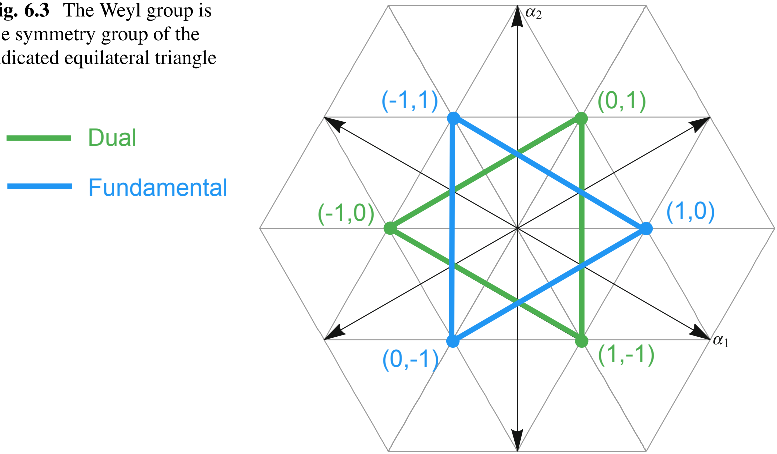

Fig. 6.3 The Weyl group is the symmetry group of the indicated equilateral triangle

of those elements μ such that $\langle \mu, H_1 \rangle$ and $\langle \mu, H_2 \rangle$ are real, since all the roots and weights have this property. Let $w_{(1,2,3)}$ denote the Weyl group element that acts by cyclically permuting the diagonal entries of each $H \in \mathfrak{h}$. Then $w_{(1,2,3)}$ takes α_1 to α_2 and α_2 to $-(\alpha_1 + \alpha_2)$, which is a counterclockwise rotation by $2\pi/3$ in Figure 6.2. Similarly, if $w_{(1,2)}$ the element that interchanges the first two diagonal entries of $H \in \mathfrak{h}$, then $w_{(1,2)}$ maps α_1 to $-\alpha_1$ and α_2 to $\alpha_1 + \alpha_2$. Thus, $w_{(1,2)}$ is the reflection across the line perpendicular to α_1. The reader is invited to compute the action of the remaining elements of the Weyl group and to verify that it is the symmetry group of the equilateral triangle in Figure 6.3.

We previously defined a pair (m_1, m_2) to be integral if m_1 and m_2 are integers and dominant if $m_1 \geq 0$ and $m_2 \geq 0$. These concepts translate into our new language as follows. If $\lambda \in \mathfrak{h}$, then λ is **integral** if $\langle \lambda, H_1 \rangle$ and $\langle \lambda, H_2 \rangle$ are integers and λ is **dominant** if $\langle \lambda, H_1 \rangle \geq 0$ and $\langle \lambda, H_2 \rangle \geq 0$. Geometrically, the set of dominant elements is a sector spanning an angle of $\pi/3$.

6.7 Weight Diagrams

In this section, we display the weights and multiplicities for several irreducible representations of $\mathsf{sl}(3;\mathbb{C})$. Figure 6.4 covers the irreducible representations with highest weighs $(1,1)$, $(1,2)$, $(0,4)$, and $(2,2)$. The first of these examples was analyzed in Sect. 6.5, and the other examples can be analyzed by the same method. In each part of the figure, the arrows indicate the roots, the two black lines indicate the boundary of the set of dominant elements, and the dashed lines indicate the boundary of the set of points lower than the highest weight. Each weight of