COMPUTING METHODS FOR PHYSICS
7 SEPTEMBER 2021

You must submit your exam by following the instructions at http://www.roma1.infn.it/people/rahatlou/cmp/

Part 1: 2D Motion with Strategy Pattern (C++)

A body of mass m moves in a two-dimensional plane (x,y) with initial velocity (v_{x0}, v_{y0}) subject to gravity along the y axis $(-mg\hat{y})$ and to friction, modelled as $-\beta\vec{v}$. The equation for the velocity is

$$m\dot{v}_x = -\beta v_x$$
$$m\dot{v}_y = -mg - \beta v_y$$

The analytical solution of the problem is

$$v_x(t) = v_{x0}e^{-\beta t/m}$$
$$v_y(t) = v_{y0}e^{-\beta t/m} - \frac{mg}{\beta}(1 - e^{-\beta t/m})$$

Provide two classes, **`Analytical`** and **`RungeKutta`**, to implement the method **`velocity()`** of a base class **`SpeedCalculator`**, with proper arguments and return type. In the **`Analytical`** class you must use the exact solutions provided. In the **`RungeKutta`** class, you can use the iterative 2nd-order Runge-Kutta method for 1st order differential equations $\frac{dy}{dx} = f(x,y)$ with $y(0) = y_0$; given a value x, y(x) is:

$$K_1 = h \cdot f(x_n, y_n)$$
$$K_2 = h \cdot f(x_n + h/2, y_n + K_1 \cdot h/2)$$
$$y_{n+1} = y_n + K_2$$

where h is the step size for the iterative method and must be one of the parameters to be passed to the method.

Evaluation will be based on: correct implementation of the Strategy pattern, correct C++ syntax, proper return type and arguments of functions, data members and interface of classes, unnecessary void functions, correct mathematical operations, correct physics units.

Part 2: Plots (ROOT or Python)

Assuming the initial values $v_{x0} = v_{y0} = 15\,\mathrm{m}/s$, m = 25 kg, and $\beta = 0.15\,\mathrm{N}s/m$, use ROOT or python to make 2 plots: plot $v_x(t)$ and $v_y(t)$ as a function of time t. In each plot, show both the analytical and the Runge-Kutta solutions. Use different line type and/or colours with proper legend.

If you use python, you must store the output of the C++ program and read the values in the python program.

Evaluation will be based on use of python features and data structures, comprehensions (instead of C-style for loops), labels, units, clarity ad correctness of plots.