$$\pi(H)\Pi(U)v = \left\langle \lambda, w^{-1} \cdot H \right\rangle \Pi(U)v = \langle w \cdot \lambda, H \rangle \, \Pi(U)v.$$

We conclude that $\Pi(U)v$ is a weight vector with weight $w \cdot \lambda$.

The same sort of reasoning shows that $\Pi(U)$ is an invertible map of the weight space with weight λ onto the weight space with weight $w \cdot \lambda$, whose inverse is $\Pi(U)^{-1}$. This means that the two weights have the same multiplicity. □

To represent the basic weights, $(1, 0)$ and $(0, 1)$, in our new approach, we look for diagonal, trace-zero matrices μ_1 and μ_2 such that

$$\begin{aligned} \langle \mu_1, H_1 \rangle &= 1, \quad \langle \mu_1, H_2 \rangle = 0 \\ \langle \mu_2, H_1 \rangle &= 0, \quad \langle \mu_2, H_2 \rangle = 1. \end{aligned}$$

These are easily found as

$$\mu_1 = \operatorname{diag}(2/3, -1/3, -1/3); \quad \mu_2 = \operatorname{diag}(1/3, 1/3, -2/3).$$

The positive simple roots $(2, -1)$ and $(-1, 2)$ are then represented as

$$\begin{aligned} \alpha_1 &= \ \ 2\mu_1 - \mu_2 = \operatorname{diag}(1, -1, 0); \\ \alpha_2 &= -\mu_1 + 2\mu_2 = \operatorname{diag}(0, 1, -1). \end{aligned} \tag{6.14}$$

Note that both α_1 and α_2 have length $\sqrt{2}$ and $\langle \alpha_1, \alpha_2 \rangle = -1$. Thus, the angle θ between them satisfies $\cos\theta = -1/2$, so that $\theta = 2\pi/3$.

Figure 6.2 shows the same information as Figure 6.1, namely, the roots and the dominant integral elements, but now drawn relative to the Weyl-invariant inner product in (6.13). We draw only the two-dimensional *real* subspace of $\mathfrak{h}$ consisting

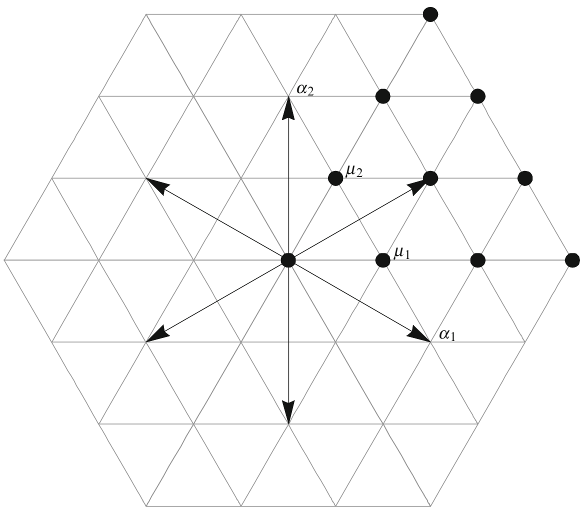

Fig. 6.2 The roots and dominant integral elements for $\mathsf{sl}(3; \mathbb{C})$, computed relative to a Weyl-invariant inner product