Stat 509/Econ 580 - HW 1

This Homework is due in the Quiz Section next Friday, October 6.

1. A and B are two events with $P(A) = 0.2$ and $P(B) = 0.6$.

(a) Can A and B be mutually exclusive? Explain.

(b) Can A^C and B^C be mutually exclusive? Explain.

(c) Can A and B be *independent*, so that $P(A \cap B) = P(A)P(B)$?

(d) In general, suppose that C and D are events that are both mutually exclusive and independent. What does this tell us about the smaller of these two probabilities, that is $\min\{P(C), P(D)\}$?

2. Suppose that A, B and C are three events. $P(A \cup B) = p$, $P(A \cup C) = q$.

(a) If we have no additional information, what can be said about $P(A)$?
Hint: What is the largest or smallest $P(A)$ *could be?*

(b) If we assume that B and C are disjoint events, so $B \cap C = \emptyset$, what can be said about $P(A)$?

Hint: consider all possible values for p *and* q *in your answers.*

3. A and B are events with $P(A) = 0.7$, $P(A \cup B) = 0.82$ and $P(A \cap B) = 0.28$.

(a) Are A and B mutually exclusive?

(b) What can be said about $P(B)$?

(c) Could A and B be dependent?

4. If X is a random variable taking integer values, if $P(X \leq 1) = s$, $P(X > 0) = t$, find $P(X = 1)$ in terms of s and t.

5. A population of n people contains k individuals with a disease. A sample of m are selected at random (without replacement) from the n in the population.

(a) For a given sample size m, find the probability that there is at least one person with the disease.
Hint: calculate the probability that the first person sampled does not have the disease; then that the second also does not, given that the first does not. . . .

(b) If $n = 1000$ and $k = 15$, find the smallest integer m such that the probability in (a) is ≥ 0.8.

(c) If $n = 10,000$ and $k = 150$, find the smallest integer m such that the probability in (a) is ≥ 0.8.

You may want to use `R` or a spreadsheet for parts (b) and (c).

In `R` you may find the functions `choose`, `factorial` and `lfactorial` useful; also recall that when finding the ratio A/B of two 'big' numbers A and B, we may use that $A/B = e^{\log A - \log B}$.

(d) Use a Binomial probability to obtain an approximate answer to (c) by a simple calculation.

Hint: Intuitively, as $n, k \to \infty$, $k/n \to 0.015$, *will it make a difference whether we are sampling with replacement or without replacement?*

6. Goldberger Qu. 2.8(b)

7. Goldberger Qu. 2.10