21-241: Matrices and Linear Transformations

Recitation Section: E

Homework Number: 1

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Problem A:

1. $Re\frac{2-i}{3+2i} = Re\frac{(2-i)*(3-2i)}{(3+2i)*(3-2i)} = Re\frac{4-7i}{13} = \frac{4}{13}$

2. $Im\frac{1+2i}{3-i} = Im\frac{(1+2i)*(3+i)}{(3-i)*(3+i)} = Im\frac{1+7i}{10} = \frac{7}{10}$

Problem B:

A. r = $\sqrt{1^2+\sqrt{3}^2}$ = 2
   1 = 2 cos Θ, Θ= $\frac{\pi}{3}$
   The two polar forms of 1 + $\sqrt{3}i$ are: 2(cos $\frac{\pi}{3}$ + i sin$\frac{\pi}{3}$) and 2$e^{i\frac{\pi}{3}}$

B. 1. a = cos$-\frac{\pi}{4}$ = $\frac{\sqrt{2}}{2}$
   b = sin$-\frac{\pi}{4}$ = -$\frac{\sqrt{2}}{2}$
   $e^{-i\frac{\pi}{4}}$ = $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

   2. a = cos$\frac{\pi}{2}$ = 0
   b = sin$\frac{\pi}{2}$ = 1
   $e^{i\frac{\pi}{2}}$ = i

   3. a = cos3π = -1
   b = sin3π = 0
   $e^{3i\pi}$ = -1

Problem C:

1. $z^2 + 2z + 26 = 0$, z = $\frac{-2 +/- \sqrt{4-104}}{2}$ = $\frac{-2 +/- 10i}{2}$ = -1 + 5i, -1-5i
2. $z^4 + 3z^2 - 4 = (z-1)(z+1)(z^2+4), \quad z = 1, -1, 2i, -2i$

Problem D:

Let U = $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, and\ Let\ V = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

This means that U+V = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, which is not in echelon form as a zero row is above a non-zero row.

Problem E:

$$\begin{bmatrix}1 & 2 & -i & 6+i\\ 2i & 1 & 0 & 2i\\ 1 & 0 & -1 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & -i & 6+i\\ 0 & 1-4i & -2 & 2-10i\\ 1 & 0 & -1 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & -i & 6+i\\ 0 & 1-4i & 2 & 2-10i\\ 0 & -2 & -1+i & -5-i\end{bmatrix} =$$

$$\begin{bmatrix}1 & 2 & -i & 6+i\\ 0 & 17 & 2+8i & 42-18i\\ 0 & -2 & -i+1 & -5-i\end{bmatrix} = \begin{bmatrix}1 & 2 & -i & 6+i\\ 0 & 0 & 10.5-\ .5i & -.5-26.5i\\ 0 & -2 & -i+1 & -5-i\end{bmatrix} =$$

$$\begin{bmatrix}1 & 2 & -i & 6+i\\ 0 & -2 & -i+1 & -5-i\\ 0 & 0 & 10.5-\ .5i & -.5-26.5i\end{bmatrix}$$

Problem F:

A. $\begin{bmatrix}2 & 1 & -1 & 1 & 4\\ 4 & -1 & 2 & 1 & 0\\ 6 & 0 & 1 & 2 & 5\end{bmatrix} = \begin{bmatrix}2 & 1 & -1 & 1 & 4\\ 0 & -3 & 4 & -1 & -8\\ 0 & -3 & 4 & -1 & -7\end{bmatrix} = \begin{bmatrix}2 & 1 & -1 & 1 & 4\\ 0 & -3 & 4 & -1 & -8\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}$

The system does not have a solution, as in the last row the coefficient matrix is all zeroes yet the right hand side vector is a non-zero value.

B. It is not a linear combination, as in part a the matrix did not have a solution. This means that there is no combination of the vectors $\begin{pmatrix}2\\4\\6\end{pmatrix}, \begin{pmatrix}1\\-1\\0\end{pmatrix}, \begin{pmatrix}-1\\2\\1\end{pmatrix}, \begin{pmatrix}1\\1\\2\end{pmatrix}$ which make $\begin{pmatrix}4\\0\\5\end{pmatrix}$.

Problem G:

A. $x_1\begin{pmatrix}0\\2\\2\end{pmatrix} + x_2\begin{pmatrix}1\\4\\5\end{pmatrix} + x_3\begin{pmatrix}1\\6\\7\end{pmatrix} + x_4\begin{pmatrix}-1\\-2\\-2\end{pmatrix} + x_5\begin{pmatrix}-2\\2\\1\end{pmatrix} = \begin{pmatrix}-5\\14\\13\end{pmatrix}$

B. $\begin{bmatrix}0 & 1 & 1 & -1 & -2 & -5\\ 2 & 4 & 6 & -2 & 2 & 14\\ 2 & 5 & 7 & -2 & 1 & 13\end{bmatrix} = \begin{bmatrix}0 & 1 & 1 & -1 & -2 & -5\\ 2 & 0 & 2 & 2 & 10 & 34\\ 2 & 0 & 2 & 3 & 11 & 38\end{bmatrix} = \begin{bmatrix}0 & 1 & 1 & -1 & -2 & -5\\ 2 & 0 & 2 & 2 & 10 & 34\\ 0 & 0 & 0 & 1 & 1 & 4\end{bmatrix} =$

$$\begin{bmatrix}2 & 0 & 2 & 2 & 10 & 34\\ 0 & 1 & 1 & -1 & -2 & -5\\ 0 & 0 & 0 & 1 & 1 & 4\end{bmatrix}$$

$$\begin{pmatrix}x_1\\x_2\\x_4\end{pmatrix} = x_3\begin{pmatrix}-2\\-1\\0\end{pmatrix} + x_5\begin{pmatrix}-10\\2\\1\end{pmatrix} + \begin{pmatrix}34\\-5\\4\end{pmatrix}$$

Problem H:

A. $\begin{bmatrix}1 & 1 & 2 & a\\ 2 & -1 & 1 & b\\ 3 & 0 & 3 & c\end{bmatrix} = \begin{bmatrix}1 & 1 & 2 & a\\ 0 & -3 & -3 & b-2a\\ 0 & -3 & -3 & c-3a\end{bmatrix} = \begin{bmatrix}1 & 1 & 2 & a\\ 0 & -3 & 3 & b-2a\\ 0 & 0 & 0 & c-b-a\end{bmatrix}$

B. c -b -a = 0. This is because if c – b – a does not equal 0, then in the last row the coefficient matrix will be all zeroes yet the right hand side vector will be a non-zero value. This means that the system will not have a solution.

C. There is not values of a, b, and c where the system has exactly one solution. This is because there is a free variable, z, so there will either be 0 or infinitely many solutions.