Coursera ML Notes

Stanford: Andrew Ng's "Machine Learning"

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1 Week 1

1.1 Introduction

Pretty simple stuff. House price example, news classification example etc. etc.

1.2 Model and cost function

Let $x^{(i)}$ denote the "input" variables/features, and $y^{(i)}$ denote the "output" or target variable we wish to predict. A pair $(x^{(i)}, y^{(i)})$ is called a **training example**, and the dataset that we'll be using to learn:

$$(x^{(i)}, y^{(i)}), \quad i = 1, \dots, m,$$

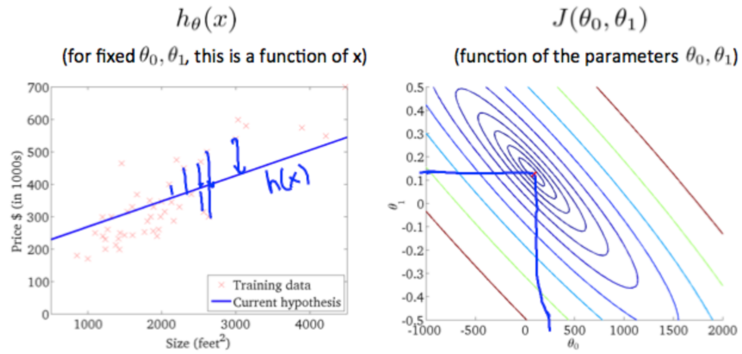
is called a **training set**. Superscript denotes an index into the training set, which isn't too bad notation when you start looking at gradient descent etc. For the purposes of this introductory course, we use X to denote the space of input values and Y to denote the space of output values, and $X = Y = \mathbb{R}$.

In supervised learning, the goal is given a training set to learn a function $h : X \to Y$ such that $h(x)$ is a good predictor for y. h is called a **hypothesis** for historical reasons. Discrete $\implies$ classification, continuous $\implies$ regression (but it can be more complicated than that.

To measure the accuracy of our hypothesis function (in linear regression), we can use a **cost function**. In the context of linear regression, this cost function is least squares.

$$J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2 \quad \equiv \quad \frac{1}{2}\|Hx - y\|^2,$$

where H is some matrix but that isn't important for this since we are using *Ng-otation* and I just wanted to show it's OLS. The function is called the "squared error function", "mean squared error" or MSE. The mean is halved as a convenience for gradient descent (because the squared carries over to the fraction, useful for the normal equation). Since this function represents the *cost*, we wish to minimise it.



1.3 Parameter learning

We want to estimate the parameters in the hypothesis function, which is where **gradient descent** comes in. Graphing our hypothesis function as a function of the parameter estimates gives a surface. Gradient descent uses the fact that the gradient is the direction towards the minimum (very roughly speaking) to perform an iterative movement towards the minimum. I would write more detail about gradient descent but I'm literally doing a summer project on optimisation so I don't think there's much point, refer to my optimisation notes for more details. This algorithm gives an iterate

$$\theta_j := \theta_j - \alpha\frac{\partial}{\partial\theta_j}J(\theta_0, \theta_1),$$

where $j = 0, 1$ represents the feature index number.

Update the parameters simultaneously, or else gradient descent won't work!

(Note: this fact would be obvious if we employed the gradient operator $\nabla J(\theta_0, \theta_1)$ instead of Ng's notation, but I digress.)

When specially applied to linear regression, we can obtain a new form of gradient descent. Replacing the cost function and hypothesis function, we obtain the following iterate:

$$\theta_0 := \theta_0 - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)$$
$$\theta_1 := \theta_1 - \alpha\frac{1}{m}\sum_{i=1}^{m}((h_\theta(x_i) - y_i)x_i)$$

These can both be verified with some calculus.

2 Week 2

2.1 Multivariate linear regression

Linear regression with multiple variables is also known as **multivariate linear regression**. In the more general case, we can have the following:

$x_j^{(i)}$	value of feature j in the ith training example
$x^{(i)}$	the input of the ith training example
m	the number of training examples
n	the number of features

The multivariate form of this is

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \ldots + \theta_n x_n \quad \equiv \quad h_\theta(x) = \theta^\top x$$

where $\theta = \begin{pmatrix}\theta_0 & \theta_1 & \ldots & \theta_n\end{pmatrix}$ and $x = \begin{pmatrix}x_0 & x_1 & \ldots & x_n\end{pmatrix}^\top$.

For gradient descent in the multivariate case using Ng's notation we have the following iterate:

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)} - y^{(i)}) \cdot x_j^{(i)}, \quad \forall j \in \{0, \ldots, n\}.$$

We introduce a practical trick for gradient descent called **feature scaling**. The idea is to ensure that features are on a similar scale, making gradient descent faster. We get every feature into *approximately* a $-1 \leq x_i \leq 1$ range. You can also perform **mean normalisation**, where we replace x_i with $x_i - \mu_i$ to make features have approximately zero mean, making sure not to apply to $x_0 = 1$ or anything like that. You can also divide by the standard deviation s_i instead of the range.

For gradient descent, we can also use something called the **automatic convergence test**. Declare convergence if $J(\theta)$ decreases by less than ε in one iteration, where ε is some small value. (It's difficult to choose this threshold value). If α is too small, we have a slow convergence rate, but if α is too large, we may not decrease on every iteration and thus may not converge.

We may also have a situation where we have polynomial regression. We can change the behaviour or curve of our hypothesis function by making it a quadratic, cubic or square root function (or any other form). For example, if our hypothesis function is $h_\theta(x) = \theta_0 + \theta_1 x$, we can create additional features based on x_1 to get a quadratic with $\theta_2 x_1^2$ or even a cubic with both $\theta_2 x_1^2$ and $\theta_3 x_1^3$ added.

2.2 Computing parameters analytically

We have been using iterative algorithms so far such as gradient descent, but for some problems we can solve for θ analytically. The **normal equation** formula (the Moore-Penrose pseudoinverse times y for the enlightened) is the solution to the problem

$$\theta = (X^\top X)^{-1} X^\top y.$$

There is no need to do feature scaling or any other bollocks with the normal equation. However, the normal equation has complexity $\mathcal{O}(n^3)$ and you may need to compute the inverse of $X^\top X$, so it can be really slow for large n.

Gradient Descent	Normal Equation
Need to choose α	No need to choose α
Needs many iterations	No need to iterate
$\mathcal{O}(kn^2)$	$\mathcal{O}(n^3)$ and inverse
Works well when n is large	Slow if n is large

But look at the normal equation; we have an inverse. Not every matrix has an inverse. Using `pinv` in MATLAB gets around this in some fancy ways, but we should still be alarmed if $X^\top X$ is noninvertible, as there may be:

- Redundant features, where two features are very closely related (linearly dependent)
- Too many features ($m \leq n$). In this case, delete some features or use "regularisation".