Artificial Intelligence

SCALAB
Grupo de Inteligencia Artificial

Universidad Carlos III de Madrid

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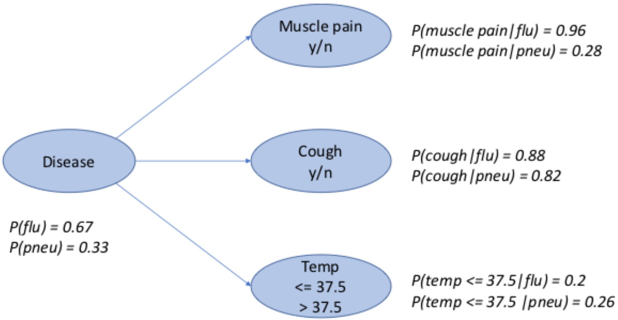
Bayesian networks – Exercises

Exercise 1

Consider the following Bayesian Network



- Given the evidence $e = \{Temp > 37.5\}$, compute the probability of *flu* (i.e. $P(flu/Temp > 37.5)$)
- Given the evidence $e = \{Temp > 37.5, Cough = yes\}$, compute the probability of *flu* (i.e. $P(flu/Temp > 37.5, cough)$)

(Bayes Nets with this particular structure are called Naive Bayes models)

Solution – Exercise 1

$$P(D, C, M, T) = P(M/D)P(C/D)P(T/D)P(D)$$

$$\begin{aligned}
P(D = flu/T > 37.5) &= \alpha \sum_{m,c} P(M = m/D = flu)P(C = c/D = flu)P(T > 37.5/D = flu)P(D = flu) = \\
&\alpha P(T > 37.5/D = flu)P(D = flu) \sum_{m} P(M = m/D = flu) \sum_{c} P(C = c/D = flu) = \\
&\alpha P(T > 37.5/D = flu)P(D = flu) = \alpha 0.8 \times 0.67 = \alpha 0.536 \approx 0.7
\end{aligned}$$

$$\begin{aligned}
P(D = pneu/T > 37.5) &= \alpha \sum_{m,c} P(M = m/D = pneu)P(C = c/D = pneu)P(T > 37.5/D = pneu)P(D = pneu) = \\
&\alpha P(T > 37.5/D = pneu)P(D = pneu) \sum_{m} P(M = m/D = pneu) \sum_{c} P(C = c/D = pneu) = \\
&\alpha P(T > 37.5/D = pneu)P(D = pneu) = \alpha 0.74 \times 0.33 = \alpha 0.2442 \approx 0.3
\end{aligned}$$

$$\alpha = \frac{1}{0.536 + 0.2442}$$

Solution – Exercise 1

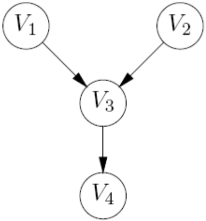
$$P(D, C, D, T) = P(D/D)P(C/D)P(T/D)P(D)$$

$$P(flu/T > 37.5, cough) = \alpha P(cough/flu)P(T > 37.5/flu)P(flu)\sum_{m} P(D = d/D = flu) =$$

$$\alpha P(cough/flu)P(T > 37.5/flu)P(flu) = \alpha 0.88 \times 0.8 \times 0.67$$

Exercise 2

Consider the following Bayesian network where all variables are binary



1. Define a probability distribution for this Bayesian network
2. Compute the marginal probability distribution of P($V4$)
3. Now, assume we observe that $V_2 = \text{true}$. What is the *a posteriori* probability distribution of V_2? What is the *a posteriori* probability distribution of V_4?
4. Next, assume we observe that $V_4 = true$ (consider V_2 is again unknown). Compute the *a posteriori* probability of V_2.
5. Compute the joint probability distribution of $V_1 = true$, $V_2 = false$, $V_3 = true$, $V_4 = true$.

Solution – Exercise 2

1. $P(V_1, V_2, V_3, V_4) = P(V_4/V_3)P(V_3/V_1, V_2)P(V_1)P(V_2)$
2. $P(V_4) = \sum_{v1,v2,v3} P(V_4/V_3 = v3)P(V_3 = v3/V_1 = v1, V_2 = v2)P(V_1 = v1)P(V_2 = v2)$
3. $P(V_2/V_2 = true) = (1, 0)$
4. $P(V_2/V_4 = true)$

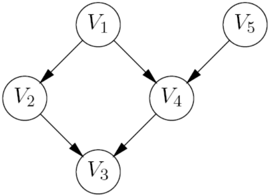
   $$P(V_2 = true/V_4 = true) =$$
   $$= \alpha P(V_2 = true, V_4 = true)$$
   $$= \alpha \sum_{v1,v3} P(V_4 = true/V_3 = v3)P(V_3 = v3/V_1 = v1, V_2 = true)P(V_1 = v1)P(V_2 = true)$$
   $$= \alpha P(V_2 = true)\sum_{v1} P(V_1 = v1)\sum_{v3} P(V_4 = true/V_3 = v3)P(V_3 = v3/V_1 = v1, V_2 = true) = \alpha \times A$$

   $P(V_2 = false/V_4 = true) = \alpha \times B$ is computed similarly. Then we can extract $\alpha = \frac{1}{A+B}$
5. $P(V_1 = true, V_2 = false, V_3 = true, V_4 = true) = P(V_4 = true/V_3 = true)P(V_3 = true/V_1 = true, V_2 = false)P(V_1 = true)P(V_2 = false)$

You can invent the probabilities for this network and try to make the computations with numbers

Exercise 3

Consider the following Bayesian network



$$
\begin{array}{ll}
P(v_1) = 0.2 & \\
P(v_5) = 0.8 & \\
P(v_2 \mid v_1) = 0.5 & P(v_2 \mid \neg v_1) = 0.4 \\
P(v_3 \mid v_2, v_4) = 0.4 & P(v_3 \mid \neg v_2, v_4) = 0.7 \\
P(v_3 \mid v_2, \neg v_4) = 0.3 & P(v_3 \mid \neg v_2, \neg v_4) = 0.6 \\
P(v_4 \mid v_1, v_5) = 0.6 & P(v_4 \mid \neg v_1, v_5) = 0.2 \\
P(v_4 \mid v_1, \neg v_5) = 0 & P(v_4 \mid \neg v_1, \neg v_5) = 1
\end{array}
$$

- Compute the probability $P(\neg v_5 / v_3)$, i.e. for V_5 equal to false given that V_3 is equal to true.

Solution – Exercise 3

$$P(\neg v_5/v_3) = \alpha \sum_{v_1,v_2,v_4} P(v_3/v_2,v_4)P(v_2/v_1)P(v_4/v_1,\neg v_5)P(\neg v_5)P(v_1)$$

$$= \alpha P(\neg v_5)\sum_{v_1} P(v_1)\sum_{v_2} P(v_2/v_1)\sum_{v_4} P(v_3/v_2,v_4)P(v_4/v_1,\neg v_5)$$

$$= \alpha P(\neg v_5)\times$$

$$[P(v_1)$$

$$[P(v_2/v_1)[P(v_3/v_2,v_4)P(v_4/v_1,\neg v_5) + P(v_3/v_2,\neg v_4)P(\neg v_4/v_1,\neg v_5)]+$$

$$P(\neg v_2/v_1)[P(v_3/\neg v_2,v_4)P(v_4/v_1,\neg v_5) + P(v_3/\neg v_2,\neg v_4)P(\neg v_4/v_1,\neg v_5)]]+$$

$$P(\neg v_1)$$

$$[P(v_2/\neg v_1)[P(v_3/v_2,v_4)P(v_4/\neg v_1,\neg v_5) + P(v_3/v_2,\neg v_4)P(\neg v_4/\neg v_1,\neg v_5)]+$$

$$P(\neg v_2/\neg v_1)[P(v_3/\neg v_2,v_4)P(v_4/\neg v_1,\neg v_5) + P(v_3/\neg v_2,\neg v_4)P(\neg v_4/\neg v_1,\neg v_5)]]]$$

All these values can be taken from CPTs

Exercise 4

- A used car sales man offers all potential customers to have a test performed on the car they are interested in buying. The test should reveal whether the car has either no defects or one (or more) defects; the prior probability that a car has no defects is 0.3.
- There are two possible tests:
  - Test1 has three possible outcomes, namely no-defects, defects and inconclusive. If the car doesn't have any defects, then the probabilities for these test results are 0.8, 0.05 and 0.15, respectively. On the other hand, if the car has defects, then the probabilities for the test results are 0.05, 0.75 and 0.2.
  - For Test2 there are only two possible outcomes (no-defects and defects). If the car doesn't have any defects, then the probabilities for the test results are 0.8 and 0.2, respectively, and if the car has defects then the probabilities are 0.25 and 0.75.
- Construct a Bayesian network (both structure and CPTs) representing the relations between the two tests and the state of the car.

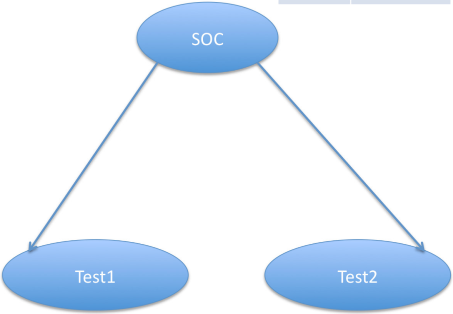
Exercise 4: Solution

- Variables:
  - `StateOfCar`: Whether the car has defects or not
    `SOC` $\in$ {defects,no-defects}
  - `Test1`: Results of Test1
    `Test1` $\in$ {defects, no-defects, inconclusive}
  - `Test2`: Results of Test2
    `Test2` $\in$ {defects,no-defects}

Exercise 4: Solution

- Structure and CPTs of the BN:

defect	no-defect
0.7	0.3



SOC	P(defect)	P(no-defect)	P(inconclusive)
defect	0,75	0,05	0,20
no-defect	0,05	0,80	0,15

SOC	P(defect)	P(no-defect)
defect	0,75	0,25
no-defect	0,20	0,80

Exercise 5

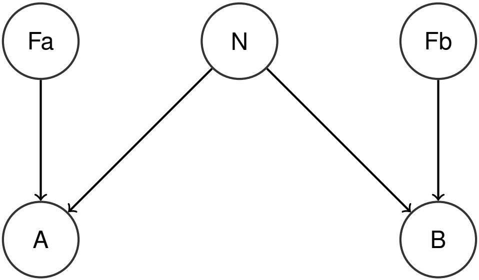
Two astronomers (A and B) are estimating the number of stars in an observable region of the universe using their respective telescopes with similar characteristics. The number of stars in each region of universe varies uniformly between 0 and 3. If the telescopes are focused correctly, which occurs in 9 out of 10 occasions, the obtained estimation corresponds to the real value 90 % of the times and in the remaining 10 % the estimation is incorrect by one star. If the telescopes are not focused correctly, the estimate differs from the real value only by one star 50 % of cases. Using all the information:

- Build a Bayesian network (both structure and CPTs) representing the domain

Exercise 5: Solution

- Random variables:
  - *N*: Real numbers of stars in that region of the universe. $N \in \{0, 1, 2, 3\}$
  - *A*: Number of stars estimated using the telescope of astronomer A. $A \in \{0, 1, 2, 3\}$
  - *B*: Number of stars estimated using the telescope of astronomer B. $B \in \{0, 1, 2, 3\}$
  - *Fa*: Telescope of Astronomer A is correctly focused. $Fa \in \{f, \neg f\}$
  - *Fb*: Telescope of Astronomer B is correctly focused. $Fb \in \{f, \neg f\}$

Exercise 5: Solution



$$P(N, A, B, Fa, Fb) = P(A \mid Fa, N)\, P(B \mid N, Fb)\, P(Fa)\, P(Fb)\, P(N)$$

Exercise 5: Solution

- $P(N = 0) = P(N = 1) = P(N = 2) = P(N = 3) = 0.25$
- $P(Fa = f) = 0.9$
- $P(Fa = \neg f) = 0.1$
- $P(Fb = f) = 0.9$
- $P(Fb = \neg f) = 0.1$

Exercise 5: Solution

- $P(A \mid Fa, N)$

	$Fa = f$				$Fa = \neg f$			
	$N = 0$	$N = 1$	$N = 2$	$N = 3$	$N = 0$	$N = 1$	$N = 2$	$N = 3$
A=0	0.9	0.05	0	0	0.5	0.25	0	0
A=1	0.1	0.9	0.05	0	0.5	0.5	0.25	0
A=2	0	0.05	0.9	0.1	0	0.25	0.5	0.5
A=3	0	0	0.05	0.9	0	0	0.25	0.5

Exercise 5: Solution

- $P(B \mid Fb, N)$

	$Fb = f$				$Fb = \neg f$			
	$N = 0$	$N = 1$	$N = 2$	$N = 3$	$N = 0$	$N = 1$	$N = 2$	$N = 3$
B=0	0.9	0.05	0	0	0.5	0.25	0	0
B=1	0.1	0.9	0.05	0	0.5	0.5	0.25	0
B=2	0	0.05	0.9	0.1	0	0.25	0.5	0.5
B=3	0	0	0.05	0.9	0	0	0.25	0.5

Exercise 6

Suppose you are working for a financial institution and you are asked to build a fraud detection system. You plan to use the following information:

- When the card holder is traveling abroad, fraudulent transactions are more likely since tourists are prime targets for thieves. More precisely, 1 % of transactions are fraudulent when the card holder is traveling, whereas only 0.2 % of the transactions are fraudulent when he is not traveling.
- On average, 5 % of all transactions happen while card holder is traveling. If a transaction is fraudulent, then the likelihood of a foreign purchase increases, unless the card holder happens to be traveling. More precisely, when the card holder is not traveling, 10 % of the fraudulent transactions are foreign purchases, whereas only 1 % of the legitimate transactions are foreign purchases.
- On the other hand, when the card holder is traveling, 90 % of the transactions are foreign purchases regardless of the legitimacy of the transactions.

Exercise 6

1. Construct a Bayesian network (both structure and CPTs) representing the relations of the fraud detection system.
2. The system has detected a foreign purchase. What is the probability of a fraud if we don't know whether the card holder is traveling or not?
3. Suppose an agent calls the client to confirm a transaction but the card holder is not at home. Her spouse confirms that she is out of town on a business trip. How does the probability of the fraud changes based on this new piece of information?

Exercise 6: Solution

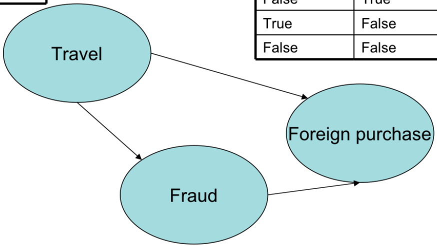
- Variables:
  - Travel: Boolean variable
  - Fraud: boolean variable
  - Foreign purchase: boolean variable
- Relations:
  - Causal inference: Increased probability of travel makes fraud more likely. Travel can cause fraud.
  - Diagnostic inference: Increased probability of foreign purchase makes fraud more likely. Foreign purchase is evidence for fraud.
  - Intercausal inference: Travel and fraud can each cause foreign purchase. Travel explains foreign purchase and so is evidence against fraud.

Exercise 6: Solution

- Structure and CPTs of BN:

True	False
0.05	0.95

Travel	Fraud	True	False
True	True	0.90	0.10
False	True	0.10	0.90
True	False	0.90	0.10
False	False	0.01	0.99



Travel	True	False
True	0.01	0.99
False	0.002	0.998

Exercise 6: Solution

- The system has detected the foreign purchase. What is the probability of a fraud if we don't know whether the card holder is traveling or not?

Exercise 6: Solution

- The system has detected the foreign purchase. What is the probability of a fraud if we don't know whether the card holder is traveling or not?
- Query: Fraud
- Evidence: Foreign purchase
- Hidden variable: Travel
- We need to compute

Exercise 6: Solution

- The system has detected the foreign purchase. What is the probability of a fraud if we don't know whether the card holder is traveling or not?
- Query: Fraud
- Evidence: Foreign purchase
- Hidden variable: Travel
- We need to compute P(Fraud=true|Foreign-purchase=true)

Exercise 6: Solution

- P(fraud|foreign-purchase)
  - =
    α * [P(fraud|travel) * P(foreign-purchase|travel, fraud) * P(travel)
    + P(fraud|¬travel) * P(foreign-purchase|¬travel,fraud)*P(¬travel)]
  - = α * [0.01 *0.90 *0.05 + 0.002* 0.10* 0.95]
  - = α * [0.00045 + 0.00019]
  - = 0.00064 α
- P(¬fraud|foreign-purchase)
  - =
    α * [P(¬fraud|travel) * P(foreign-purchase|travel,¬fraud) * P(travel)
    + P(¬fraud|¬travel) * P(foreign-purchase|¬travel,¬fraud)*P(¬travel)]
  - = α * [0.99 *0.90 *0.05 + 0.998* 0.01* 0.95]
  - = α * [0.04455+0.009481]
  - =0.054031 α
- α=1/(0.00064+ 0.054031)
- P(fraud|foreign-purchase)=1.1 %

Exercise 6: Solution

- An agent calls but the card holder is not at home. Her spouse confirms that she is out of town on a business trip. How does the probability of the fraud changes based on this new piece of information?
  - P(fraud|foreign-purchase,travel)
    = α * [P(fraud|travel) * P(foreign-purchase|travel, fraud) * P(travel)]
    = α * (0.01) (0.90) (0.05)
    = α * 0.00045
  - P(¬fraud|foreign-purchase,travel)
    = α * [P(¬fraud|travel) * P(foreign-purchase|travel,¬ fraud) * P(travel)]
    = α * (0.99) (0.90) (0.05)
    = α * * 0.04455
  - α = 1/(0.00045+ 0.04455)
  - P(fraud|foreign-purchase,travel) = 0.01

Exercise 7

When a person allergic to cats is visiting the house of a friend and starts to sneeze, it may be because he is cold. It may be also because his friend has a cat. Usually, the furniture of cat owners is scratched.

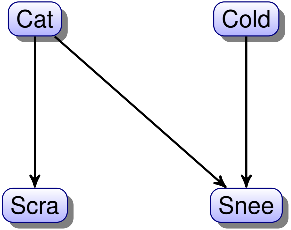
1. We want to build a system able of reasoning with the information in the text for persons allegic to cats. Design a bayesian network to represent this infomation.
2. Which is the number of parameters of the network?
3. Jonh is allergic to cats. Now, he is in the house of a friend and he observes that the furniture is scratched. Which is the expression to compute probability that his friend has cat?
4. Now, Jonh starts to sneeze. Which is the expression to compute probability that his friend has cat? Which is the expression to compute the probability that Jonh is cold?

Exercise 7 - solution

1. Random variables (all binary):
   - *Cat*. There is a cat in the house.
   - *Cold*. The person is cold.
   - *Scra*. The furniture is scratched.
   - *Snee*. The person sneezes.

Exercise 7 - solution

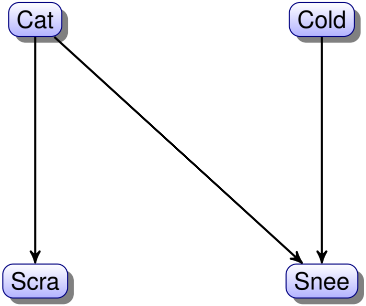
- Cats cause Scratches
- Cats cause Sneezing
- Be cold causes sneezing



Exercise 7: parameters

P(Cat=true)
p_a

P(Cold=true)
p_b



Cat	P(Scra=true/Cat)
true	pc_1
false	pc_2

Cat	Cold	P(Snee=true/Cat,Cold)
true	true	pd_{11}
false	true	pd_{12}
true	false	pd_{21}
false	false	pd_{22}

8 parameters

Ejercise 7: part 3 - P(Cat = true / Scra = true)

$$
\begin{aligned}
P(Cat = t / Scra = t) &= \alpha P(Cat = t, Scra = t) \\
&= \alpha \sum_{cold} \sum_{snee} P(Scra = t, Cat = t, Cold = cold, Snee = snee)) \\
&= \alpha \sum_{cold} \sum_{snee} P(Scra = t / Cat = t) P(Cat = t) P(Snee = snee / Cat = t, Cold = cold) P(Cold = cold) \\
&= \alpha P(Scra = t / Cat = t) P(Cat = t) \sum_{cold} [P(Cold = cold) \sum_{snee} P(Snee = snee / Cat = t, Cold = cold)] \\
&= \alpha P(Scra = t / Cat = t) P(Cat = t) \sum_{cold} [P(Cold = cold) \sum_{snee} P(Snee = snee / Cat = t, Cold = cold)] \\
&= \alpha P(Scra = t / Cat = t) P(Cat = t)
\end{aligned}
$$

Trick: all variables in the network that are not ancestors of query or evidence variables are irrelevant!

Ejercise 7: part 4 - P(Cat = true / Scra = true, Snee = true) and P(Cold= true / Scra = true, Snee = true)

$$
\begin{aligned}
P(Cat = t / Scra = t, Snee = t) &= \alpha P(Cat = t, Scra = t, Snee = t) \\
&= \alpha \sum_{cold} P(Scra = t, Cat = t, Cold = cold, Snee = t)) \\
&= \alpha \sum_{cold} P(Scra = t / Cat = t) P(Cat = t) P(Snee = t / Cat = t, Cold = cold) P(Cold = cold) \\
&= \alpha P(Scra = t / Cat = t) P(Cat = t) \sum_{cold} P(Cold = cold) P(Snee = t / Cat = t, Cold = cold)]
\end{aligned}
$$

$$
\begin{aligned}
P(Cold = t / Scra = t, Snee = t) &= \alpha P(Cold = t, Scra = t, Snee = t) \\
&= \alpha \sum_{cat} P(Scra = t, Cat = cat, Cold = t, Snee = t)) \\
&= \alpha \sum_{cat} P(Scra = t / Cat = cat) P(Cat = cat) P(Snee = t / Cat = cat, Cold = t) P(Cold = t) \\
&= \alpha P(Cold = t) \sum_{cat} P(Scra = t / Cat = cat) P(Cat = cat) P(Snee = t / Cat = cat, Cold = t)
\end{aligned}
$$

Exercise 8

We want to build a naïve Bayes classifier to detect fires a from the following historic annotated cases, where the existence of fire (Yes, No) has been annotated for different cases of the variables: existence of smoke (Yes, No), CO and CO_2 levels (range 0 to 2) and the Temperature (range 0 to 2).

1. Define the structure and probability distributions of the classifier.
2. What is the probability of fire for the cases marked as *Test*?

Humo	CO	CO_2	Temp	Fuego
Y	N	1	2	Y
Y	Y	2	0	Y
N	Y	2	1	Y
N	N	1	2	Y
Y	Y	1	1	Y
Y	Y	0	2	Y
N	N	2	2	Y
Test				
Y	N	1	1	?

Humo	CO	CO_2	T	Fuego
Y	N	2	1	N
Y	N	1	0	N
N	N	1	0	N
Y	N	0	1	N
N	Y	1	0	N
Y	N	0	2	N
Y	Y	0	1	N
Test				
N	Y	2	0	?

Exercise 8: solution

Smoke	CO	CO_2	Temp	Fire
Y	N	1	2	Y
Y	Y	2	0	Y
N	Y	2	1	Y
N	N	1	2	Y
Y	Y	1	1	Y
Y	Y	0	2	Y
N	N	2	2	Y
		Test		
Y	N	1	1	?

Humo	CO	CO_2	T	Fire
Y	N	2	1	N
Y	N	1	0	N
N	N	1	0	N
Y	N	0	1	N
N	Y	1	0	N
Y	N	0	2	N
Y	Y	0	1	N
		Test		
N	Y	2	0	?

- A priori probability of Fire is: $P(F = Y) = 0.5$
- CPTs for Fire=Y:

$$P(S|F = Y) = \langle \frac{4}{7}\ (Y),\ \frac{3}{7}\ (N)\rangle$$
$$P(CO|F = Y) = \langle \frac{4}{7}\ (Y),\ \frac{3}{7}\ (N)\rangle$$
$$P(CO_2|F = Y) = \langle \frac{1}{7}\ (0),\ \frac{3}{7}(1)\frac{3}{7}\ (2)\rangle$$
$$P(T|F = Y) = \langle \frac{1}{7}\ (0),\ \frac{2}{7}\ (1)\frac{4}{7}\ (2)\rangle$$

- CPTs for Fire=N:

$$P(S|F = N) = \langle \frac{5}{7}\ (Y),\ \frac{2}{7}\ (N)\rangle$$
$$P(CO|F = N) = \langle \frac{2}{7}\ (Y),\ \frac{5}{7}\ (N)\rangle$$
$$P(CO_2|F = N) = \langle \frac{3}{7}\ (0),\ \frac{3}{7}\ (1)\frac{1}{7}\ (2)\rangle$$
$$P(T|F = N) = \langle \frac{3}{7}\ (0),\ \frac{3}{7}\ (1)\frac{1}{7}\ (2)\rangle$$

- $P(F|S, CO, CO_2, T) = \alpha P(F)P(S|F)P(CO|F)P(CO_2|F)P(T|F)$

Exercise 8: solution

Smoke	CO	CO_2	Temp	Fire
Y	N	1	2	Y
Y	Y	2	0	Y
N	Y	2	1	Y
N	N	1	2	Y
Y	Y	1	1	Y
Y	Y	0	2	Y
N	N	2	2	Y
Test				
Y	N	1	1	?

Humo	CO	CO_2	T	Fire
Y	N	2	1	N
Y	N	1	0	N
N	N	1	0	N
Y	N	0	1	N
N	Y	1	0	N
Y	N	0	2	N
Y	Y	0	1	N
Test				
N	Y	2	0	?

- $P(F|S, CO, CO_2, T) = \alpha P(F)P(S|F)P(CO|F)P(CO_2|F)P(T|F)$
- For the first Test,

  $P(F = Y|Y, N, 1, 1) = \alpha 0.5P(S = Y|F = Y)P(CO = N|F = Y)P(CO_2 = 1|F = Y)P(T = 1|F = Y)$

  $P(F = Y|Y, N, 1, 1) = \alpha 0.5 \times \frac{4}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{2}{7} = \alpha 0.5 \times \frac{72}{7^4}$

  $P(F = N|Y, N, 1, 1) = \alpha 0.5P(S = Y|F = N)P(CO = N|F = N)P(CO_2 = 1|F = N)P(T = 1|F = N)$

  $P(F = N|Y, N, 1, 1) = \alpha 0.5 \times \frac{5}{7} \times \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} = \alpha 0.5 \times \frac{225}{7^4}$
- Then $P(F|Y, N, 1, 1) \propto \langle 72, 225\rangle$, and the most probable class is Fire=N.

Exercise 8: solution

Smoke	CO	CO_2	Temp	Fire
Y	N	1	2	Y
Y	Y	2	0	Y
N	Y	2	1	Y
N	N	1	2	Y
Y	Y	1	1	Y
Y	Y	0	2	Y
N	N	2	2	Y
Test				
Y	N	1	1	?

Humo	CO	CO_2	T	Fire
Y	N	2	1	N
Y	N	1	0	N
N	N	1	0	N
Y	N	0	1	N
N	Y	1	0	N
Y	N	0	2	N
Y	Y	0	1	N
Test				
N	Y	2	0	?

- $P(F|S, CO, CO_2, T) = \alpha P(F)P(S|F)P(CO|F)P(CO_2|F)P(T|F)$
- For the second Test,

$$P(F = Y|N, Y, 2, 0) = \alpha 0.5 P(S = N|F = Y)P(CO = Y|F = Y)P(CO_2 = 2|F = Y)P(T = 0|F = Y)$$
$$P(F = Y|N, Y, 2, 0) = \alpha 0.5 \times \frac{3}{7} \times \frac{4}{7} \times \frac{3}{7} \times \frac{1}{7} = \alpha 0.5 \times \frac{72}{7^4}$$
$$P(F = N|N, Y, 2, 0) = \alpha 0.5 P(S = N|F = N)P(CO = Y|F = N)P(CO_2 = 2|F = N)P(T = 0|F = N)$$
$$P(F = N|N, Y, 2, 0) = \alpha 0.5 \times \frac{2}{7} \times \frac{2}{7} \times \frac{1}{7} \times \frac{3}{7} = \alpha 0.5 \times \frac{12}{7^4}$$

- Then, $P(F|N, Y, 2, 0) \propto \langle 72, 12 \rangle$, and the most probable class is Fire=Y .