

Econ Problem Set 4

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Problem 1. The first step is to substitute the form of the policy function ($K_{t+1} = e^{z_t} AK_t^\alpha$) into the euler equation. We see that on the left side:

$$\frac{1}{e^{z_t} K_t^\alpha - K_{t-1}} = \frac{1}{e^{z_t} K_t^\alpha (1 - A)} \quad (1)$$

And on the right side, we have

$$\beta \mathbb{E} \left(\frac{\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1}}{e^{z_{t+1}} K_{t+1}^\alpha - K_{t+2}} \right) = \beta \mathbb{E} \left(\frac{\alpha e^{z_{t+1}} (e^{z_t} AK_t^\alpha)^{\alpha-1}}{e^{z_{t+1}} (e^{z_t} AK_t^\alpha)^\alpha - (e^{z_{t+1}} A (e^{z_t} AK_t^\alpha))^\alpha} \right) \quad (2)$$

$$= \beta \mathbb{E} \left(\frac{\alpha e^{z_{t+1}} (e^{z_t} AK_t^\alpha)^{\alpha-1}}{e^{z_{t+1}} (e^{z_t} AK_t^\alpha)^\alpha (1 - A)} \right) \quad (3)$$

$$= \beta \mathbb{E} \left(\frac{\alpha}{(e^{z_t} AK_t^\alpha) (1 - A)} \right) \quad (4)$$

Since we are in period t and there is no uncertainty about anything inside the expected value, we can say

$$\frac{1}{e^{z_t} K_t^\alpha (1 - A)} = \frac{\alpha \beta}{(e^{z_t} AK_t^\alpha) (1 - A)} \quad (5)$$

Which is true if and only if $\alpha \beta = A$.

Problem 2. The characterizing equations are:

- **Household**

$$w_t (1 - \tau) \frac{1}{c_t} = \frac{a}{1 - l_t} \quad (6)$$

$$\frac{1}{c_t} = \beta \mathbb{E} \left[\frac{1}{c_{t+1}} [(r_{t+1} - \delta)(1 - \tau) + 1] \right] \quad (7)$$

- **Firm**

$$R_t = \alpha e^{z_t} K^{\alpha-1} L^{1-\alpha} \quad (8)$$

$$W_t = (1 - \alpha) e^{z_t} K^\alpha L^{-\alpha} \quad (9)$$

- **Government**

$$\tau[w_t l_t (r_t - \delta) k_t] = T_t \quad (10)$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \quad (11)$$

$$k_t = K_t \quad (12)$$

$$w_t = W_t \quad (13)$$

$$r_t = R_t \quad (14)$$

We can't use the same tricks to solve for A, because the functional form from problem 1 does not yield the same nice cancellation.

Problem 3. Characterizing equations:

- **Household**

$$w_t(1 - \tau)c_t^{-\gamma} = \frac{a}{1 - l_t} \quad (15)$$

$$c_t^{-\gamma} = \beta \mathbb{E}[c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1 - \tau) + 1]] \quad (16)$$

- **Firm**

$$R_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha} \quad (17)$$

$$W_t = (1 - \alpha) e^{z_t} K_t^{\alpha} L_t^{-\alpha} \quad (18)$$

- **Government**

$$\tau[w_t l_t (r_t - \delta) k_t] = T_t \quad (19)$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \quad (20)$$

$$k_t = K_t \quad (21)$$

$$w_t = W_t \quad (22)$$

$$r_t = R_t \quad (23)$$

Problem 4. Characterizing Equations:

- **Household**

$$w_t(1 - \tau)c_t^{-\gamma} = -a(1 - l_t)^{-\gamma} \quad (24)$$

$$c_t^{-\gamma} = \beta \mathbb{E}[c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1 - \tau) + 1]] \quad (25)$$

- **Firm**

$$R_t = e_t^z [\alpha K_t^\eta + (1 - \alpha) L_t^\eta]^{\frac{1}{\eta}-1} \alpha K_t^{\eta-1} \quad (26)$$

$$W_t = e_t^z [\alpha K_t^\eta + (1 - \alpha) L_t^\eta]^{\frac{1}{\eta}-1} (1 - \alpha) L_t^{\eta-1} \quad (27)$$

- **Government**

$$\tau[w_t l_t (r_t - \delta)k_t] = T_t \quad (28)$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \quad (29)$$

$$k_t = K_t \quad (30)$$

$$w_t = W_t \quad (31)$$

$$r_t = R_t \quad (32)$$

Problem 5. Characterizing Equations:

- **Household**

$$c_t^{-\gamma} = \beta \mathbb{E}[c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1 - \tau) + 1]] \quad (33)$$

- **Firm**

$$R_t = \alpha e^{\alpha z_t} K_t^{\alpha-1} L_t^{1-\alpha} \quad (34)$$

$$W_t = (1 - \alpha) e^{\alpha z_t} K_t^{\alpha} L_t^{-\alpha} \quad (35)$$

- **Government**

$$\tau[w_t (r_t - \delta)k_t] = T_t \quad (36)$$

- **Market Clearing and Price Equivalence**

$$1 = L_t \quad (37)$$

$$k_t = K_t \quad (38)$$

$$w_t = W_t \quad (39)$$

$$r_t = R_t \quad (40)$$

In steady state we get:

$$c^{-\gamma} = \beta \mathbb{E}[c^{-\gamma}[(r - \delta)(1 - \tau) + 1]] \quad (41)$$

$$r = \alpha e^{\alpha z} k^{\alpha-1} \quad (42)$$

$$w = (1 - \alpha) e^{\alpha z} k^{\alpha} \quad (43)$$

$$\tau[w(r - \delta)k] = T \quad (44)$$