#### MACHINE LEARNING

Universidad Carlos III de Madrid

ΑI

uc3m





#### Outline

- Motivation
- 2 Introduction to Machine Learning concepts
- Naïve Bayes

- Definition
  - programs that improve automatically based on the experience according to some measure

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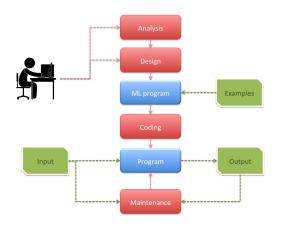
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  - adaptability



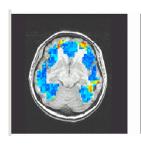
## Programming by machine learning



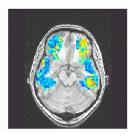
### Some examples

- Medicine: how to diagnose, medicate
- Domotics: when to switch-off lights, open shades
- Smart cities: how to efficiently regulate energy consumption
- Bank: whom to loan, when to accept a credit card charge
- Marketing: what profile of people buy beers
- Personalization: what kinds of music do you listen to
- Robots: how to program a robot to grasp any object
- Astronomy: does an image contain something interesting
- Image understanding: does a given image include my wife

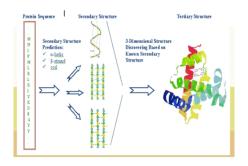
# Analysis of brain images







### Prediction of protein secondary structure



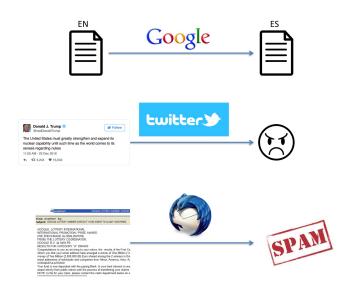
### Data mining → Big data

- Data mining
  - What is the profile of costumers that spend more than X?
  - Should we invest on X tomorrow?
  - What book should we recommend after reading Y?
  - Should we allow a charge on a credit card?
  - What numbers do people call?
  - What do people buy together with beers?

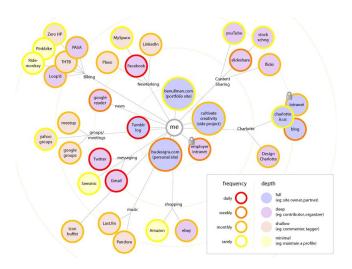
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- Big data
  - What public transport services people use over one year?
  - How to control traffic lights to reduce energy consumption?
  - What do images with traffic congestions have in common?
  - How does traffic behave at all city junctions over time?
  - What are uncommon traffic patterns from GPS data?

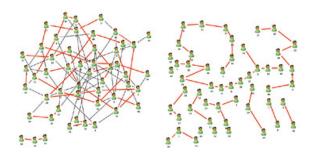
### Text mining



## Social mining



# Social mining



### Customization



#### Customization









## RALPH. Learning to drive



### Sensors. Image recognition



### Sensors. Image recognition









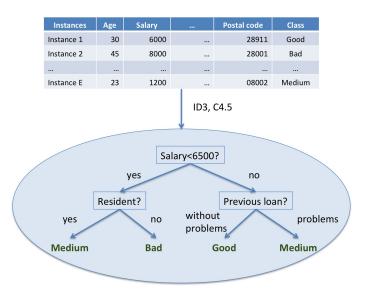
### Some companies that use it



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#### **Decision trees**



#### Rules

Instances	Age	Salary	 Postal code	Class
Instance 1	30	6000	 28911	Good
Instance 2	45	8000	 28001	Bad
Instance E	23	1200	 08002	Medium

C4.5, AQ, CN2

If Salary<6500=yes AND Resident=yes
Then Class=Medium
If Salary<6500=yes AND Resident=no
Then Class=Bad
If Salary<6500=no AND Previous Loan=yes
Then Class=Good
If Salary<6500=no AND Previous Loan=no
Then Class=Medium

# Bayesian classifier

Instances	Age	Salary	 Postal code	Class
Instance 1	30	6000	 28911	Good
Instance 2	45	8000	 28001	Bad
Instance E	23	1200	 08002	Medium

Naïve Bayes

/	Class	D(Class)	Resident	Class	P(Resident=V  Class)	
	ciass	P(Class)	yes	Good	0.7	
	Good	0.6	yes	Bad	0.1	
	Bad	0.2	yes	Medium	0.2	
	Medium	0.2	no	Good	0.3	
			no	Bad	0.3	,
			no	Medium	0.4	

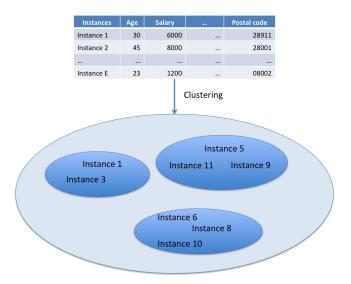
## Instance-based learning

Instances	Age	Salary	 Postal code	Class
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## Clustering



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## Bayesian classifier. Naïve Bayes

$$p(C \mid X) = \frac{p(X \mid C)p(C)}{p(X)}$$

- Class random variable: C
- Example random variable: X
- "A priori": *p*(*C*)
- Likelihood:  $p(X \mid C)$
- Evidence: p(X)
- "A posteriori": p(C | X)

### Bayesian classifier. Naïve Bayes

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- Class random variable: C
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- "A priori": *p*(*C*)
- Likelihood: p(X | C)
- Evidence: p(X)
- "A posteriori": p(C | X)
- In case of a particular class and example:

$$p(C = c_k \mid X = x_i) = \frac{p(X = x_i \mid C = c_k)p(C = c_k)}{p(X = x_i)}$$



## Naïve Bayes

Output of classifier

$$\begin{aligned} \mathsf{class}(x_i) &= & \arg\max_{c_k \in C} p(C = c_k \mid X = x_i) \\ &= & \arg\max_{c_k \in C} \frac{p(X = x_i \mid C = c_k) p(C = c_k)}{p(X = x_i)} \end{aligned}$$

Normalization factor is constant with respect to C = c<sub>k</sub>:

$$class(x_i) = \arg \max_{C_k \in C} p(X = x_i \mid C = c_k) p(C = c_k)$$

### Naïve Bayes assumption

• Each instance is a vector with values for *n* attributes

$$x_i = \langle x_i(1), x_i(2), \dots, x_i(n) \rangle$$

 It can also be seen as values of a set of random variables (attributes)

$$x_i = \langle A_1 = x_i(1), A_2 = x_i(2), \dots, A_n = x_i(n) \rangle$$

### Naïve Bayes assumption

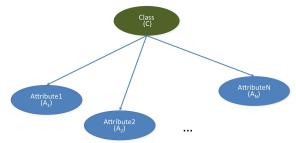
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$$x_i = \langle A_1 = x_i(1), A_2 = x_i(2), \dots, A_n = x_i(n) \rangle$$

Naïve Bayes assumption: all attribute values are independent



### Attributes independence

• Thus,

$$p(X = x_i \mid C = c_k) = p(A_1 = x_i(1), \dots, A_n = x_i(n) \mid C = c_k)$$

$$= \prod_{i=1}^{n} p(A_i = x_i(j) \mid C = c_k)$$

### Attributes independence

Thus,

$$p(X = x_i \mid C = c_k) = p(A_1 = x_i(1), \dots, A_n = x_i(n) \mid C = c_k)$$

$$= \prod_{j=1}^{n} p(A_j = x_i(j) \mid C = c_k)$$

And

class
$$(x_i)$$
 =  $\underset{c_k \in C}{\text{arg max}} p(X = x_i \mid C = c_k) p(C = c_k)$   
 =  $\underset{c_k \in C}{\text{arg max}} p(C = c_k) \prod_{j=1}^n p(A_j = x_i(j) \mid C = c_k)$ 

## Generating the classifier

- For each class  $c_k \in C$ 
  - $p(C = c_k) = \frac{\text{number examples of class } c_k}{\text{total number of examples}}$
  - For each attribute A<sub>j</sub> and value a<sub>jl</sub>, compute p(A<sub>j</sub> = a<sub>jl</sub> | C = c<sub>k</sub>):
    - if  $A_j$  is discrete,  $p(A_j = a_{jl} \mid C = c_k) = \frac{\text{number examples of class } c_k \text{ that } A_j = a_{jl}}{\text{number of examples of class } c_k}$
    - if A<sub>j</sub> is continuous, compute μ<sub>jk</sub> (average) y σ<sub>jk</sub> (standard deviation)

#### Classification

Given a new example x<sub>i</sub>, classify it as:

$$class(x_i) = \arg \max_{c_k \in C} p(C = c_k) \prod_{j=1}^n p(A_j = x_i(j) \mid C = c_k)$$

- If A<sub>j</sub> is discrete, p(A<sub>j</sub> = x<sub>i</sub>(j) | C = c<sub>k</sub>) has been stored for all j, values of j and k
- If  $A_j$  is continuous,

$$p(A_j = x_i(j) \mid C = c_k) = \frac{1}{\sqrt{2\pi\sigma_{jk}^2}} e^{-\frac{1}{2}\frac{(x_i(j) - \mu_{jk})^2}{\sigma_{jk}^2}}$$

#### Classification

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Correction

if any 
$$p(A_j = x_i(j) \mid C = c_k) = 0$$
  
then  $p(A_j = x_i(j) \mid C = c_k) = \epsilon, 0 < \epsilon < 1$ 



# Example

Age	Income	Fixed job	Client	Class
Young	Small	No	No	No
Adult	None	Yes	Yes	No
Adult	Medium	No	No	Yes
Young	Small	No	No	No
Young	High	Yes	Yes	Yes
Adult	Medium	Yes	Yes	Yes
Young	Small	Yes	Yes	Yes
Adult	Small	No	Yes	No

### Example

- Class a priori probability
  - $P(Yes) = \frac{4}{8} = 0.5$
  - $P(No) = \frac{4}{8} = 0.5$
- Conditional probabilities

	Class		
Age	Yes	No	
Young	$\frac{2}{4} = 0.5$	$\frac{2}{4} = 0.5$	
Adult	$\frac{2}{4} = 0.5$	$\frac{2}{4} = 0.5$	

	Class		
Income	Yes	No	
None	$\frac{0}{4} = 0.0$	$\frac{1}{4} = 0.25$	
Small	$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$	
Medium	$\frac{2}{4} = 0.5$	$\frac{0}{4} = 0.0$	
High	$\frac{1}{4} = 0.25$	$\frac{\dot{0}}{4} = 0.0$	

Fixed	Cla	ass
job	Yes	No
Yes	$\frac{3}{4} = 0.75$	$\frac{1}{4} = 0.25$
No	$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$

	Class				
Client	Yes	No			
Yes	$\frac{3}{4} = 0.75$	$\frac{2}{4} = 0.5$			
No	$\frac{1}{4} = 0.25$	$\frac{2}{4} = 0.5$			
	7 E L 7 E L 7 E				

#### Classification

• Given  $X_i$  to classify

Age	Income	Fixed job	Client
Adult	High	Yes	No

• what is its most probable class? ( $\epsilon = 0.01$ )

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Age	Income	Fixed job	Client
Adult	High	Yes	No

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$$p(C = \text{Yes} \mid X_i) = p(C = \text{Yes}) \times p(\text{Age=Adult} \mid C = \text{Yes}) \times \\ p(\text{Income=High} \mid C = \text{Yes}) \times \\ p(\text{Fixed job=Yes} \mid C = \text{Yes}) \times \\ p(\text{Client=No} \mid C = \text{No}) \times \\ p(\text{Income=High} \mid C = \text{No}) \times \\ p(\text{Fixed job=Yes} \mid C = \text{No}) \times \\ p(\text{Client=No} \mid C = \text{No}) \times \\ p(\text{Client=No} \mid C = \text{No}) \times \\ p(\text{Single operator}) \times$$

Most probable class: Yes

