Econ Problem Set 1

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Problem 1.

- (i). The state variables are the future sequence of prices and the number of barrels that are available to the owner.
- (ii). The control variable is the number of barrels sold.
- (iii). The transition equation is

$$b_{t+1} = b_t - k_t$$

  where $b_t =$ the number of barrels available at the start of period t and $k_t =$ the number of barrels sold in period t. Define also that $p_t =$ the price in period t.

- (iv). The sequence problem is:

$$max_{\{k_t\}_{t=1}^{\infty}} \left\{ \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} p_t k_t \right\} \text{ such that } b_0 = \sum_{t=1}^{\infty} k_t$$

  while the Bellman equation is:

$$V(B) = max_{0 \leq k \leq B} \left\{ pk + \frac{1}{1+r} V(B') \right\}$$

  where $B' = B - k$

- (v). I have a few constraints. First of all, the lifetime budget constraint given above:

$$b_0 = \sum_{t=1}^{\infty} k_t$$

  The $=$ is actually a $\geq$, but it is always optimal to sell more, assuming positive price. We also have the constraint that for all t,

$$k_t \geq 0$$

  . Therefore, the Lagrangian looks like this:

$$\mathcal{L}(k_1, ..., k_n, ..., \lambda) = \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} p_t k_t + \lambda(b_0 - \sum_{t=1}^{\infty} k_t)$$

The first order conditions are:

$$\frac{1}{(1+r)^{t-1}}^{t-1} p_t = \lambda \text{ for all } t \tag{1}$$

$$b_0 = \sum_{t=1}^{\infty} k_t \tag{2}$$

Combining two periods of (1),

$$\frac{1}{(1+r)^{t-1}} p_t = \frac{1}{(1+r)^t} p_{t+1}$$
$$\implies p_t = \frac{1}{(1+r)} p_{t+1}$$

This is the Euler equation, which has a weird look to it because there is no k_t choice term, but this is because the problem is effectively describing linear utility.

- (vi). If $p_{t+1} = p_t$ for all t, it is optimal to sell all the oil in the first period, because there is no discount term on the first period sales. If $p_{t+1} > \frac{1}{1+r}$ for all t, it would never be optimal to sell the oil and the owner would continue to hold it. ***

Problem 2.

- (i) The state variables are the capital stock k_t, and the size of the shock z_t. Income y_t could also maybe be considered a state variable, but it is completely determined by these other two.
- (ii) The control variables are consumption and investment, (c_t) and (i_t).
- (iii) The Bellman equation is:

$$V(k_t, z_t) = max_{c_t}\{E_0(c_t) + \beta E_{z_{t+1}|z_t} V(k_{t+1}, z_{t+1})\} \tag{3}$$

  where $k_{t+1} = (1-\delta)k_t + (y_t - c_t) = (1-\delta)k_t + z_t k_t - c_t$ (law of motion). Also, $E_{z_{t+1}|z_t} = E_{z_{t+1}}$ because the shocks are independently distributed. This should be enough to solve the model!
- (iv) I've done this part of the exercise in the jupyter notebook titled "PS1Ex2 - Neoclassical Growth"

Problem 3.

- (i) The bellman equation is identical to the bellman equation we used in Problem 2 (See equation (3)). The difference here is that we cannot simply ignore the difference between $E_{z_{t+1}|z_t}$ and E_{z_t} - because it is an autoregressive process, the shocks are correlated.
- (ii) I've done this in the jupyter notebook titled "PS1Ex3 - Neoclassical Growth with Serial Correlation"

Problem 4.

- (i) The Bellman equation is:

$$V(w) = max_{d\in\{1,0\}} V^d(w)$$

where w is the wage that is offered in the current period, w' is the wage offered in the next period, b is the value of unemployment benefits, $d = 1$ is the decision to work, $d = 0$ is the decision not to work, and

$$V^1(w) = \sum_{t=0}^{\infty} \beta^t w$$

$$V^0(w) = \beta E_{w'|w}(V(w'))$$

- (ii)