Fig. 6.4   Weight diagrams for representations with highest weights $(1, 1)$, $(1, 2)$, $(0, 4)$, and $(2, 2)$
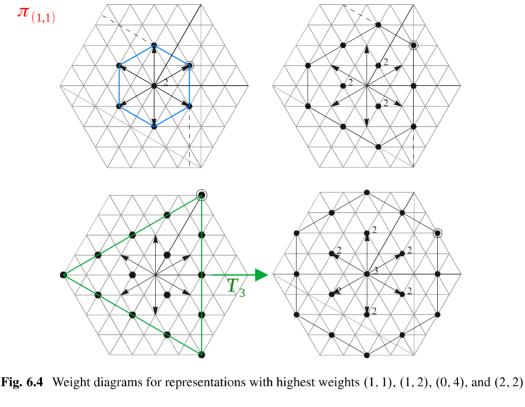
a particular representation is indicated by a black dot, with a number next to a dot indicating its multiplicity. A dot without a number indicates a weight of multiplicity 1.

Our last example is the representation with highest weight $(9, 2)$ (Figure 6.5), which cannot feasibly be analyzed using the method of Sect. 6.5. Instead, the weights are determined by the results of Sect. 6.8 and the multiplicities are computed using the Kostant multiplicity formula. (See Figure 10.8 in Sect. 10.6.) See also Exercises 11 and 12 for another approach to computing multiplicities.

6.8   Further Properties of the Representations

Although we now have a classification of the irreducible representations of $\mathsf{sl}(3; \mathbb{C})$ by means of their highest weights, there are other things we might like to know about the representations, such as (1) the other weights that occur, besides the highest weight, (2) the multiplicities of those weights, and (3) the dimension of the representation. In this section, we establish which weights occur and state without proof the formula for the dimension. A formula for the multiplicities and a proof of the dimension formula are given in Chapter 10 in the setting of general semisimple Lie algebras.