Comparison of deep learning algorithms for forecasting stock returns and portfolio optimization

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Introduction

- Can deep learning techniques effectively forecast future returns or sign of returns from past information?
- How can we build, based on those forecast, a tradable portfolio?
- Can this portfolio outperform the market?
- How can we evaluate the quality of a given model?

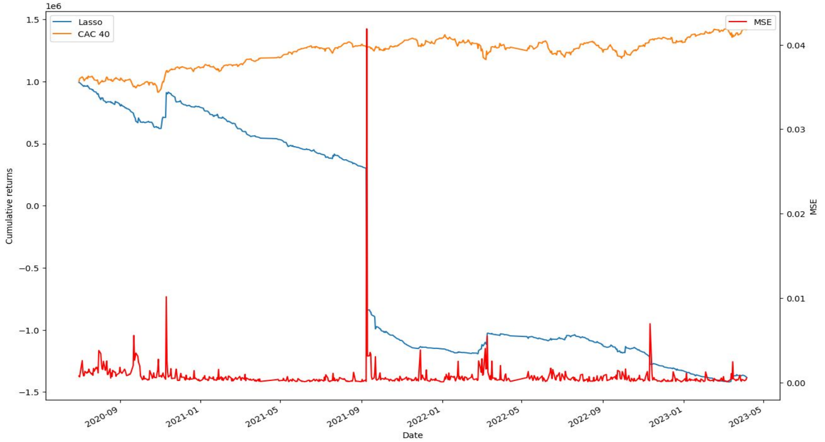
Data Preparation

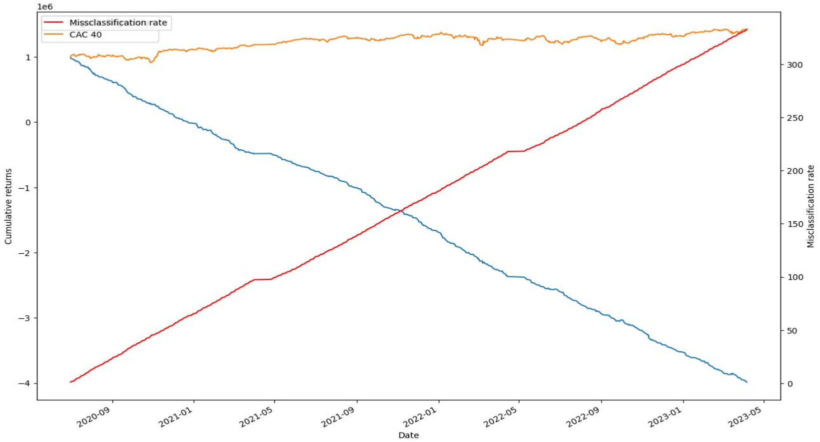
- CAC 40 stock data for June 1, 2009 to March 29, 2023
- Three types of signals: fundamental, macroeconomic and technical
- Drop stocks with >80% missing values
- Normalize with standard scaling

Baseline models

- Linear and logistic regression with LASSO penalty

Model	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total mean-squared error
LASSO	-92.2%	72.7%	-1.27	0.46
Model	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total misclassification error
Logistic Reg	-182.3%	13.4%	-13.6	50.4%





Models Used

- **Baseline models:** Linear and Logistic Regression
- **Deep learning models:**
  - Multi-layer Perceptron (MLP)
  - Convolutional Neural Network (CNN)
  - Recurrent Neural Network (RNN)
    - Long-Short Term Memory (LSTM)
    - Gated Recurrent Unit (GRU)
  - Time-Fusion Transformer (TFT)

MLP Architecture

- Varied dimensions of hidden layers and depth of network
- Mathematical model of hidden layers:

$$\mathbf{H} = \sigma_1(\mathbf{X}\mathbf{W}^{(1)} + \mathbf{b}^{(1)})$$
$$\mathbf{O} = \sigma_2(\mathbf{H}\mathbf{W}^{(2)} + \mathbf{b}^{(2)})$$

- Hidden layers: ReLU activations
- Final layer:
    - Sigmoid activation for classification
    - Identity activation for regression

MLP Results

Regression Models	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total mean-squared error
10 layers of dimension 32	**-13.14%**	**204.2%**	**-0.06**	**11.74**
10 layers of dimension 64	-20.5%	207.3%	-0.09	11.43
50 layers of dimension 64	-2.9%	3.3%	-0.87	0.28
Classification Models	**Yearly returns**	**Yearly volatility**	**Yearly Sharpe ratio**	**Misclassification Error**
10 layers of dimension 32	-198.3%	19.5%	-10.15	0.51
10 layers of dimension 64	-167.6%	17.0%	-9.85	0.51
50 layers of dimension 64	**1.6%**	**12.1%**	**0.13**	**0.49**

CNN Architecture

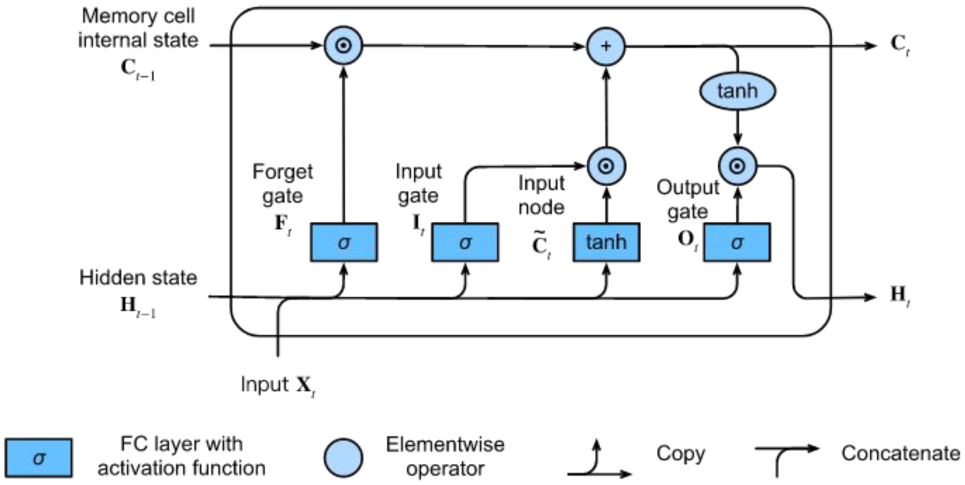
- Initial layer: linear layer mapping from feature space d to latent space of dimension p >> d with ReLu activation
- Subsequent hidden layers: Varying number of 1D-convolutions with identity activation
- Output linear layer:
    - Sigmoid activation for classification
    - Identity activation for regression

CNN Results

Regression Models	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total mean-squared error
1 layer of 64 channels	123.1%	265.3%	0.46	27.2
1 layer of 128 channels	-247.4%	564.3%	0.43	33.4
2 layers of 32, 64 channels	**404.1%**	**414.5%**	**0.97**	**25.9**
3 layers of 32, 64, 128 channels	243.1%	302.1%	0.80	29.0
Classification Models	**Yearly returns**	**Yearly volatility**	**Yearly Sharpe ratio**	**Misclassification Error**
1 layer of 64 channels	-7.1%	17.2%	-0.41	0.48
1 layer of 128 channels	**64.4%**	**66.1%**	**0.97**	**0.47**
2 layers of 32, 64 channels	-397.7%	33.4%	-11.9	0.54
3 layers of 32, 64, 128 channels	-147.9%	20.2%	-7.33	0.52

LSTM Architecture

- Input is passed through a gated memory cell with its own internal state
- Counteracts vanishing gradient problem
- Input gate, forget gate, and output gate are computed as:



$$
\begin{aligned}
\mathbf{I}_t &= \sigma(\mathbf{X}_t \mathbf{W}_{xi} + \mathbf{H}_{t-1} \mathbf{W}_{hi} + \mathbf{b}_i) \\
\mathbf{F}_t &= \sigma(\mathbf{X}_t \mathbf{W}_{xf} + \mathbf{H}_{t-1} \mathbf{W}_{hf} + \mathbf{b}_f) \\
\mathbf{O}_t &= \sigma(\mathbf{X}_t \mathbf{W}_{xo} + \mathbf{H}_{t-1} \mathbf{W}_{ho} + \mathbf{b}_o)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\mathbf{C}}_t &= \mathbf{tanh}(\mathbf{X}_t \mathbf{W}_{xc} + \mathbf{H}_{t-1} \mathbf{W}_{hc} + \mathbf{b}_c) \\
\mathbf{C}_t &= \mathbf{F}_t \odot \mathbf{C}_{t-1} + \mathbf{I}_t \odot \mathbf{C}_t \\
\mathbf{H}_t &= \mathbf{O}_t \odot \tanh(\mathbf{C}_t)
\end{aligned}
$$

Source: https://d2l.ai/chapter_recurrent-modern/lstm.html

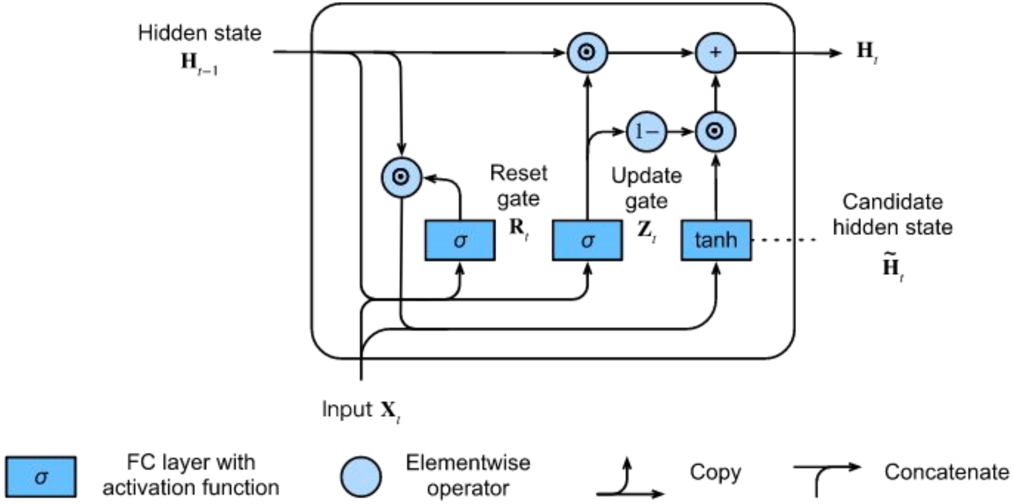
LSTM Results

Regression Models	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total mean-squared error
1 layer	**60.6%**	**188.4%**	**0.32**	**12.78**
2 layers	32.8%	140.8%	0.23	9.95
Classification Models	**Yearly returns**	**Yearly volatility**	**Yearly Sharpe ratio**	**Misclassification Error**
1 layer	**-376.3%**	**14.7%**	**-25.5**	**0.53**
2 layers	-412.9%	14.4%	-28.7	0.52

GRU Architecture

- Less computationally complex than LSTM
- Reset and update gates computed according to:

$$\mathbf{R}_t = \sigma(\mathbf{X}_t\mathbf{W}_{xr} + \mathbf{H}_{t-1}\mathbf{W}_{hr} + \mathbf{b}_r)$$
$$\mathbf{Z}_t = \sigma(\mathbf{X}_t\mathbf{W}_{xz} + \mathbf{H}_{t-1}\mathbf{W}_{hz} + \mathbf{b}_z)$$
$$\tilde{\mathbf{H}}_t = \tanh(\mathbf{X}_t\mathbf{W}_{xh} + (\mathbf{R}_t \odot \mathbf{H}_{t-1})\,\mathbf{W}_{hh} + \mathbf{b}_h)$$
$$\mathbf{H}_t = \mathbf{Z}_t \odot \mathbf{H}_{t-1} + (1 - \mathbf{Z}_t) \odot \tilde{\mathbf{H}}_t$$



GRU Results

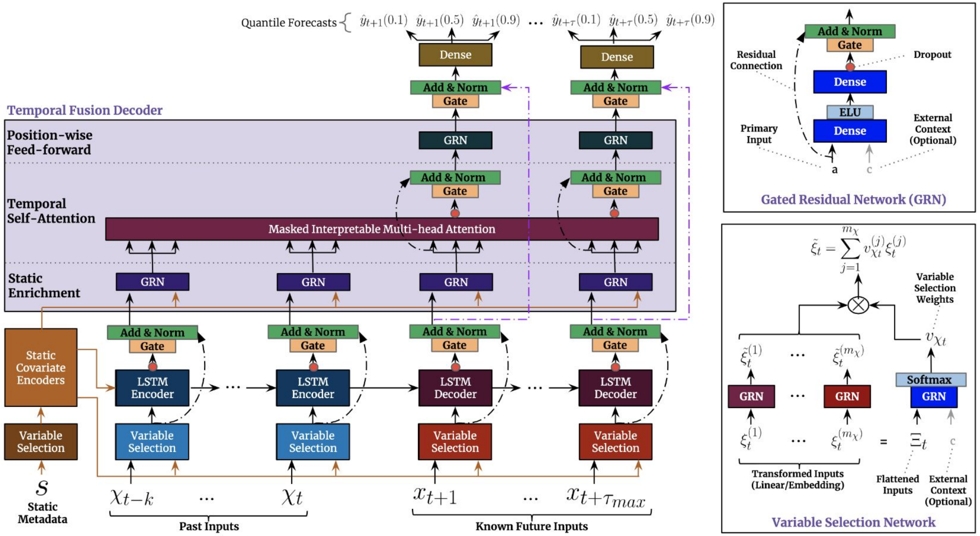
Regression Models	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total mean-squared error
1 layer	**-37.5%**	**125.7%**	**-0.29**	**13.33**
2 layers	-95.6%	76.5%	-1.25	10.42
Classification Models	**Yearly returns**	**Yearly volatility**	**Yearly Sharpe ratio**	**Misclassification Error**
1 layer	**-423.7%**	**17.2%**	**-24.6**	**0.55**
2 layers	-399.9%	14.4%	-27.8	0.49

TFT Architecture

$$\mathbf{D} = \{(\mathbf{k}_1, \mathbf{v}_1), \ldots, (\mathbf{k}_m, \mathbf{v}_m)\} \qquad \mathbf{f}(\mathbf{q}, \mathbf{D}) = \sum_{i=1}^{m} \alpha(\mathbf{q}, \mathbf{k}_i)\mathbf{v}_i$$

$$\forall i \in (1, \ldots, h), \mathbf{h}_i = f(\mathbf{W}_i^{(q)}\mathbf{q}, \mathbf{W}_i^{(k)}\mathbf{k}, \mathbf{W}_i^{(v)}\mathbf{v}) \in R^{p_v}$$

- Gating mechanisms
- Variable selection networks
- Static covariate encoders
- Interpretable multi-head attention
- Temporal processing
- Prediction intervals



Source: Lim, Arik, Loeff and Pfister, Temporal Fusion Transformers for Interpretable Multi-horizon Time Series Forecasting, 2020

TFT results

	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total mean-squared error
TFT (regression)	-32.3%	10%	-3.21	0.39
	Yearly returns	**Yearly volatility**	**Yearly Sharpe ratio**	**Misclassification Error**
TFT (classification)	0.5%	9.4%	0.0055	49.1%

Portfolio Construction

For regression:

- Solve the portfolio optimization problem

$$\max_{\omega_t \in R^n} \alpha_t^T \omega_t - \frac{\lambda}{2} \omega_t{}^T \Sigma_t \omega_t \Rightarrow \omega_t^* = \frac{1}{\lambda} \Sigma_t^{-1} \alpha_t$$

- Use **exponential smoothing** to obtain the the historical covariance matrix of the returns
- Obtain precision matrix by using graphical LASSO:

$$\hat{\Theta} = \mathrm{argmin}_{\Theta \geq 0} \left(\mathrm{tr}(S\Theta) - \log \det(\Theta) + \lambda \sum_{j \neq k} |\Theta_{jk}| \right)$$

For classification: long stock with expected positive returns else short

Overall Results

- In regression
  - Highest Sharpe ratio and highest MSE for CNN
  - Beats market index
  - But **unreliable model**
- In classification
  - Highest Sharpe and lowest misclassification error for CNN
  - Beats market index
  - **Best overall model**

Table 7: Summary statistics - benchmark

	Yearly returns	Yearly volatility	Yearly Sharpe ratio
CAC 40	13.2%	18.9%	0.7

Table 8: Summary statistics - regression

Model	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Total mean-squared error
LASSO	-92.2%	72.7%	-1.27	0.46
MLP	-2.9%	3.3%	-0.87	0.28
CNN	**404.1%**	**414.5%**	**0.97**	**25.94**
LSTM	32.8%	140.8%	0.23	9.95
GRU	-37.5%	125.7%	-0.29	13.33
TFT	-32.3%	10.0%	-3.2	0.39

Table 9: Summary statistics - classification

Model	Yearly returns	Yearly volatility	Yearly Sharpe ratio	Misclassification rate
Logistic Reg	-182.3%	13.4%	-13.6	50.4%
MLP	1.6%	12.1%	-0.13	48.5%
CNN	**64.4%**	**66.1%**	**0.97**	**47.2%**
LSTM	-376.3%	14.7%	-25.5	52.8%
GRU	-399.9%	14.4%	-27.8	49.0%
TFT	0.5%	9.3%	0.0055	49.1%

Future Work

- Further hyperparameter optimization
- Improve covariance matrix estimation methodology
- Perform 3-class classification to avoid predicting noise
- Application to different datasets or markets
- Comparison of combined models

Conclusion

- ***Can deep learning techniques effectively forecast future returns from past information?***
  - Deep learning models for regression can be used to predict market returns but we **could not find a regression model which has an acceptable mean-squared error while outperforming the market index**
  - They **significantly outperform standard linear models when it comes to predicting the sign of those returns** and we find that the CNN has the best overall prediction accuracy and Sharpe ratio
- ***How can we build, based on those forecast, a tradable portfolio? Can this portfolio outperform the market?***
  - Using portfolio optimization to build our portfolio leads to **good performance overall**
  - We were able to **outperform the CAC 40 benchmark** based on our portfolio construction methodologies
  - Our **best performing model yields a Sharpe ratio of 0.97 while the benchmark has a Sharpe ratio of 0.7 over the same period**
- ***How can we evaluate the quality of a given model?***
  - Statistical and risk-return performance metrics are **not always aligned**
  - They should therefore **both be considered** when assessing the portfolio performance

Questions?

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