Econ Problem Set 4

Matthew Brown
OSM Boot Camp 2018

July 10, 2018

Problem 1. The first step is to substitute the form of the policy function ($K_{t+1} = e^{z_t}AK_t^\alpha$) into the euler equation. We see that on the left side:

$$\frac{1}{e^{z_t}K_t^\alpha - K_{t-1}} = \frac{1}{e^{z_t}K_t^\alpha(1-A)} \tag{1}$$

And on the right side, we have

$$\beta\mathbb{E}\Big(\frac{\alpha e^{z_{t+1}}K_{t+1}^{\alpha-1}}{e^{z_t+1}K_{t+1}^\alpha - K_{t+2}}\Big) = \beta\mathbb{E}\Big(\frac{\alpha e^{z_{t+1}}(e^{z_t}AK_t^\alpha)^{\alpha-1}}{e^{z_{t+1}}(e^{z_t}AK_t^\alpha)^\alpha - (e^{z_t+1}A(e^{z_t}AK_t^\alpha))^\alpha}\Big) \tag{2}$$

$$= \beta\mathbb{E}\Big(\frac{\alpha e^{z_{t+1}}(e^{z_t}AK_t^\alpha)^{\alpha-1}}{e^{z_{t+1}}(e^{z_t}AK_t^{\alpha^2})(1-A)}\Big) \tag{3}$$

$$= \beta\mathbb{E}\Big(\frac{\alpha}{(e^{z_t}AK_t^\alpha)(1-A)}\Big) \tag{4}$$

Since we are in period t and there is no uncertainty about anything inside the expected value, we can say

$$\frac{1}{e^{z_t}K_t^\alpha(1-A)} = \frac{\alpha\beta}{(e^{z_t}AK_t^\alpha)(1-A)} \tag{5}$$

Which is true if and only if $\alpha\beta = A$.

Problem 2. The characterizing equations are:

- **Household**

$$w_t(1-\tau)\frac{1}{c_t}) = \frac{a}{1-l_t} \tag{6}$$

$$\frac{1}{c_t} = \beta\mathbb{E}\Big[\frac{1}{c_{t+1}}[(r_{t+1}-\delta)(1-\tau)+1]\Big] \tag{7}$$

- **Firm**

$$R_t = \alpha e^{z_t}K^{\alpha-1}L^{1-\alpha} \tag{8}$$

$$W_t = (1-\alpha)e^{z_t}K^\alpha L^{-\alpha} \tag{9}$$

- **Government**

$$\tau[w_t l_t (r_t - \delta) k_t] = T_t \tag{10}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{11}$$
$$k_t = K_t \tag{12}$$
$$w_t = W_t \tag{13}$$
$$r_t = R_t \tag{14}$$

We can't use the same tricks to solve for A, because the functional form from problem 1 does not yield the same nice cancellation.

Problem 3. Characterizing equations:

- **Household**

$$w_t(1-\tau)c_t^{-\gamma} = \frac{a}{1-l_t} \tag{15}$$
$$c_t^{-\gamma} = \beta\mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\big] \tag{16}$$

- **Firm**

$$R_t = \alpha e^{z_t} K_t^{\alpha-1} L_T^{1-\alpha} \tag{17}$$
$$W_t = (1-\alpha)e^{z_t} K_t^{\alpha} L_t^{-\alpha} \tag{18}$$

- **Government**

$$\tau[w_t l_t (r_t - \delta) k_t] = T_t \tag{19}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{20}$$
$$k_t = K_t \tag{21}$$
$$w_t = W_t \tag{22}$$
$$r_t = R_t \tag{23}$$

Problem 4. Characterizing Equations:

- **Household**

$$w_t(1-\tau)c_t^{-\gamma} = -a(1-l_t)^{-\gamma} \tag{24}$$
$$c_t^{-\gamma} = \beta\mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\big] \tag{25}$$

- **Firm**

$$R_t = e_t^z[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1}{\eta}-1}\alpha K_t^{\eta-1} \tag{26}$$
$$W_t = e_t^z[\alpha K_t^\eta + (1-\alpha)L_t^\eta]^{\frac{1}{\eta}-1}(1-\alpha)L_t^{\eta-1} \tag{27}$$

- **Government**

$$\tau[w_t l_t (r_t - \delta)k_t] = T_t \tag{28}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{29}$$
$$k_t = K_t \tag{30}$$
$$w_t = W_t \tag{31}$$
$$r_t = R_t \tag{32}$$

Problem 5. Characterizing Equations:

- **Household**

$$c_t^{-\gamma} = \beta \mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1} - \delta)(1-\tau) + 1]\big] \tag{33}$$

- **Firm**

$$R_t = \alpha e^{\alpha z_t} K_t^{\alpha-1} L_T^{1-\alpha} \tag{34}$$
$$W_t = (1-\alpha) e^{\alpha z_t} K_t^{\alpha} L_t^{-\alpha} \tag{35}$$

- **Government**

$$\tau[w_t (r_t - \delta)k_t] = T_t \tag{36}$$

- **Market Clearing and Price Equivalence**

$$1 = L_t \tag{37}$$
$$k_t = K_t \tag{38}$$
$$w_t = W_t \tag{39}$$
$$r_t = R_t \tag{40}$$

In steady state we get:

$$c^{-\gamma} = \beta\big[c^{-\gamma}[(r - \delta)(1-\tau) + 1]\big] \tag{41}$$
$$r = \alpha k^{\alpha-1} \tag{42}$$
$$w = (1-\alpha)k^{\alpha} \tag{43}$$
$$\tau[w(r-\delta)k] = T \tag{44}$$

Solving analytically, we see that

$$r = \frac{\frac{1}{\beta} - 1}{1-\tau} + \delta = 7.287 \tag{45}$$
$$k = \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}} = .121 \tag{46}$$
$$w = (1-\alpha)k^{\alpha} = 1.328 \tag{47}$$
$$Y = k^{\alpha} = .729 \tag{48}$$
$$I = \delta k = 2.213 \tag{49}$$

The values of r, w, k, and c are solved numerically in the jupyter notebook, and they confirm these numbers.

Problem 6. Characterizing Equations (household from problem 4, firm from problem 5):

- **Household**

$$w_t(1-\tau)c_t^{-\gamma} = -a(1-l_t)^{-\gamma} \tag{50}$$

$$c_t^{-\gamma} = \beta\mathbb{E}\big[c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\big] \tag{51}$$

- **Firm**

$$R_t = \alpha e^{\alpha z_t} K_t^{\alpha-1} L_T^{1-\alpha} \tag{52}$$

$$W_t = (1-\alpha)e^{\alpha z_t} K_t^{\alpha} L_t^{-\alpha} \tag{53}$$

- **Government**

$$\tau[w_t l_t(r_t-\delta)k_t] = T_t \tag{54}$$

- **Market Clearing and Price Equivalence**

$$l_t = L_t \tag{55}$$

$$k_t = K_t \tag{56}$$

$$w_t = W_t \tag{57}$$

$$r_t = R_t \tag{58}$$

Steady state gives us:

$$w(1-\tau)c^{-\gamma} = -a(1-l)^{-\gamma} \tag{59}$$

$$c^{-\gamma} = \beta\big[c^{-\gamma}[(r-\delta)(1-\tau)+1]\big] \tag{60}$$

$$r = \alpha k^{\alpha-1} \tag{61}$$

$$w = (1-\alpha)k^{\alpha} \tag{62}$$

$$\tau[w(r-\delta)k] = T \tag{63}$$

These are solved numerically in the Jupyter notebook.