REASONING UNDER UNCERTAINTY

Universidad Carlos III de Madrid

AI

Outline

1. Introduction
2. Probabilistic reasoning
3. Bayesian networks
4. Markov Models
5. Fuzzy logic

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5. Fuzzy logic
   - Introduction
   - Representation: fuzzy sets
   - Fuzzy inference
   - Final remarks

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Fuzzy Logic

- Classical logic: `true` or `false`
- Fuzzy logic: concepts are not always true or false
  - fuzzy concept: close to door (distance)
  - fuzzy modifiers: very close, quite close, close

Fuzzy Logic

- Classical logic: `true` or `false`
- Fuzzy logic: concepts are not always true or false
  - fuzzy concept: close to door (distance)
  - fuzzy modifiers: very close, quite close, close
- Computation (needs numerical values):
  - fuzzy sets: membership values
  - values on the range [0.0,1.0]
  - 0.0 represents absolute falseness
  - 1.0 represents absolute truth
- Fuzzy vs. probability:
  young person, close to wall, hot temperature
- Defined by Lofti Zadeh [Zadeh, 1973]

Linguistic imprecision

- Descriptions:
  - the weather was rainy with 98% humidity and hot with temperature of 35.5^{o}C
  - the weather was very humid and really hot

Linguistic imprecision

- Descriptions:
  - the weather was rainy with 98% humidity and hot with temperature of 35.5^{o}C
  - the weather was very humid and really hot
- Instructions:
  - when you are at 10 metres from the junction, start braking at 50% pedal level
  - when you are near the junction, start braking slowly

Driving problem. Fuzzy solution

- Fuzzy (natural) description of a problem
  - if near the junction, brake slowly
  - if very close to the right, turn fast to the left
  - if near the left lane and not passing, turn fast to the right

Driving problem. Fuzzy solution

- Fuzzy (natural) description of a problem
  - if near the junction, brake slowly
  - if very close to the right, turn fast to the left
  - if near the left lane and not passing, turn fast to the right
- Computing fuzzy solution, we need to find out how to:
  - define terms such as *near, right, slowly, very, etc.*
  - combine terms using *AND*, *OR* and other connectives
  - use a single rule
  - combine all rules into one final output

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Fuzzy sets

- Boolean/Crisp set A
  - $A : S \rightarrow \{0, 1\}$
  - Characteristic function:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \text{ is an element of set } A \\ 0 & \text{if } x \text{ is not an element of set } A \end{cases}$$

- Can define: *John is a person*

Fuzzy sets

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$$\mu_A(x) = \begin{cases} 1 & \text{if } x \text{ is an element of set } A \\ 0 & \text{if } x \text{ is not an element of set } A \end{cases}$$

- Can define: *John is a person*
- What about *John is tall*?
- Example: crisp set `tall`
  - crisp definition of Tall:

$$\text{Tall} = \{x | \text{height}(x) > 1.8 \text{ meters}\}$$

  - but what about a person with a height = 1.79 meters?
  - what about 1.78 meters?

    ...
  - what about 1.30 meters?

Linguistic Variables

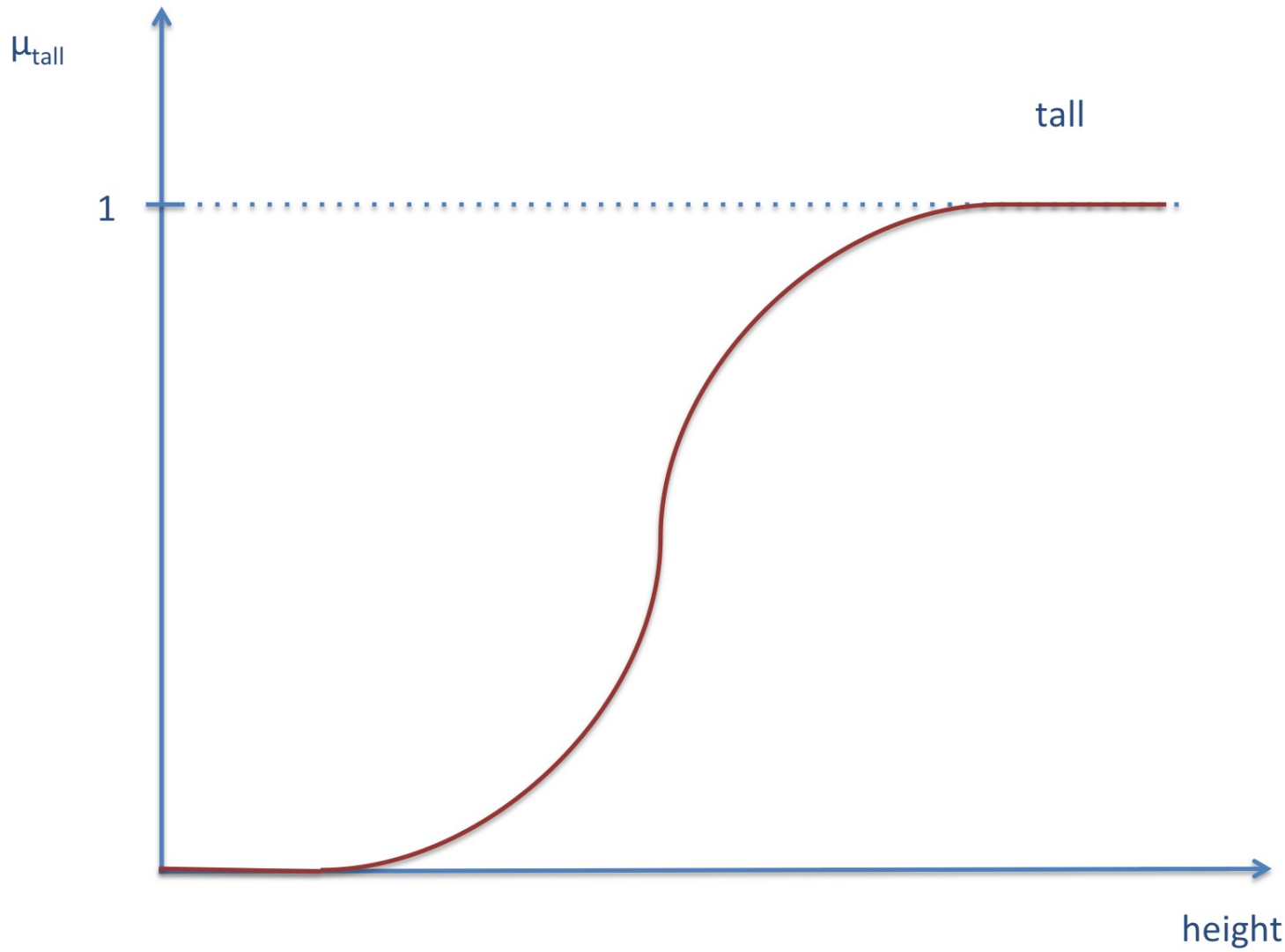
- Universe of discourse: distance, height, temperature, ...
- Variables used in fuzzy systems
  - to express qualities such as height
  - can take values: `tall`, `medium` or `short`
  - these values define subsets of the universe of discourse
- They are characterized by the membership function

$$\mu_{\text{tall}}(x) : \text{height}(x) \rightarrow [0, 1]$$

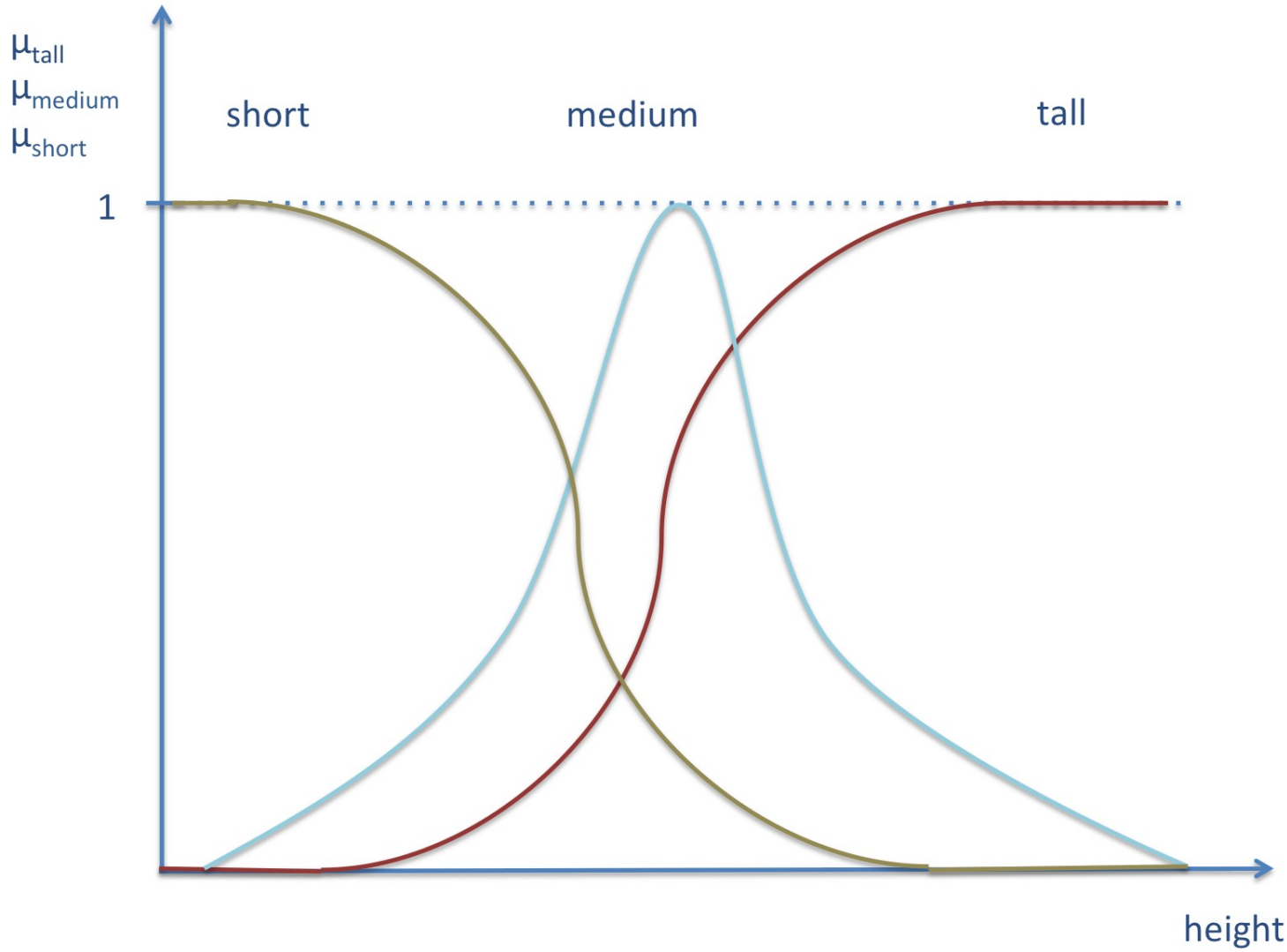
  - $\mu_{\text{tall}}(x)$ is a grade (degree) of membership of x in set `tall`
  - characteristic function: $0 \leq \mu_{\text{tall}}(x) \leq 1$
  - full membership: 1
  - no membership: 0
  - graded membership: anything in between

Linguistic variables

Linguistic variables



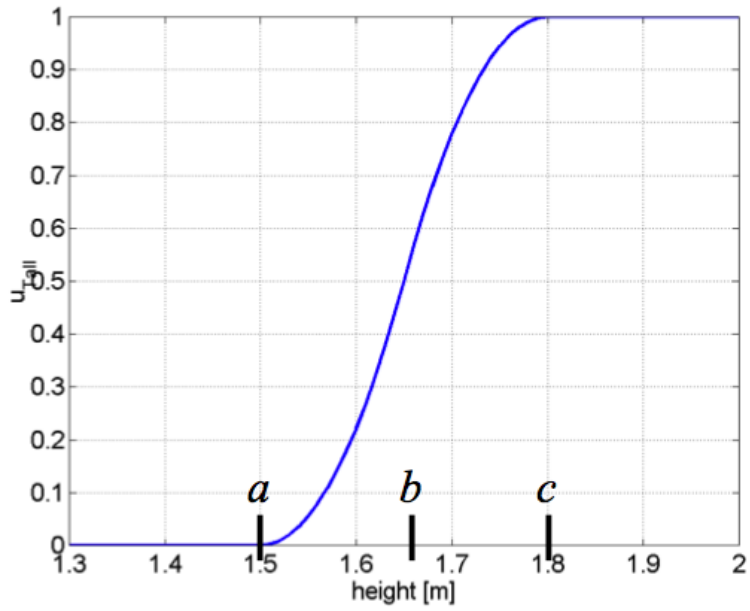
Linguistic variables



Membership functions: S-function

- The S-function can be used to define fuzzy sets

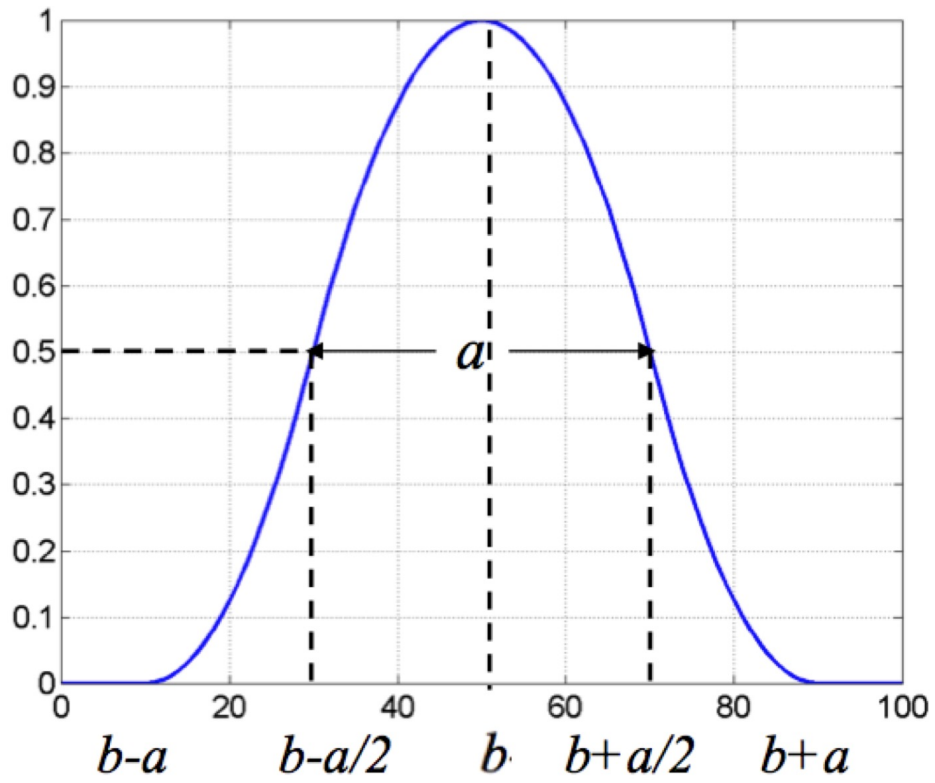
$$S(x, a, b, c) = \begin{cases} 0 & \text{if } x \leq a \\ 2(x - a/c - a)^2 & \text{if } a \leq x \leq b \\ 1 - 2(x - a/c - a)^2 & \text{if } b \leq x \leq c \\ 1 & \text{if } x \geq c \end{cases}$$



Membership functions: π Function

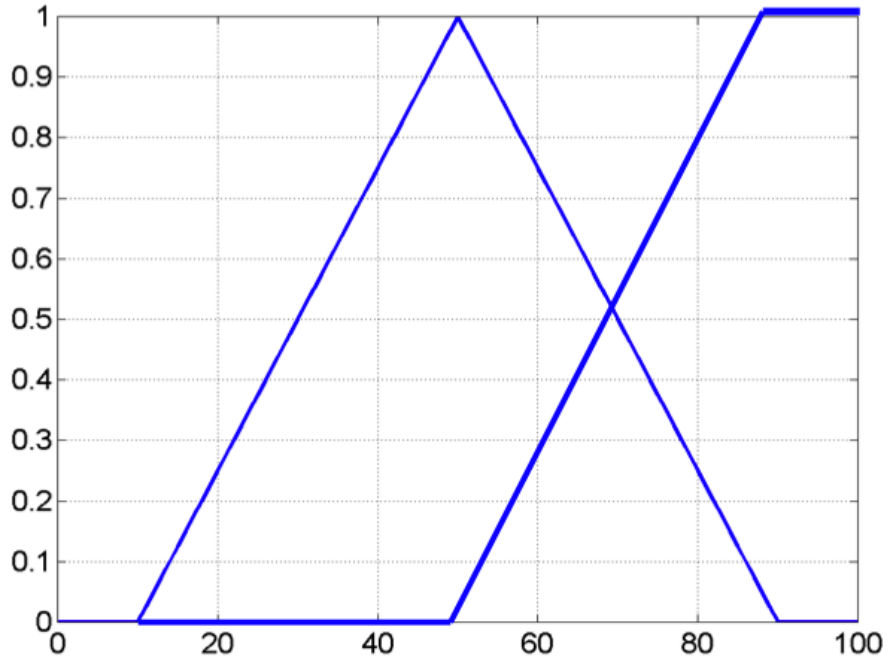
$$\pi(x, a, b) = \begin{cases} S(x, b-a, b-a/2, b) & \text{if } x \leq b \\ 1 - S(x, b, b+a/2, a+b) & \text{if } x \geq b \end{cases}$$

E.g., close (to b)



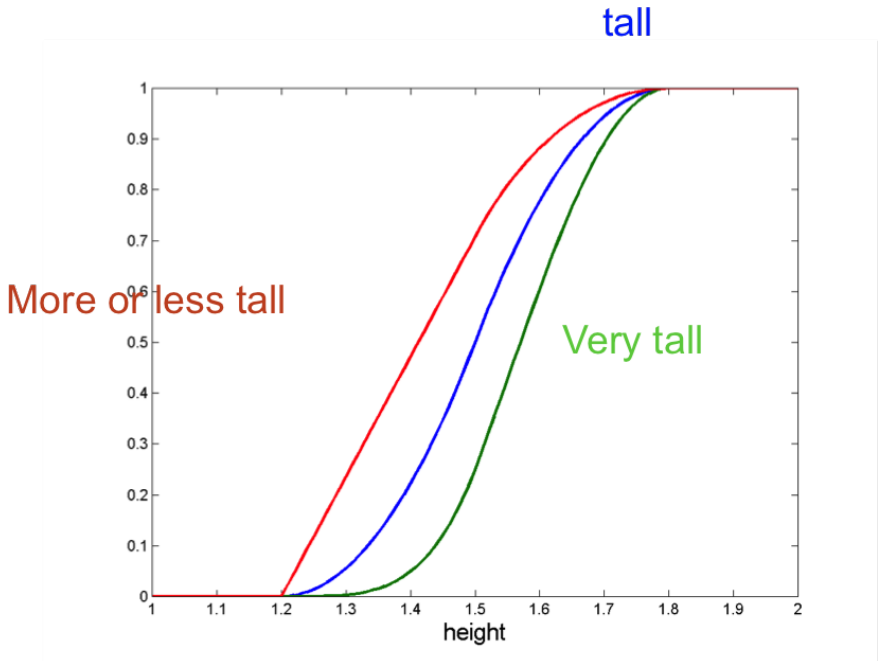
Simple membership functions

- Piecewise linear: triangular etc.
- Easier to represent and compute



Linguistic Hedges

- Hedges modify the meaning of a fuzzy set
  - Very: $\mu(x)^2$
  - More or less: $\mu(x)^{1/2}$
  - Slightly: $\mu(x)^{1/3}$
  - Extremely: $\mu(x)^3$

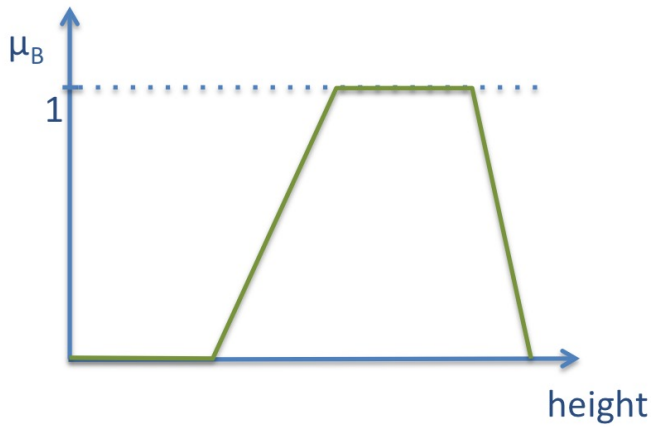


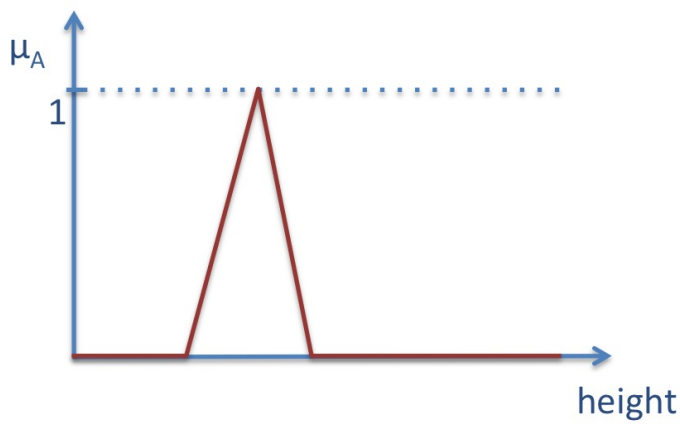
Fuzzy Set Operators

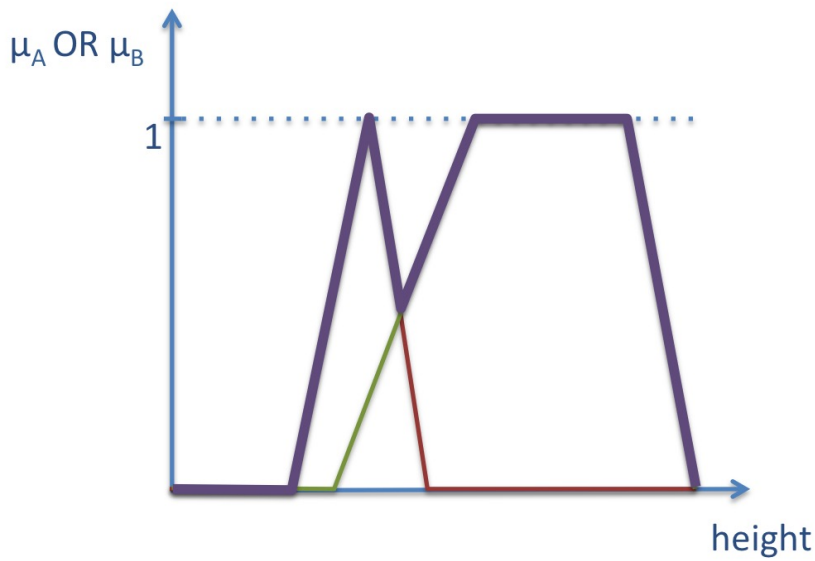
- Fuzzy Set operators:
  - union
  - intersection
  - complement
- Many possible definitions

Fuzzy Set Union (OR operator)

$$\mu_A(x) \vee \mu_B(x) = \max(\mu_A(x), \mu_B(x))$$

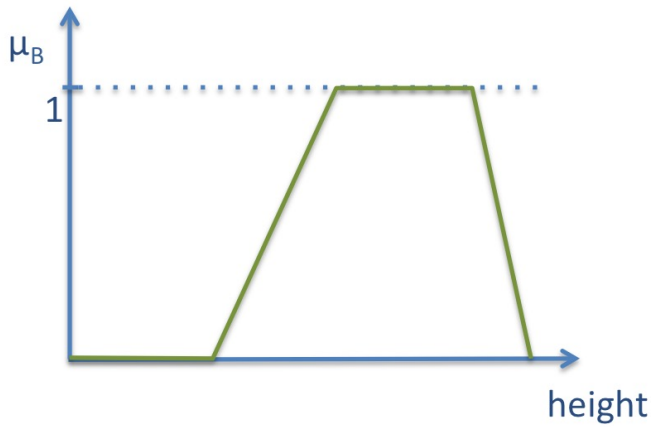


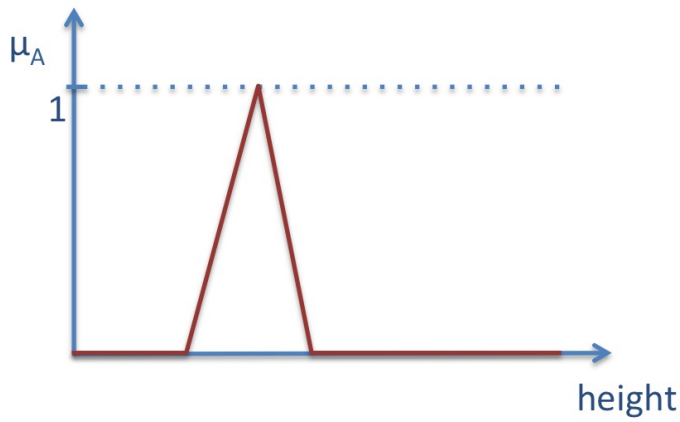


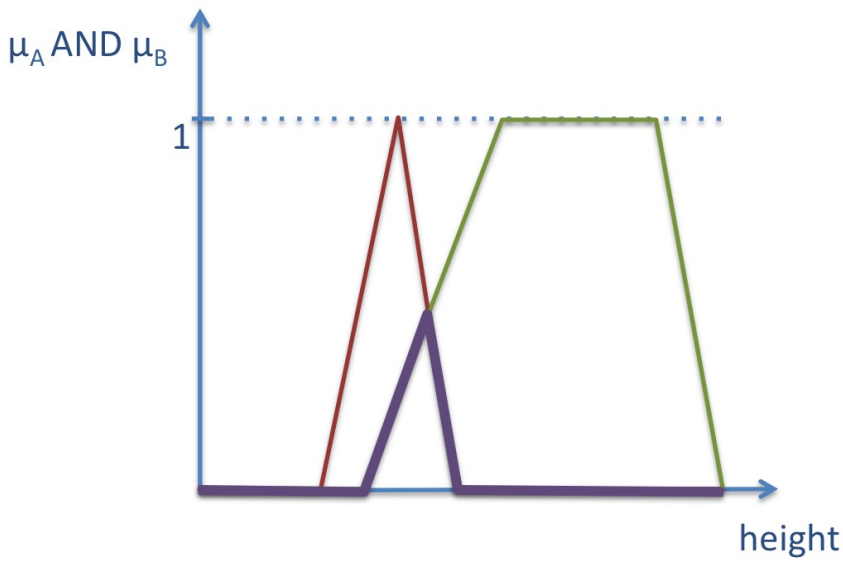


Fuzzy Set Intersection (AND operator)

$$\mu_A(x) \wedge \mu_B(x) = \min(\mu_A(x), \mu_B(x))$$

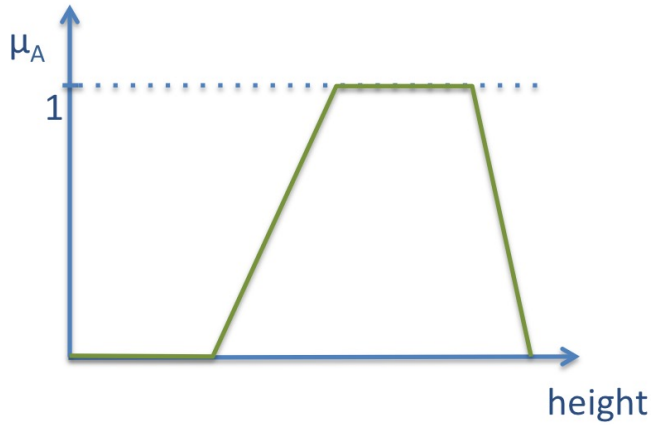


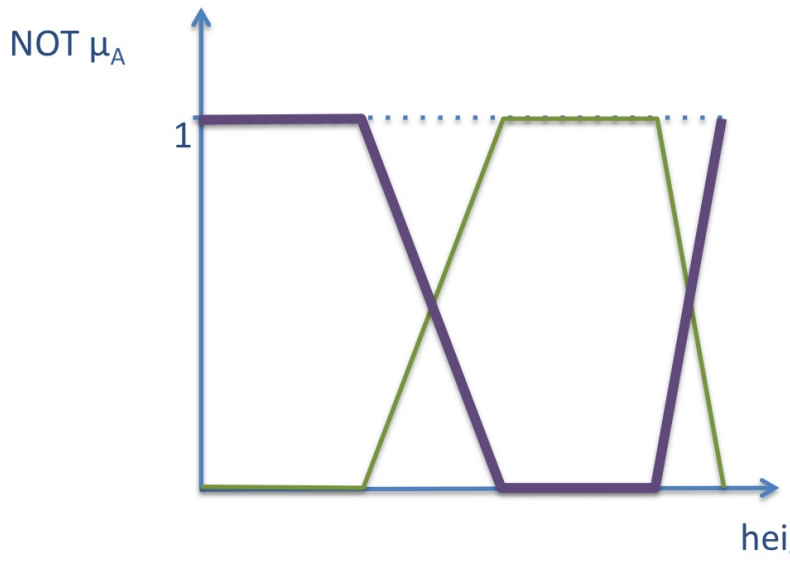




Fuzzy Set Complement (NOT operator)

$$\neg \mu_A(x) = 1 - \mu_A(x)$$





If-Then Rules

- Fuzzy operators:
  - NOT (A) = 1 - A
  - A AND B = min(A, B)
  - A OR B = max(A, B)

If-Then Rules

- Fuzzy operators:
  - NOT (A) = 1 - A
  - A AND B = min(A, B)
  - A OR B = max(A, B)
- Use fuzzy sets and fuzzy operators as the subjects and verbs of fuzzy logic to form rules

  if X is A then Y is B
- A and B are linguistic terms defined by fuzzy sets on the sets X and Y respectively

Evaluation of fuzzy rules

- In Boolean logic: $p \Rightarrow q$
  if p is true then q is true
  - Example:
    If the light is red then I stop the car
    The light is red
    Therefore I stop the car

Evaluation of fuzzy rules

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  if p is true then q is true
  - Example:
    If the light is red then I stop the car
    The light is red
    Therefore I stop the car
- In fuzzy logic: $p \Rightarrow q$
  if p is true to some degree then q is true to some degree
  - Example:
    If street is wet then speed is very slow
    The street is damp
    Therefore speed is slow

How?

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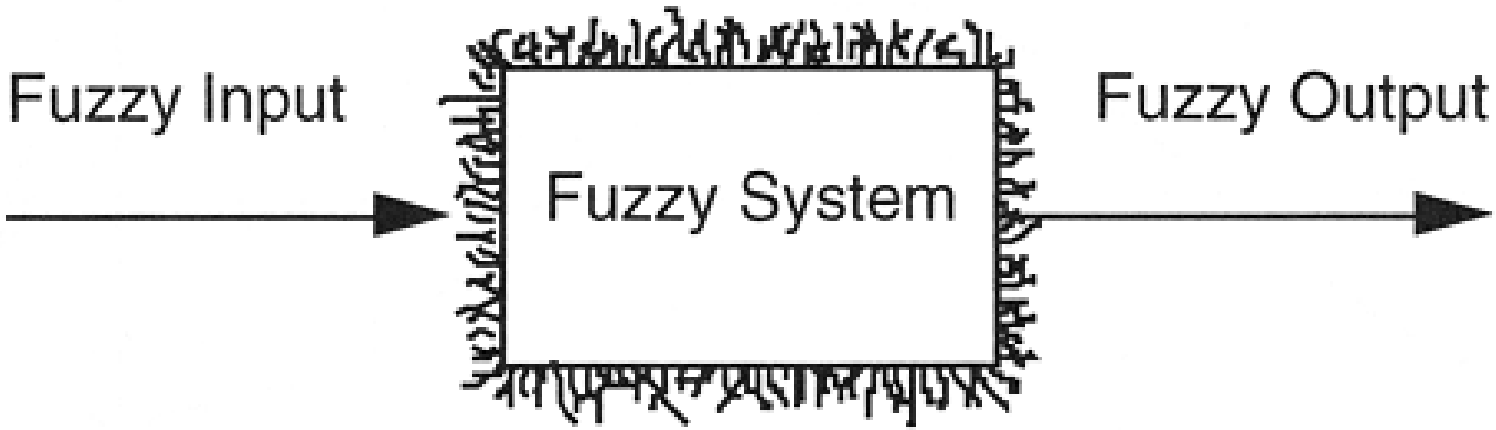
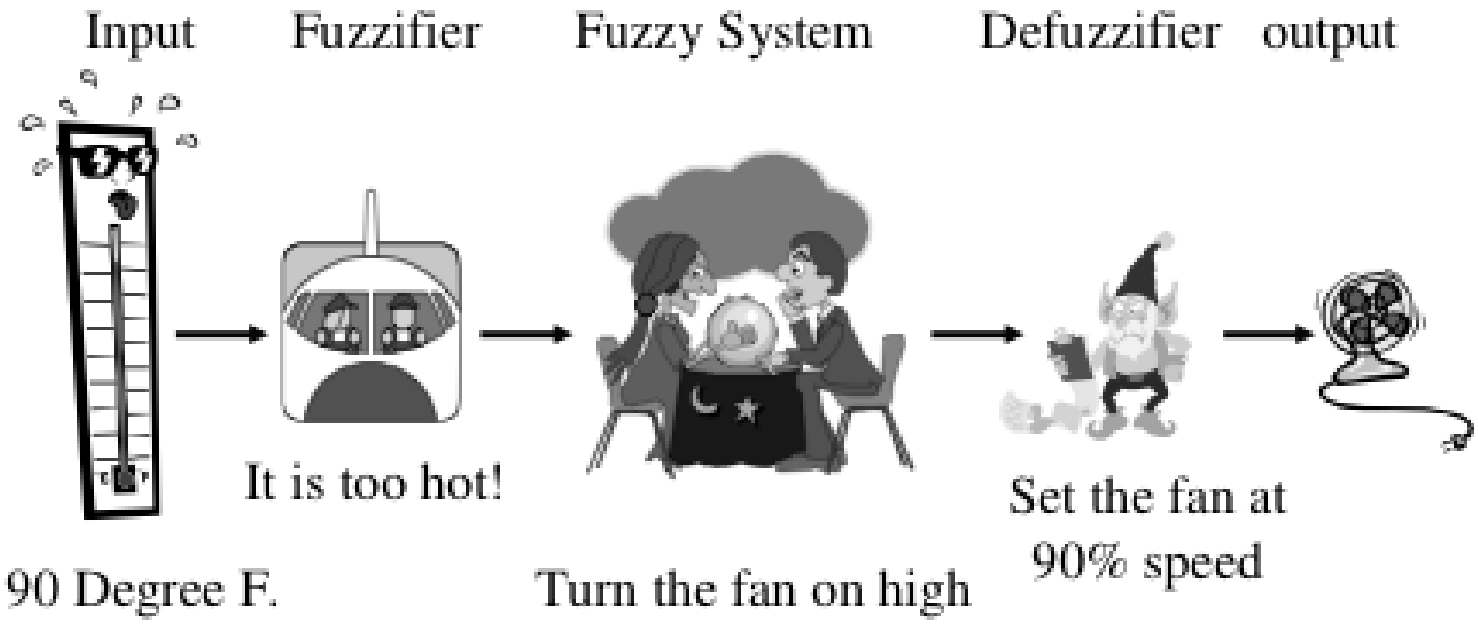
Fuzzy systems

How can fuzzy systems be used in a world where measurements and actions are expressed as crisp values?



Fuzzy systems



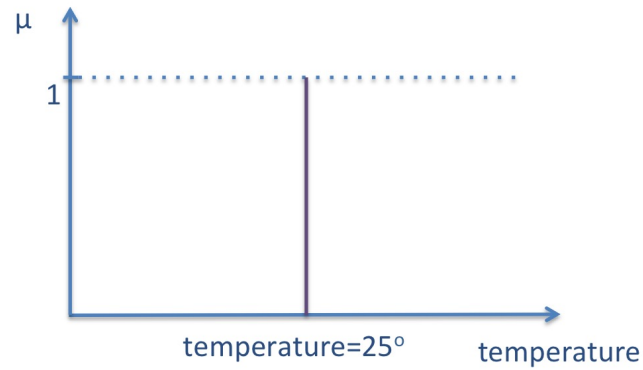
Mamdani fuzzy inference

- Mamdani method: most commonly used fuzzy inference technique
  - he built one of the first fuzzy systems to control a steam engine and boiler combination
  - he applied a set of fuzzy rules supplied by experienced human operators
- Four steps:
  - fuzzification of the input variables
  - rule evaluation: single rule inference
  - composition: aggregation of inference on single rules
  - defuzzification of the output variables

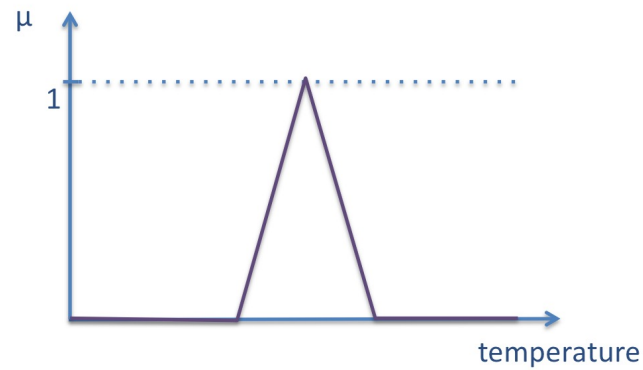
Step 1: Fuzzification

- Transform crisp inputs into membership degree of fuzzy sets

- fuzzy singleton: if input data do not contain noise nor vagueness



- fuzzy set: if data are vague or perturbed by noise



Step 2: Rule Evaluation

Computing the antecedent

- Match fuzzified inputs with antecedents of fuzzy rules
- Computes similarity S between antecedent and fact
  - *if temperature is hot then fan is high*
  - *temperature is lukewarm*

$$S = \max_{T} \{\min(\text{hot}(T), \text{lukewarm}(T))\}$$

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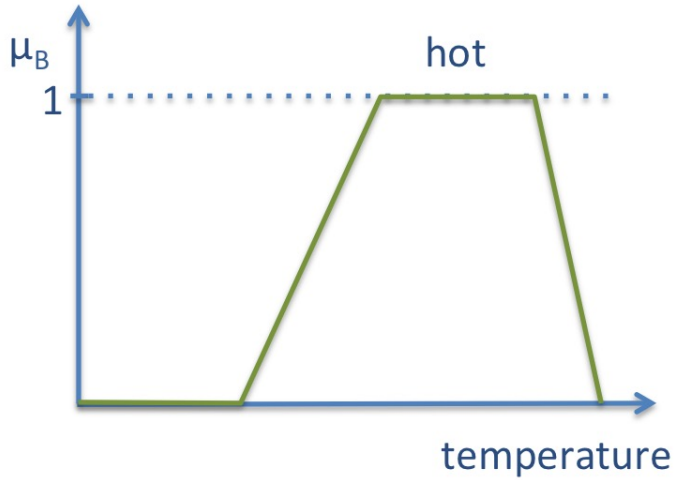
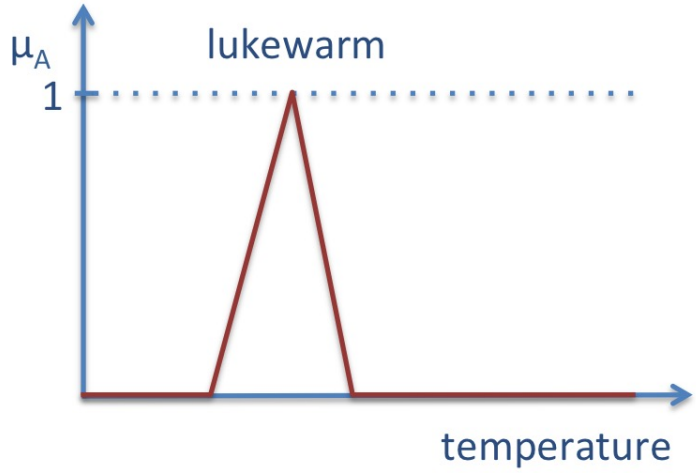
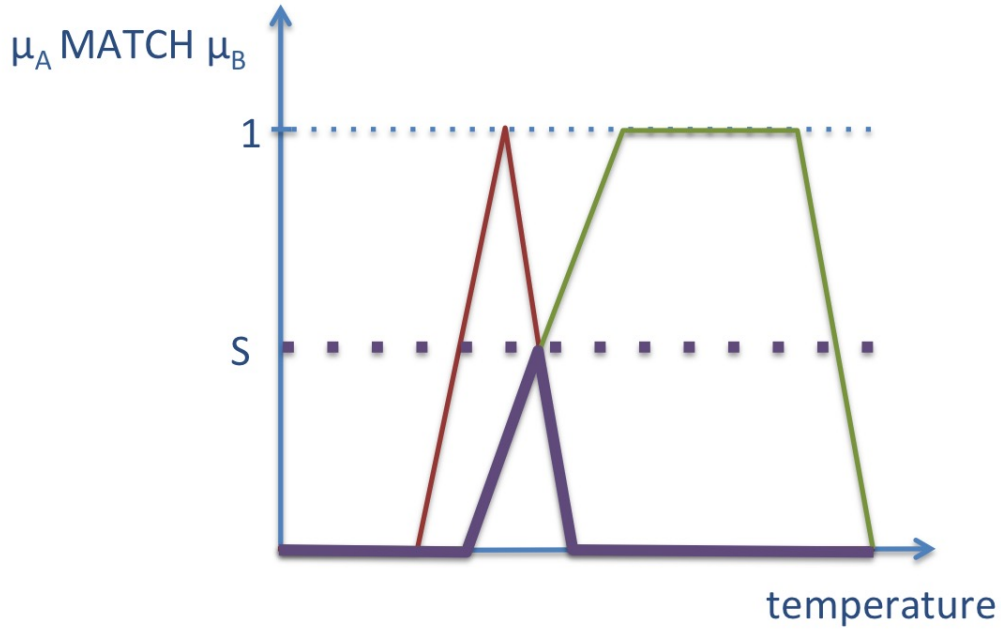
$$S = \max_{T}\{\min(\text{hot}(T), \text{lukewarm}(T))\}$$

- Multiple antecedents:
  - OR: evaluate the disjunction of the rule antecedents (max)
  - AND: evaluate the conjunction of the rule antecedents (min)

Rule evaluation



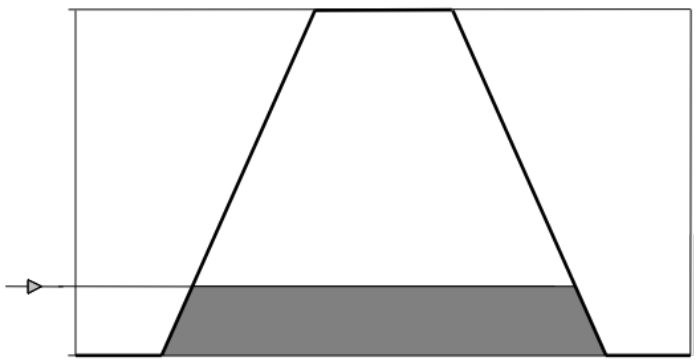




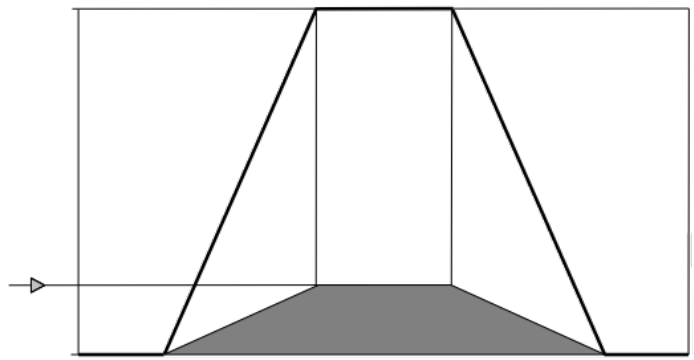
Step 2: Rule Evaluation

Computing the consequent

- Apply result of the antecedent evaluation to membership function of the consequent
- Two common methods:
  - clipping
  - scaling



clipping



scaling

Step 2: Rule Evaluation

Computing the consequent: Clipping

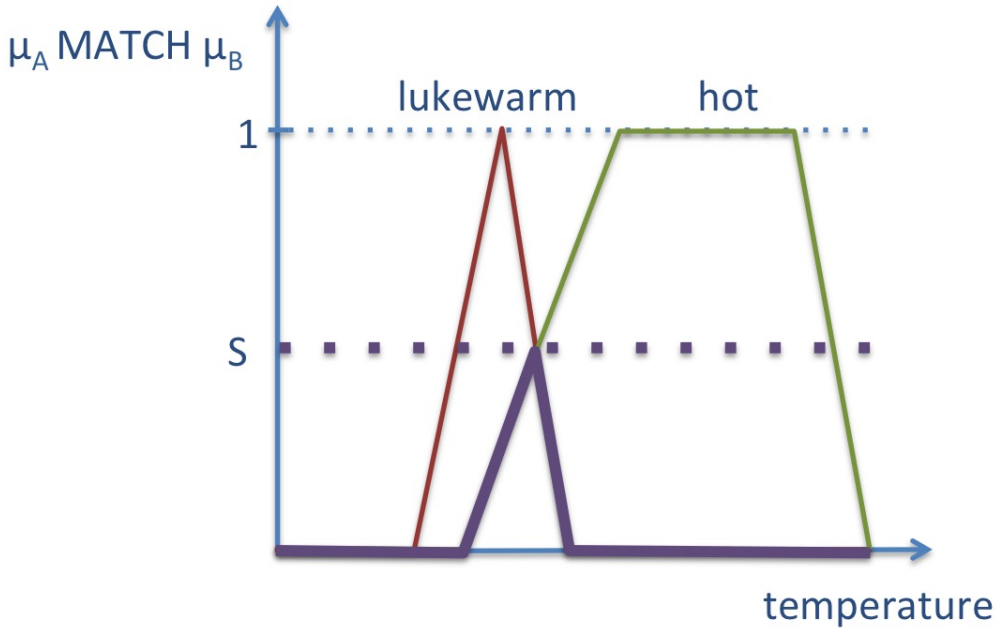
- Most common method: cut the consequent membership function at the level of the antecedent truth (alpha-cut)

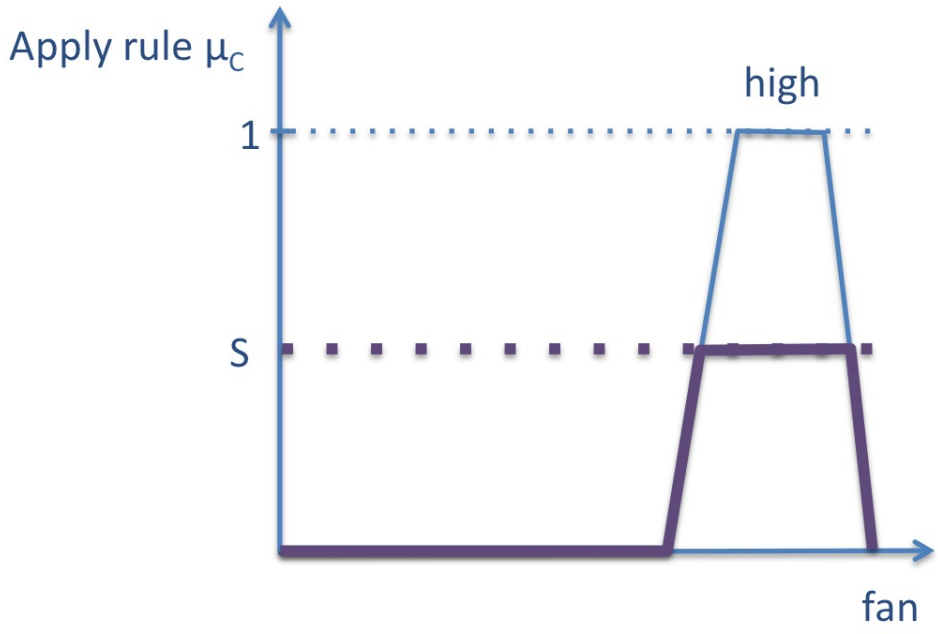
$$Q(x) = \min(S, \mu_c(x))$$

where c is the fuzzy set of consequent

  - since the top of the membership function is sliced, the clipped fuzzy set loses some information
  - involves less complex and faster mathematics
  - generates an aggregated output surface that is easier to defuzzify

Clipping


μ_A MATCH μ_B
lukewarm
hot
1
S
temperature


Apply rule μ_C
high
1
S
fan

Step 2: Rule Evaluation

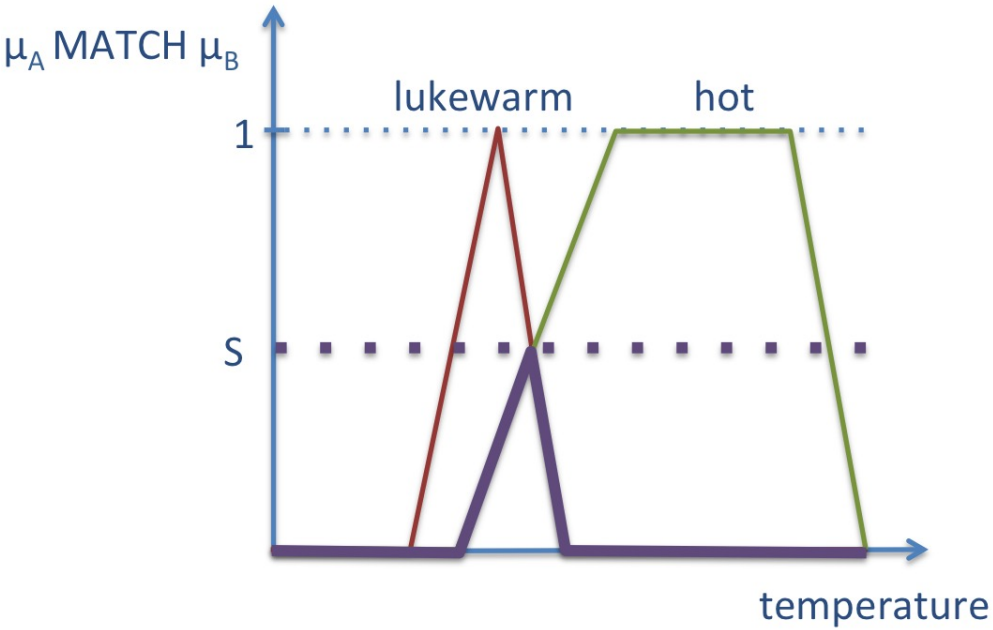
Computing the consequent: Scaling

- offers a better approach for preserving the original shape of the fuzzy set

$$Q(x) = S \cdot \mu_c(x)$$

  - the original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent
  - generally, it loses less information
  - can be very useful in fuzzy expert systems

Scaling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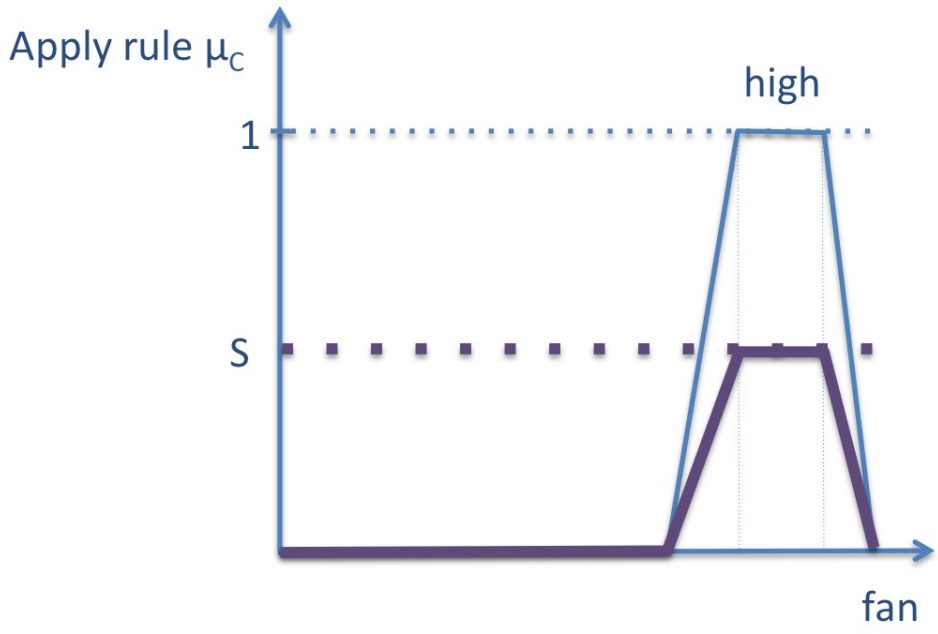
Apply rule μ_C
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1
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Step 3: Composition

- Aggregation: unification of the outputs of all rules
  - Combines membership functions of all rule consequents previously clipped or scaled into a single fuzzy set
  - Input: list of clipped or scaled consequent membership functions
  - Output: a fuzzy set for each output variable
- Aggregation rules: max, sum, etc.
  - we will use max, which is the UNION of two consequents

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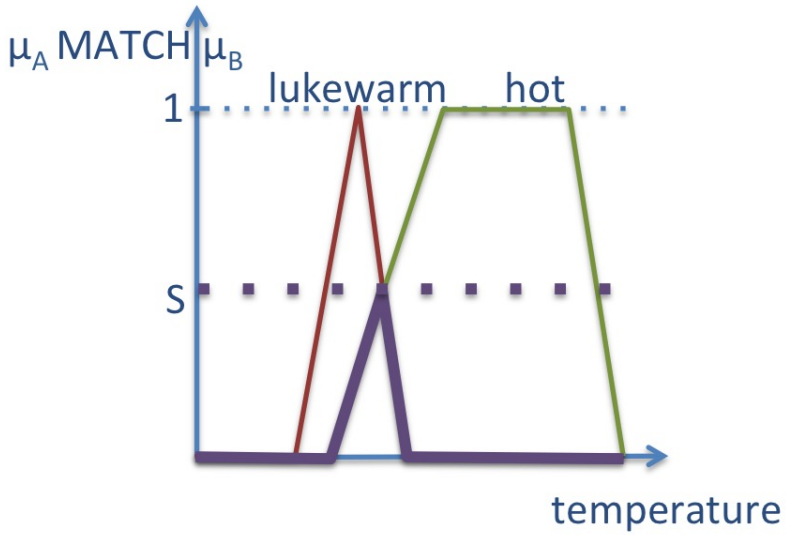
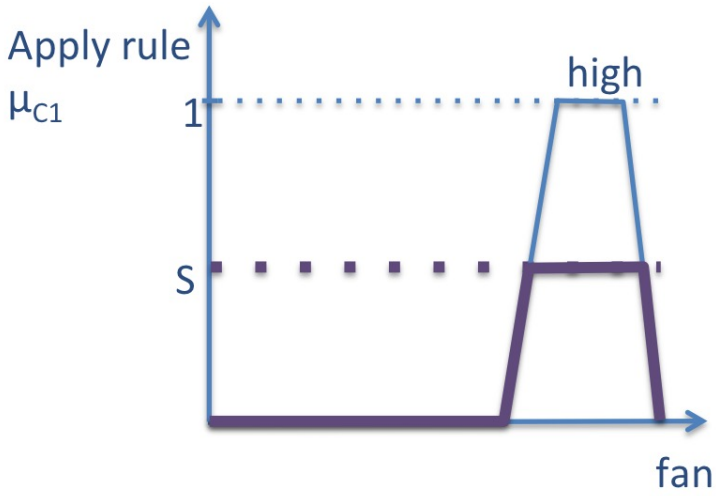
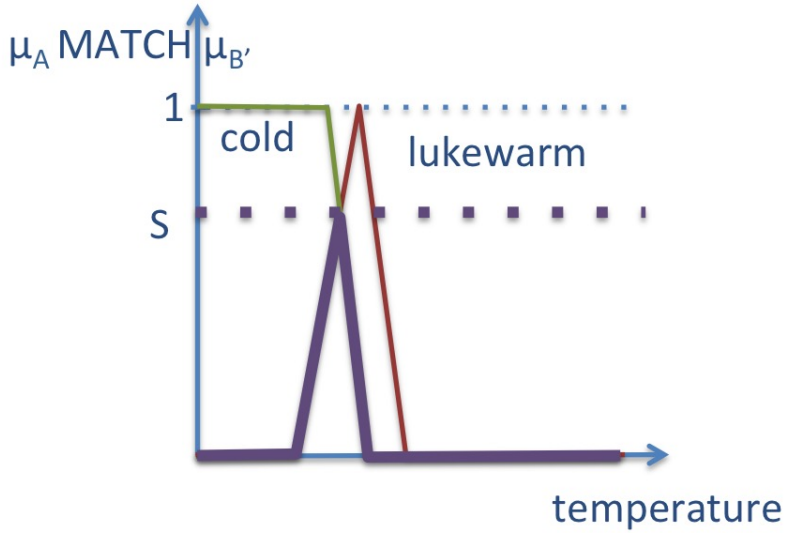
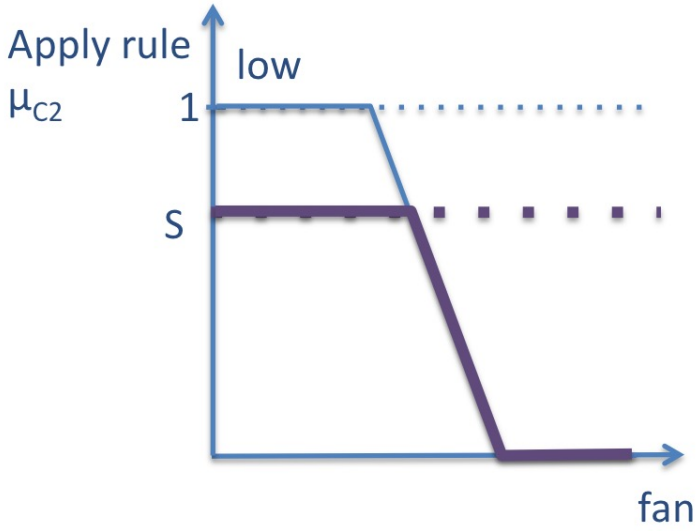
- Example
  - *if temperature is hot then fan is high*
  - *if temperature is cold then fan is low*
  - *temperature is lukewarm*

Aggregation

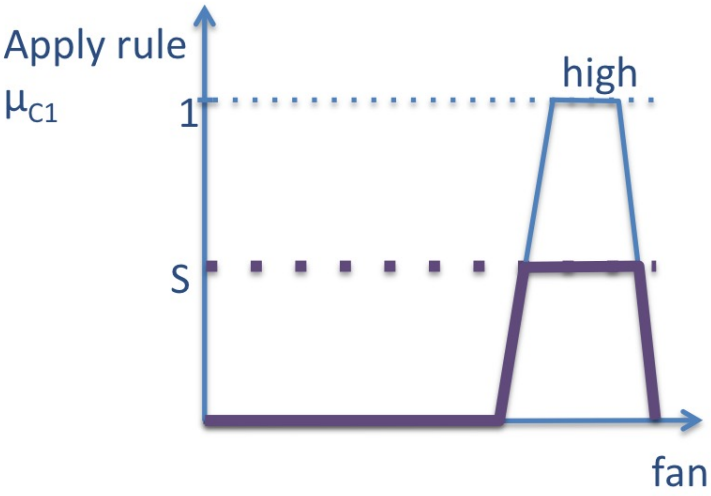


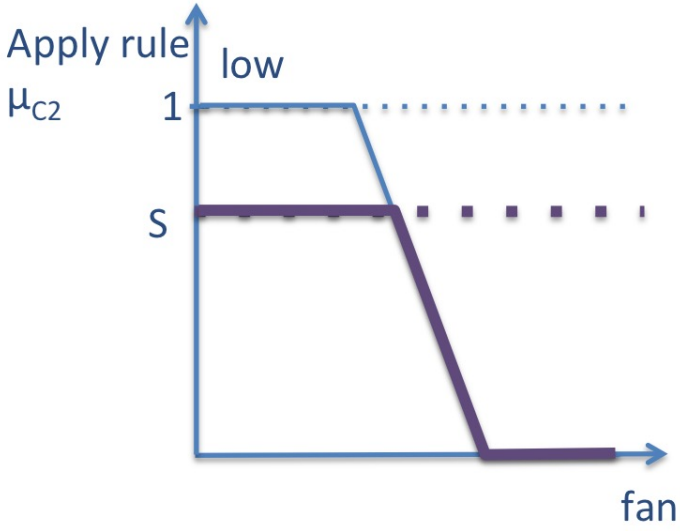


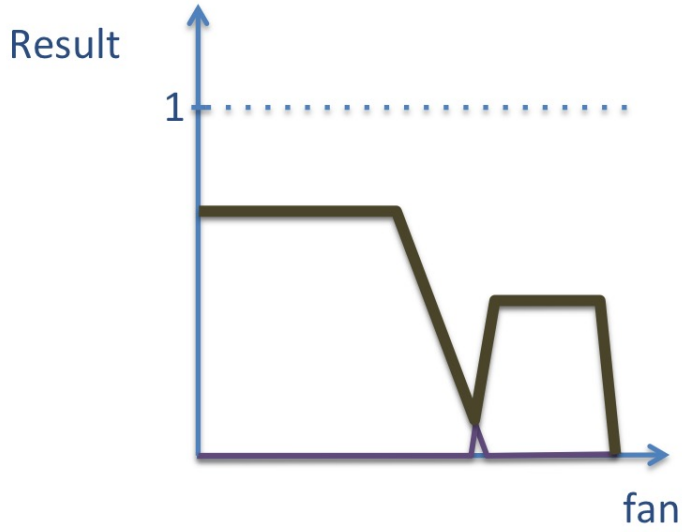




Aggregation



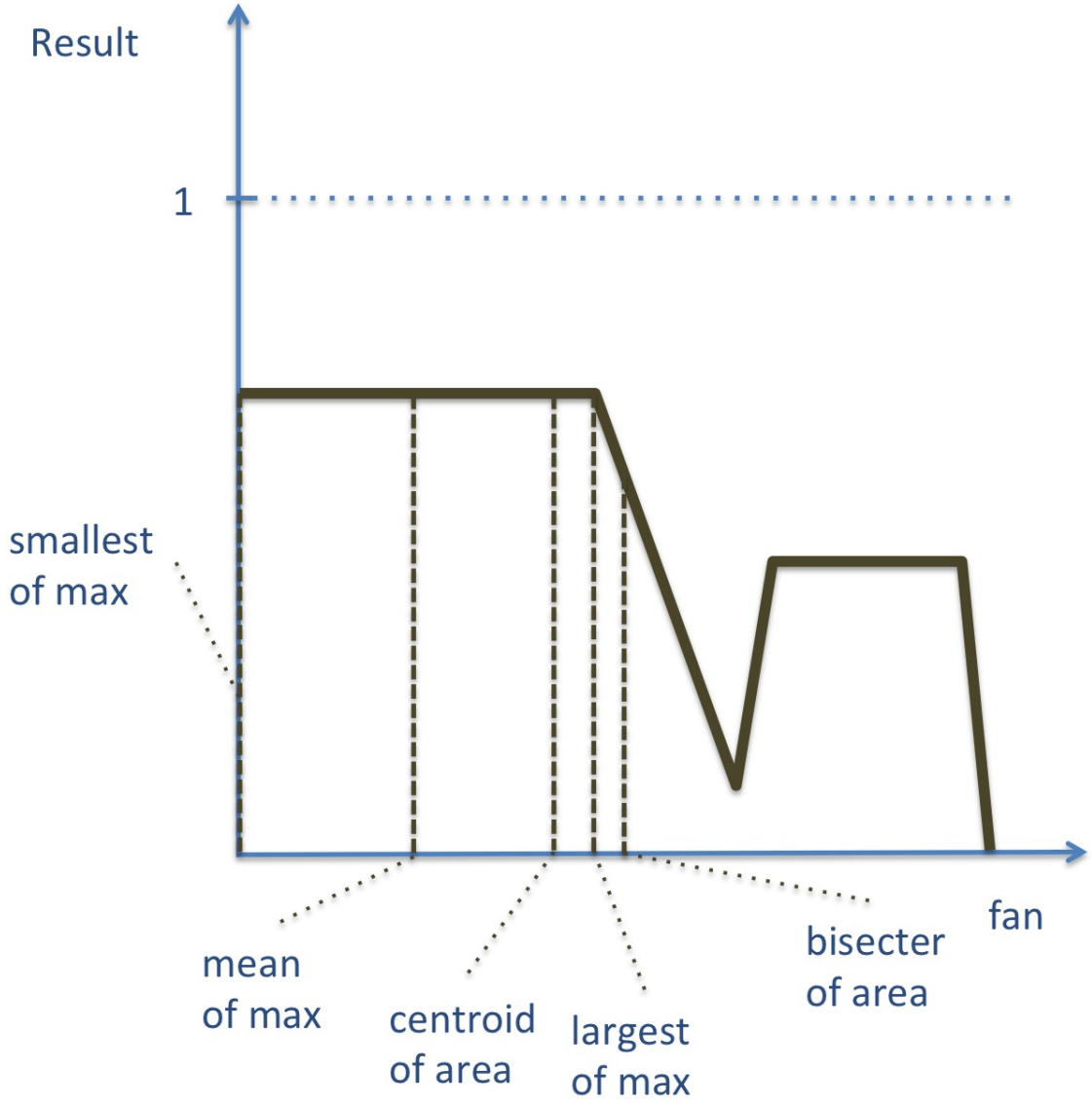




Step 4: Defuzzification

- Input: fuzzy set
- Output: single crisp number that represents the set
- Practical when making a decision, taking an action, etc.
- Defuzzifying methods:
  - Centroid Of Area (COA)
  - Bisector Of Area (BOA)
  - Mean Of Maximum (MOM)
  - Smallest Of Maximum (SOM)
  - Largest Of Maximum (LOM)

Step 4: Defuzzification



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Fuzzy Logic

- Advantages:
  - represents vague language naturally
  - enriches (not replaces) crisp sets
  - allows flexible engineering design
  - improves model performance
  - is simple to implement
  - it often works!
- Limitations:
  - how to determine the membership functions?
  - usually requires fine-tuning of parameters
  - defuzzification can produce undesired results
- Fuzzy tools and shells:
  - Matlab's Fuzzy Toolbox
  - FuzzyClips

References

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