Econ Problem Set 4

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1 DSGE

Problem 1. The first step is to substitute the form of the policy function ($K_{t+1} = e^{z_t} A K_t^{\alpha}$) into the euler equation. We see that on the left side:

$$\frac{1}{e^{z_t} K_t^{\alpha} - K_{t-1}} = \frac{1}{e^{z_t} K_t^{\alpha}(1-A)} \tag{1}$$

And on the right side, we have

$$\beta \mathbb{E}\Big(\frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_t+1} K_{t+1}^{\alpha} - K_{t+2}}\Big) = \beta \mathbb{E}\Big(\frac{\alpha e^{z_{t+1}} (e^{z_t} A K_t^{\alpha})^{\alpha-1}}{e^{z_{t+1}} (e^{z_t} A K_t^{\alpha})^{\alpha} - (e^{z_t+1} A (e^{z_t} A K_t^{\alpha}))^{\alpha}}\Big) \tag{2}$$

$$= \beta \mathbb{E}\Big(\frac{\alpha e^{z_{t+1}} (e^{z_t} A K_t^{\alpha})^{\alpha-1}}{e^{z_{t+1}} (e^{z_t} A K_t^{\alpha^2})(1-A)}\Big) \tag{3}$$

$$= \beta \mathbb{E}\Big(\frac{\alpha}{(e^{z_t} A K_t^{\alpha})(1-A)}\Big) \tag{4}$$

Since we are in period t and there is no uncertainty about anything inside the expected value, we can say

$$\frac{1}{e^{z_t} K_t^{\alpha}(1-A)} = \frac{\alpha\beta}{(e^{z_t} A K_t^{\alpha})(1-A)} \tag{5}$$

Which is true if and only if $\alpha\beta = A$.

Problem 2. The characterizing equations are:

- **Household**

$$w_t(1-\tau)\frac{1}{c_t}) = \frac{a}{1-l_t} \tag{6}$$

$$\frac{1}{c_t} = \beta \mathbb{E}\big[\frac{1}{c_{t+1}}[(r_{t+1} - \delta)(1-\tau) + 1]\big] \tag{7}$$

- **Firm**

$$R_t = \alpha e^{z_t} K^{\alpha-1} L^{1-\alpha} \tag{8}$$

$$W_t = (1-\alpha) e^{z_t} K^{\alpha} L^{-\alpha} \tag{9}$$

- **Government**

$$\tau[w_t l_t (r_t - \delta) k_t] = T_t \tag{10}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{11}$$
$$k_t = K_t \tag{12}$$
$$w_t = W_t \tag{13}$$
$$r_t = R_t \tag{14}$$

We can't use the same tricks to solve for A, because the functional form from problem 1 does not yield the same nice cancellation.

Problem 3. Characterizing equations:

- **Household**

$$w_t(1-\tau)c_t^{-\gamma} = \frac{a}{1-l_t} \tag{15}$$
$$c_t^{-\gamma} = \beta \mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\big] \tag{16}$$

- **Firm**

$$R_t = \alpha e^{z_t} K_t^{\alpha-1} L_T^{1-\alpha} \tag{17}$$
$$W_t = (1-\alpha) e^{z_t} K_t^{\alpha} L_t^{-\alpha} \tag{18}$$

- **Government**

$$\tau[w_t l_t (r_t - \delta) k_t] = T_t \tag{19}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{20}$$
$$k_t = K_t \tag{21}$$
$$w_t = W_t \tag{22}$$
$$r_t = R_t \tag{23}$$

Problem 4. Characterizing Equations:

- **Household**

$$w_t(1-\tau)c_t^{-\gamma} = -a(1-l_t)^{-\xi} \tag{24}$$
$$c_t^{-\gamma} = \beta \mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\big] \tag{25}$$

- **Firm**

$$R_t = e_t^z[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1}{\eta}-1} \alpha K_t^{\eta-1} \tag{26}$$
$$W_t = e_t^z[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1}{\eta}-1} (1-\alpha) L_t^{\eta-1} \tag{27}$$

- **Government**

$$\tau[w_t l_t (r_t - \delta) k_t] = T_t \tag{28}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{29}$$
$$k_t = K_t \tag{30}$$
$$w_t = W_t \tag{31}$$
$$r_t = R_t \tag{32}$$

Problem 5. Characterizing Equations:

- **Household**

$$c_t^{-\gamma} = \beta \mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\big] \tag{33}$$

- **Firm**

$$R_t = \alpha e^{\alpha z_t} K_t^{\alpha-1} L_T^{1-\alpha} \tag{34}$$
$$W_t = (1-\alpha) e^{\alpha z_t} K_t^{\alpha} L_t^{-\alpha} \tag{35}$$

- **Government**

$$\tau[w_t (r_t - \delta) k_t] = T_t \tag{36}$$

- **Market Clearing and Price Equivalence**

$$1 = L_t \tag{37}$$
$$k_t = K_t \tag{38}$$
$$w_t = W_t \tag{39}$$
$$r_t = R_t \tag{40}$$

In steady state we get:

$$c^{-\gamma} = \beta\big[c^{-\gamma}[(r-\delta)(1-\tau) + 1]\big] \tag{41}$$
$$r = \alpha k^{\alpha-1} \tag{42}$$
$$w = (1-\alpha) k^{\alpha} \tag{43}$$
$$\tau[w(r-\delta)k] = T \tag{44}$$

Solving analytically, we see that

$$r = \frac{\frac{1}{\beta} - 1}{1-\tau} + \delta = 7.287 \tag{45}$$
$$k = \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}} = .121 \tag{46}$$
$$w = (1-\alpha) k^{\alpha} = 1.328 \tag{47}$$
$$Y = k^{\alpha} = .729 \tag{48}$$
$$I = \delta k = 2.213 \tag{49}$$

The values of r, w, k, and c are solved numerically in the jupyter notebook, and they confirm these numbers.

Problem 6. Characterizing Equations (household from problem 4, firm from problem 5):

- **Household**

$$w_t(1-\tau)c_t^{-\gamma} = -a(1-l_t)^{-\xi} \tag{50}$$
$$c_t^{-\gamma} = \beta\mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\big] \tag{51}$$

- **Firm**

$$R_t = \alpha e^{\alpha z_t} K_t^{\alpha-1} L_T^{1-\alpha} \tag{52}$$
$$W_t = (1-\alpha)e^{\alpha z_t} K_t^{\alpha} L_t^{-\alpha} \tag{53}$$

- **Government**

$$\tau[w_t l_t (r_t-\delta)k_t] = T_t \tag{54}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{55}$$
$$k_t = K_t \tag{56}$$
$$w_t = W_t \tag{57}$$
$$r_t = R_t \tag{58}$$

Steady state gives us:

$$w(1-\tau)c^{-\gamma} = -a(1-l)^{-\xi} \tag{59}$$
$$c^{-\gamma} = \beta\big[c^{-\gamma}[(r-\delta)(1-\tau)+1]\big] \tag{60}$$
$$r = \alpha k^{\alpha-1} l^{1-\alpha} \tag{61}$$
$$w = (1-\alpha)k^{\alpha} l^{-\alpha} \tag{62}$$
$$\tau[wl(r-\delta)k] = T \tag{63}$$

These are solved numerically in the Jupyter notebook.

2 Linearization

Problem 1. I am very confused about how to log-linearize, so I didn't manage this or number 2.

Problem 3. I shifted everything up a period since this made more sense to me. The epsilons can disappear because they turn to 0 in expected value. Also every variable is supposed to have a tilde, but I might have missed some.

$$
\begin{aligned}
\mathbb{E}\{F\tilde{X}_{t+2} + G\tilde{X}_{t+1} + H\tilde{X}_t + L\tilde{Z}_{t+2} + M\tilde{Z}_{t+1}\} &= 0 \\
\mathbb{E}\{F(P\tilde{X}_{t+1} + Q\tilde{Z}_{t+2}) + G(P\tilde{X}_t + Q\tilde{Z}_{t+1}) + H\tilde{X}_t + L(N\tilde{Z}_{t+1} + \epsilon_{t+1}) + M\tilde{Z}_{t+1}\} &= 0 \\
\mathbb{E}\{((FP)\tilde{X}_{t+1} + FQ\tilde{Z}_{t+2} + (GP + H)\tilde{X}_t + (GQ + LN + M)\tilde{Z}_{t+1}\} &= 0 \\
\mathbb{E}\{(FP)(P\tilde{X}_t + Q\tilde{Z}_{t+1}) + FQ(N\tilde{Z}_{t+1} + \epsilon_{t+1}) + (GP + H)\tilde{X}_t + (GQ + LN + M)\tilde{Z}_{t+1} &= 0 \\
((FP + G)P + H)X_t + [(FQ + L)N + (FP + G)Q + M]\tilde{Z}_{t+1} &= 0
\end{aligned}
$$