Assignment 1 STA457H1F

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1 Exercise - Canadian/U.S. dollar exchange rates

1.1 Original Data

First, we plot the correlogram and periodogram of the log-transformed data.

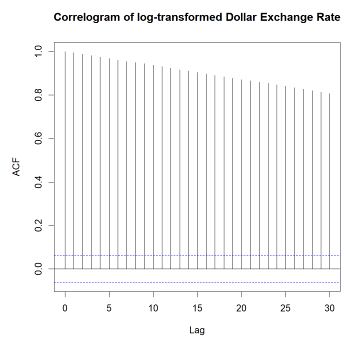

Figure 1:

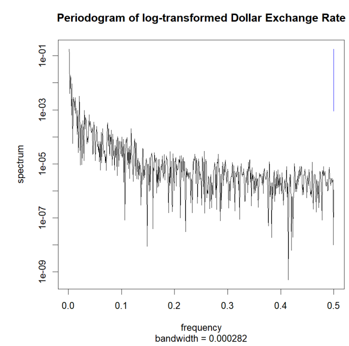

Figure 2:

The correlogram allows us to analyze the relationship between each pair of numeric variables of a dataset. Figure 1 shows high autocorrelation that slowly declines as the lag increases. This indicates long-term memory in the time series, which implies that past values of the exchange rate are significantly influencing future values over several periods. This can be interpreted as economic factors and events affect for a long period of time the exchange rate.

The periodogram allows us to identify dominant cyclical behavior in a series. Figure 2 shows that the periodogram is highest at low frequencies, which reflects the fact that the exchange rate evolves slowly over time.

1.2 First Differences

Second, we plot the correlogram and periodogram of the first differences.

The first differences represent the changes in the log exchange rate from one time period to the next.

In Figure 3, the correlogram of first differences shows no significant autocorrelations with lags hovering around zero.

Figure 4 shows that the periodogram resembles a relatively flat but noisy line, indicating that the time series has no dominant cycles or periodic behaviour.

The correlograms in Figure 1 and in Figure 3 differ significantly. The first one shows long-term memory of the original series with high values of autocorrelation. The second one shows that differences have short-term memory with autocorrelation values dropping towards zero immediately.

The correlograms support the assumption of a random walk model for the series. Since we have extremely high autocorrelation that decreases linearly as the lag increases and low correlation for the first differences.

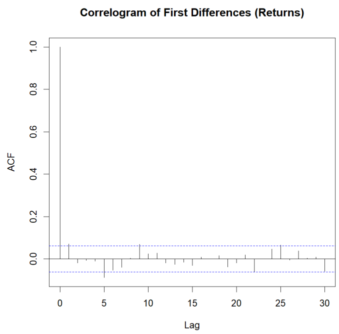

Figure 3:

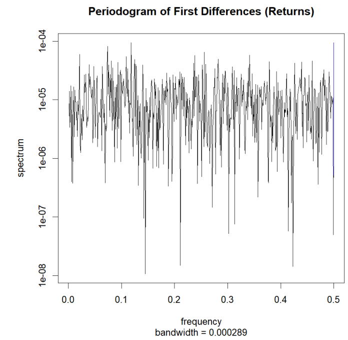

Figure 4:

The periodograms in Figure 2 and in Figure 4 also show some differences. The original time series displays peaks towards lower frequencies whereas the first differences series looks overall flat and more noisy. This is also consistent with the random walk hypothesis.

Overall, the assumption of a random walk model for this series seems correct. The results we have obtained are consistent with the model.

1.3 Absolute Values of First Differences

We now look at the correlogram and periodogram of the absolute values of the first differences.

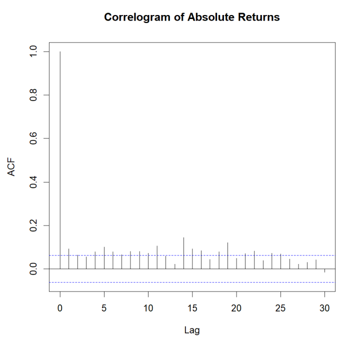

Figure 5:

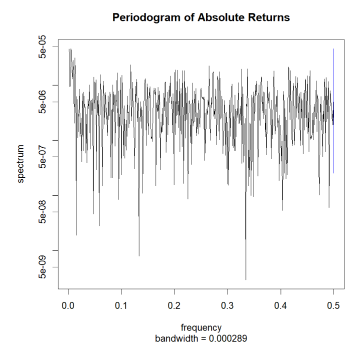

Figure 6:

The values in Figure 3 are much closer to zero than those in Figure 5. In fact, the correlogram in Figure 5 describes the volatility of the exchange rates. Financial time series typically exhibit volatility clustering and indeed we can see in Figure 5 that there is significant autocorrelation at several lags in the plot. This is a partial violation of the simple random walk assumption and suggests the need for a model that accounts for volatility dynamics.

The periodogram in Figure 6 shows higher power at low frequencies, reflecting long-term dependence in volatility. Such structure is also inconsistent with the random walk model, which assumes no long-term memory or autocorrelation in the absolute returns.

Overall, a more advanced model might be more appropriate than a random walk for modelling the volatility of this time series.

2 Carbon Dioxide Concentrations

2.1 Original Data

First, we plot the periodogram of the time series.

The periodogram in Figure 7 shows distinct peaks at low frequencies, which represent long-term trends and seasonal cycles. We do see peaks around the frequency of 1 cycle per year and multiples

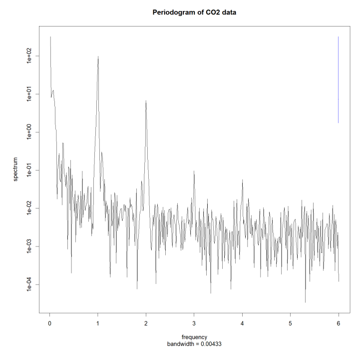

Figure 7:

of it. The peaks correspond to seasonal patterns in the CO2 concentrations, as we expect to observe yearly cycles in atmospheric CO2, driven by natural processes.

2.2 Detrended Data

Second, we plot the periodogram of the detrended time series.

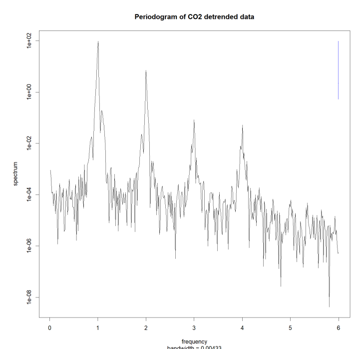

Figure 8:

After detrending, we are left with more focused information about the cyclic or seasonal behaviour. Having removed the trend component, the low-frequency peaks have been filtered out. What remains are the peaks associated with the seasonal variations and higher frequencies.

The differences in Figure 7 and Figure 8 are pretty straightforward. We see higher, more established peaks at the same multiples of 1. The peaks at low frequency disappeared once the data was detrended.

These interpretations align with the expectations for a dataset like the CO2 concentrations, where we expect both long-term increases (trend) and regular seasonal variations.

3 Exercise 3

3.1 Derive the spectral density of $\{Z_t\}$

By filtering theorem

$$f_Y(w) = |\sum_{u=-\infty}^{+\infty} c_u exp(2\pi wu)|^2 f_X(w) = |\Psi(w)|^2 f_X(w)$$

Then

$$\begin{aligned} Z_t &= \sum_{u=-\infty}^{+\infty} c_u X_{t-u} + \sum_{u=-\infty}^{+\infty} c_u (X_{t-u} - \sum_{u=-\infty}^{+\infty} c_u X_{t-u}) \\ &= \sum_{u=-\infty}^{+\infty} 2c_u X_{t-u} - \sum_{u=-\infty}^{+\infty} (c_u)^2 X_{t-u} \\ &= \sum_{u=-\infty}^{+\infty} [1-(1-c_u)^2] X_{t-u} \end{aligned} \tag{1}$$

Hence

$$f_Z(w) = |2\Psi(w) - \Psi(w)^2|^2 f_X(w) = |1-(1-\Psi(w))^2|^2 \ f_X \ (w)$$

3.2 Compute $\{d_u\}$

The HP filter is designed to smooth out the series, so we expect to see that $\{d_u\}$ is a modified version of $\{c_u\}$ that incorporates the boosting process.

We can obtain $\{c_u\}$ as was done in the lecture and then derive $\{d_u\}$ with the following formula:

$$d_u = [1-(1-c_u)^2]$$

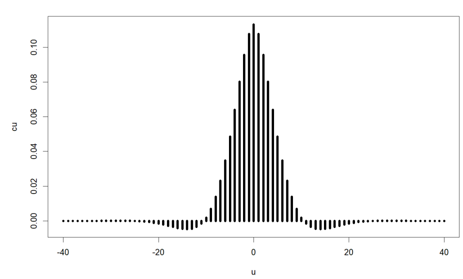

Figure 9:

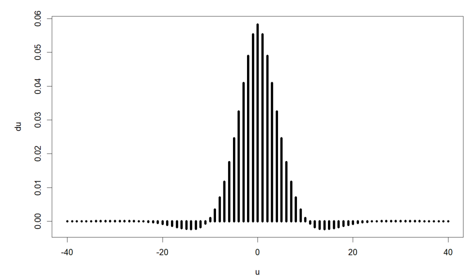

Figure 10:

Indeed, $\{d_u\}$ has similar values with respect to $\{c_u\}$. As we can see $\{d_u\}$ values are smaller in magnitude as the previous derivation had shown.

A RScript for Exercise 1

Figure 11

B RScript for Exercise 2

Figure 12

C RScript for Exercise 3

Figure 13

```r
# load the data and analyze the data on the log-scale
dollar <- scan(file.choose())
dollar <- ts(log(dollar))

# define the first differences of the time series
returns <- diff(dollar)

# a) plot the correlogram and periodogram of the original data
acf(dollar, main="Correlogram of log-transformed Dollar Exchange Rate")
spec.pgram(dollar, main="Periodogram of log-transformed Dollar Exchange Rate")

# b) plot the correlogram and periodogram of the first differences
acf(returns, main="Correlogram of First Differences (Returns)")
spec.pgram(returns, main="Periodogram of First Differences (Returns)")

# d) plot the correlogram and periodogram of the absolute values of the
#    first differences
abs_returns <- abs(returns)
acf(abs_returns, main="Correlogram of Absolute Returns")
spec.pgram(abs_returns, main="Periodogram of Absolute Returns")

```

Figure 11:

```r
# load the data and analyze the data
co2 <- scan(file.choose())
co2 <- ts(co2,start=c(1958,3),end=c(2024,7),frequency=12)

# a) Plot the periodogram of the time series
spec.pgram(co2, main="Periodogram of co2 data")

# b) Load the co2-trend data and calculate the differences
co2Trend <- scan(file.choose())
co2Trend <- ts(co2Trend,start=c(1958,3),end=c(2024,7),frequency=12)
co2deTrend <- co2 - co2Trend

spec.pgram(co2deTrend, main="Periodogram of co2 detrended data")
spec.pgram(co2, main="Periodogram of co2 data")

```

Figure 12:

```r
M <- 1024
lambda <- 100
omega <- c(0:(M-1))/M
trans <- 1/(1+16*lambda*sin(omega*pi)^4)
cu <- Re(fft(trans))/M
u <- c(-40:40)
plot(u, c(cu[41:2], cu[1:41]), ylab="cu", type="h", lwd=5)

du <- - sqrt(1 - cu) + 1
plot(u, c(du[41:2], du[1:41]), ylab="du", type="h", lwd=5)

```

Figure 13: