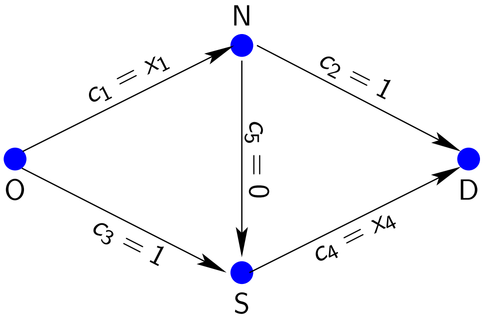
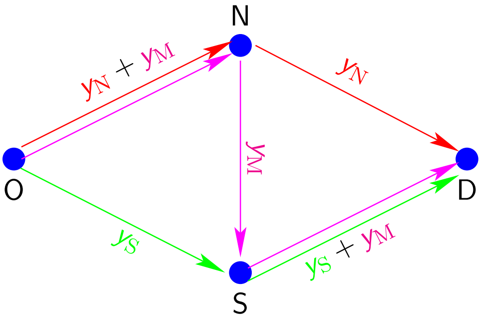
First network example: Braess network

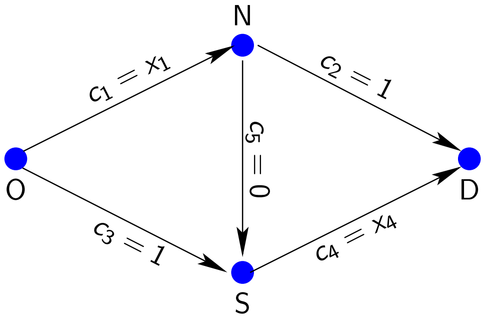


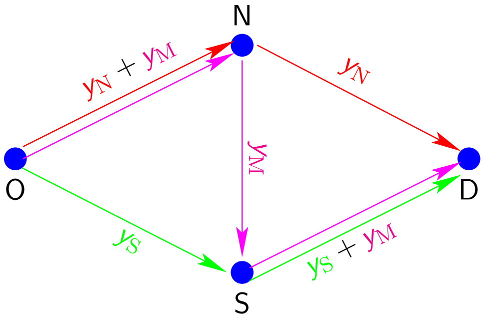


- Traffic flows from O to D via two junctions (**N**orth and **S**outh).
- Note: extreme choices for link costs — link 5 is a kind of perfect short cut!!!!
- Note: symmetry in the network: link 1 $\equiv$ link 4 and link 2 $\equiv$ link 3

- Three distinct routes, **N**orth, **S**outh and **M**idtown.
- **Link flows** x can be expressed in terms of **route flows** y.
- Low demand: all traffic takes the short-cut midtown route.

Braess network: UE solution





- Route costs (per user) are:

$c_\mathrm{N} = 1 + y_\mathrm{N} + y_\mathrm{M}$
$c_\mathrm{S} = 1 + y_\mathrm{S} + y_\mathrm{M}$
$c_\mathrm{M} = y_\mathrm{N} + y_\mathrm{S} + 2y_\mathrm{M}$

- Suppose demand $d = 1$.

- UE solution is

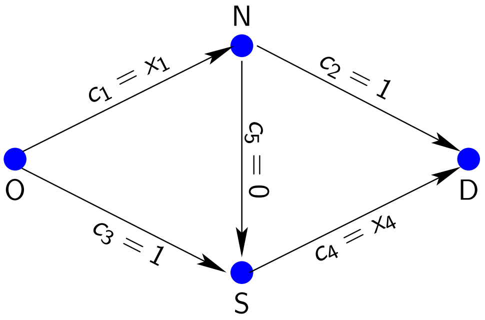
$$y_\mathrm{M} = 1 \quad \text{and} \quad y_\mathrm{N} = y_\mathrm{S} = 0.$$

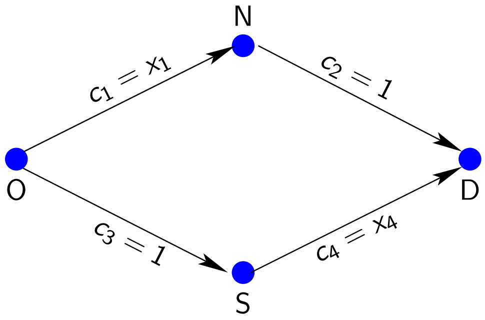
- Check route costs (per user):

$c_\mathrm{M} = 2 \leq$ (actually equals) $c_\mathrm{N}, c_\mathrm{S}$

- UE System cost $f = 2$ (remember this).

Braess network: SO solution and the paradox





- Recall: for $d = 1$, the UE solution has all traffic using the midtown route ($y_{\mathrm{M}} = 1$) with system cost $f = 2$.
- If instead $y_{\mathrm{N}} = y_{\mathrm{S}} = 1/2$ and $y_{\mathrm{M}} = 0$, we get $f = 3/2$, and this is the System Optimal (SO) assignment — with $c_{\mathrm{N}} = c_{\mathrm{S}} = 3/2$ and $c_{\mathrm{M}} = 1$.
- But $y_{\mathrm{N}} = y_{\mathrm{S}} = 1/2$ is the solution of the UE problem with the midtown link deleted.
- Upshot: closing the midtown link improves the traffic!!! (reduces system cost of UE solution)