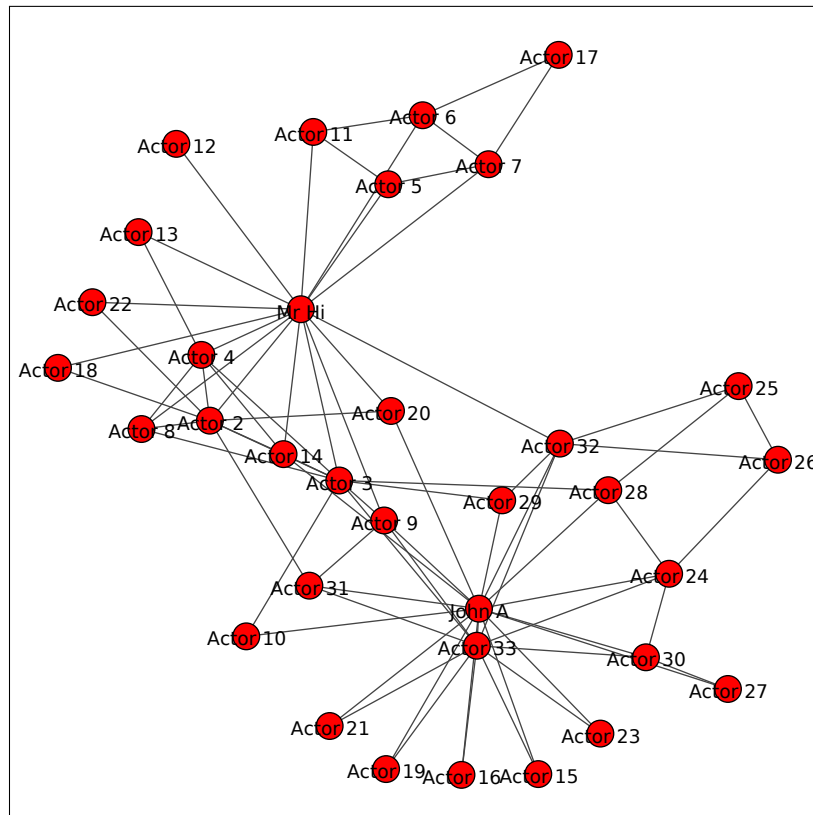

Figure 1: The Existing Graph

The actual structure of the two resulting clubs after the split are shown in Figure 2.
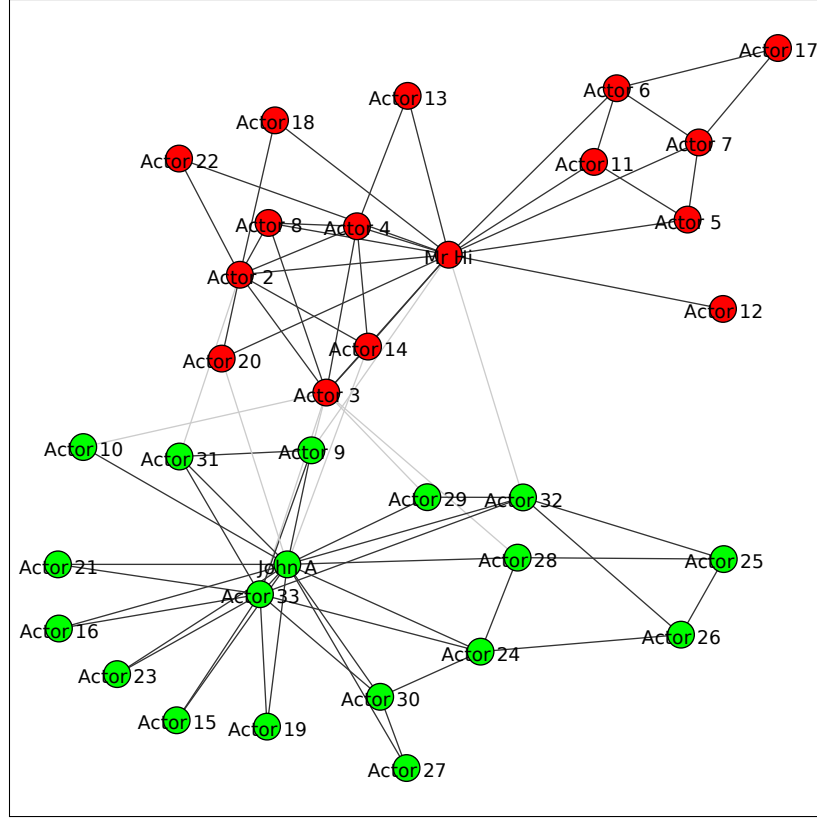

Figure 2: Actual Graph After Split

To predict a separation of the existing graph into two or more distinct community components a pair of community detection algorithms will be employed and the results will be compared to the actual results of the split from Zachary's original study [1]. Community Detection was chosen as a means for predicting the results of fission events because it is logical that a given community would less likely be split along strong inter-community edges than those weaker, community-spanning edges.

The first algorithm used was the Edge Betweenness algorithm, developed by Girvan and Newman [4]. This is a divisive algorithm that removes edges that have the highest betweenness measure because these tend to be community-spanning edges.
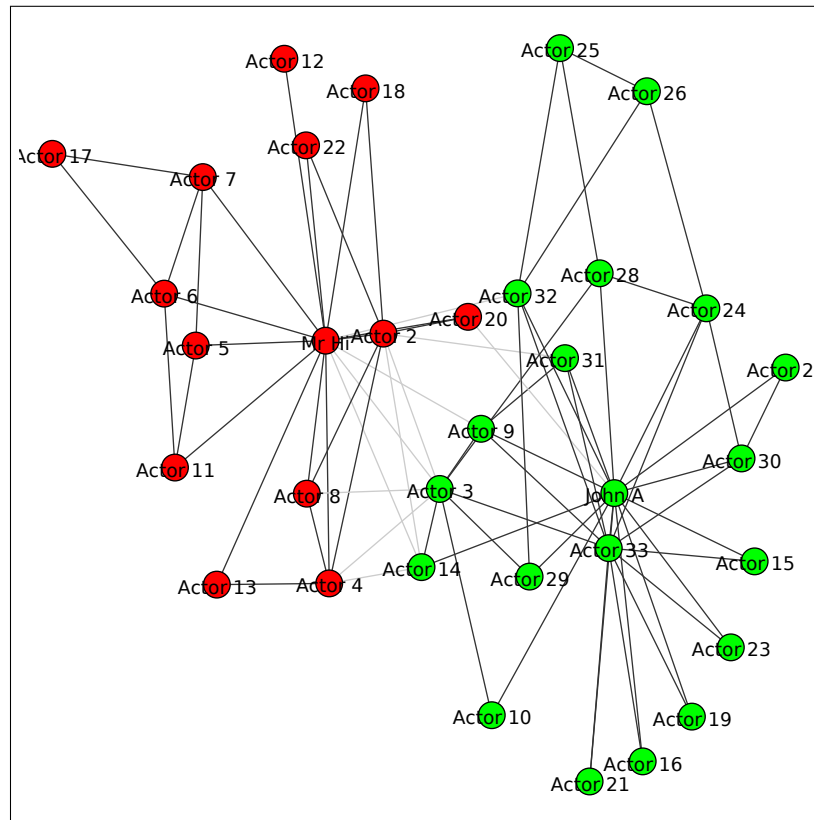

Figure 3: Prediction of Edge Betweenness Algorithm

```
Edge Betweenness method results:
Variant elements:
[3, 14]
94.12% accuracy
```

The second method used was the Leading Eigenvector algorithm developed my M. Newman [5]. This method uses a special matrix, called the modularity matrix, to
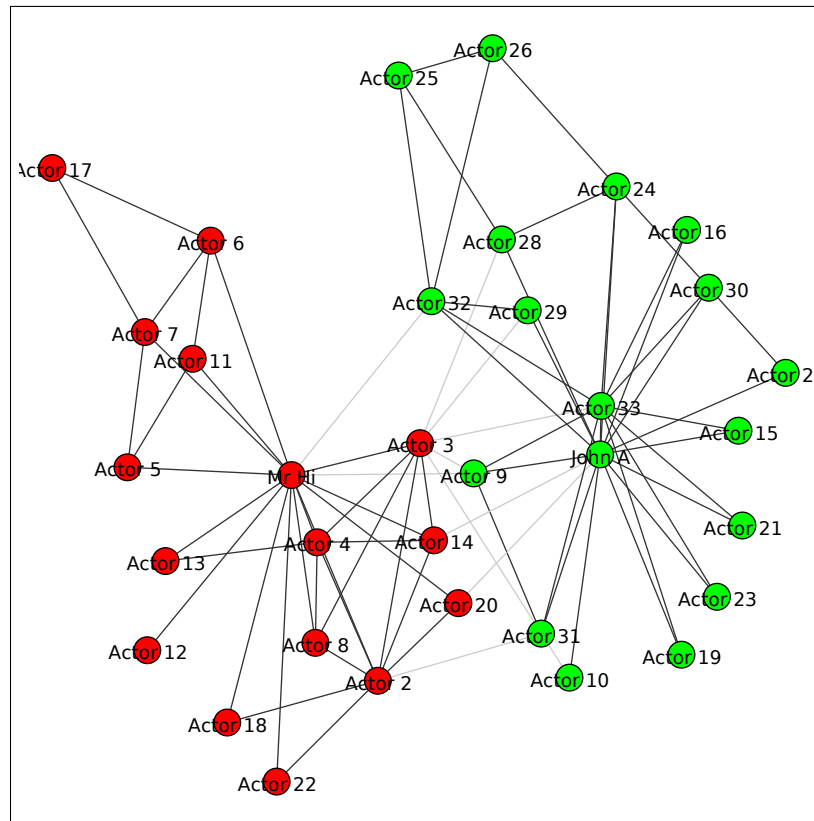

Figure 4: Prediction of Leading Eigenvector Algorithm

```
Leading Eigenvector method results:
Variant elements:
[]
100.0 % accuracy
```

The python code to produce these graphs is shown in Listing 1.

```python
#! /usr/bin/env python

import pickle
import igraph
from igraph import plot

FILENAME = 'karate.pickle'

# load the graph
data = pickle.loads(open(FILENAME).read())['karate']

# Create faction list of actual group membership after split,
# translated to values that would mirror a cut.membership list
factions_after_split = map(lambda x: int(x-1), data.vs['Faction'])

def compare(predicted):
    """
    Compares the predicted and actual numerical lists and
    returns a list of variant members and the accuracy of the prediction
    """
    res = []
    for idx, val in enumerate(factions_after_split):
        if predicted[idx] != val:
            # Translate index to match original dataset
            res.append(idx + 1)
    return res, 100 - round(float(len(res)) / float(len(factions_after_split)) * 100, 2)

def print_results(method, res, acc):
    print("{} method results: \nVariant elements:\n\t{}\n{} % accuracy".format(method, res,
        acc))

if __name__ == '__main__':
    # Plot existing graph
    layout = data.layout('fr')
    plot(data, "initial_karate_graph.pdf", layout=layout, vertex_label=data.vs['name'],
        margin=30)

    # Girvan-Newman Edge Betweenness method
    com_eb = data.community_edge_betweenness(
        clusters=2,
        directed=False,
        weights=data.es['weight'])
    clust_eb = com_eb.as_clustering()
    res_eb, acc_eb = compare(clust_eb.membership)
    plot(clust_eb, "clust_eb.pdf", vertex_label=data.vs['name'], margin=25)
    print_results("Edge Betweenness", res_eb, acc_eb)

    # Newman Leading Eigenvector method
    clust_le = data.community_leading_eigenvector(clusters=2, weights=data.es['weight'])
    res_le, acc_le = compare(clust_le.membership)
    plot(clust_le, "clust_le.pdf", vertex_label=data.vs['name'], margin=25)
    print_results("Leading Eigenvector", res_le, acc_le)

    # Plot  3..5 community predictions
    for i in xrange(3, 6):
        cluster = data.community_edge_betweenness(
            clusters=i,
            directed=False,
            weights=data.es['weight']).as_clustering()
        plot(cluster, "cluster" + str(i) + ".pdf", vertex_label=data.vs['name'], margin=25)
```

Listing 1: Finding Communities in Zachary's Karate Club

2 Question 2

2.1 Question

```
We know the group split in two different groups.  Suppose the
disagreements in the group were more nuanced -- what would the clubs
look like if they split into groups of 3, 4, and 5?
```

2.2 Answer

The Edge Betweenness algorithm was run for target clusterings of three, four and five and the results are in Figures 5, 6, and 7.
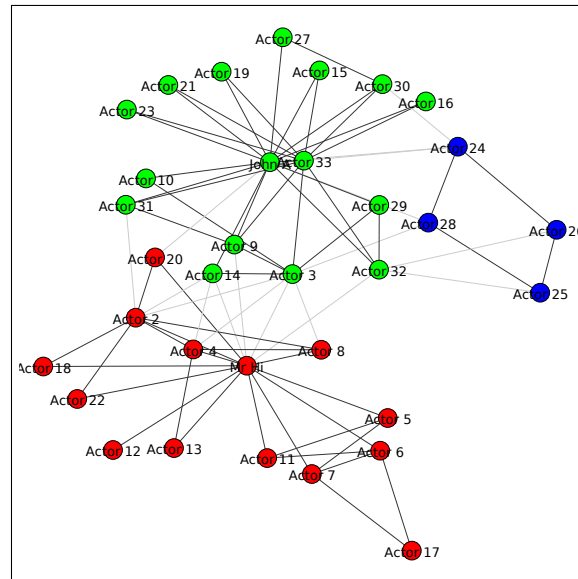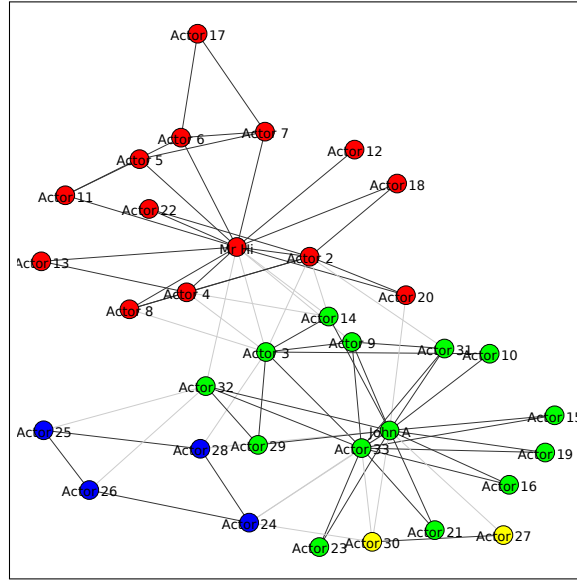

Figure 5: 3-Cluster Prediction
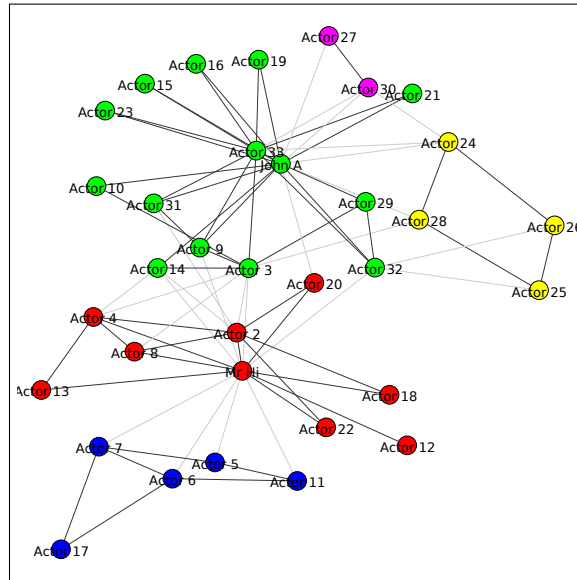
Figure 6: 4-Cluster Prediction

Figure 7: 5-Cluster Prediction

3 References

[1] Wayne W. Zachary. An Information Flow Model for Conflict and Fission in Small Groups, 1977.

[2] L. R. Ford, Jr. and D. R. Fulkerson. *Flows in Networks*. 1962.

[3] The Python Software Foundation. Python pickle module. https://docs.python.org/2/library/pickle.html, 2014.

[4] M. E. J. Newman and M. Girvan. Finding and Evaluating Community Structure in Networks. 69, 2004.

[5] M. E. J. Newman. Finding Community Structure in Networks Using the Eigenvectors of Matrices. 74, 2006.