

# Econ Problem Set 1

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**Problem 1.** • (i). The state variables are the future sequence of prices and the number of barrels that are available to the owner.

• (ii). The control variable is the number of barrels sold.

• (iii). The transition equation is

$$b_{t+1} = b_t - k_t$$

where  $b_t$  = the number of barrels available at the start of period  $t$  and  $k_t$  = the number of barrels sold in period  $t$ . Define also that  $p_t$  = the price in period  $t$ .

• (iv). The sequence problem is:

$$\max_{\{k_t\}_{t=1}^{\infty}} \left\{ \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} p_t k_t \right\} \text{ such that } b_0 = \sum_{t=1}^{\infty} k_t$$

while the Bellman equation is:

$$V(B) = \max_{0 \leq k \leq B} \left\{ pk + \frac{1}{1+r} V(B') \right\}$$

where  $B' = B - k$

• (v). I have a few constraints. First of all, the lifetime budget constraint given above:

$$b_0 = \sum_{t=1}^{\infty} k_t$$

The  $=$  is actually a  $\geq$ , but it is always optimal to sell more, assuming positive price. We also have the constraint that for all  $t$ ,

$$k_t \geq 0$$

. Therefore, the Lagrangian looks like this:

$$\mathcal{L}(k_1, \dots, k_n, \dots, \lambda) = \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} p_t k_t + \lambda(b_0 - \sum_{t=1}^{\infty} k_t)$$

The first order conditions are:

$$\frac{1}{(1+r)^{t-1}} p_t = \lambda \text{ for all } t \quad (1)$$

$$b_0 = \sum_{t=1}^{\infty} k_t \quad (2)$$

Combining two periods of (1),

$$\begin{aligned} \frac{1}{(1+r)^{t-1}} p_t &= \frac{1}{(1+r)^t} p_{t+1} \\ \implies p_t &= \frac{1}{(1+r)} p_{t+1} \end{aligned}$$

This is the Euler equation, which has a weird look to it because there is no  $k_t$  choice term, but this is because the problem is effectively describing linear utility.

- (vi). If  $p_{t+1} = p_t$  for all  $t$ , it is optimal to sell all the oil in the first period, because there is no discount term on the first period sales. If  $p_{t+1} > \frac{1}{1+r}$  for all  $t$ , it would never be optimal to sell the oil and the owner would continue to hold it. \*\*\*

**Problem 2.** • (i) The state variables are the capital stock  $k_t$ , and the size of the shock  $z_t$ . Income  $y_t$  could also maybe be considered a state variable, but it is completely determined by these other two.

- (ii) The control variables are consumption and investment,  $(c_t)$  and  $(i_t)$ .
- (iii) The Bellman equation is:

$$V(k_t, z_t) = \max_{c_t} \{ E_0(c_t) + \beta E_{z_{t+1}|z_t} V(k_{t+1}, z_{t+1}) \} \quad (3)$$

where  $k_{t+1} = (1 - \delta)k_t + (y_t - c_t) = (1 - \delta)k_t + z_t k_t - c_t$  (law of motion). Also,  $E_{z_{t+1}|z_t} = E_{z_{t+1}}$  because the shocks are independently distributed. This should be enough to solve the model!

- (iv) I've done this part of the exercise in the jupyter notebook titled "PS1Ex2 - Neo-classical Growth"

**Problem 3.** • (i) The bellman equation is identical to the bellman equation we used in Problem 2 (See equation (3)). The difference here is that we cannot simply ignore the difference between  $E_{z_{t+1}|z_t}$  and  $E_{z_t}$  - because it is an autoregressive process, the shocks are correlated.

- (ii)