

Assignment Part 2

In []: 1 #| default_exp assignment_part_2

Nyquist Plot

Q1. Consider a close loop system with unity feedback.

- For the $G(s) = s - 1$, hand sketch the Nyquist diagram
- Determine $Z = N + P$, algebraically find the closed-loop pole location, and show that the closed loop pole location is consistent with the Nyquist diagram calculation.
- Use the controller $D(s) = k = 2$.

Q2. Consider the following controller D and system G

For controller $D(s) = k$ and $G(s) = \frac{s+1}{s^2(s+10)}$

- Hand sketch the asymptotes of the Bode plot magnitude and phase for the open-loop transfer function
- Hand sketch Nyquist diagram.
- Discuss stability margins

Gain and Phase margins

Q3. Given a closed loop system with unity gain with the following loop transfer function:

$$G(s) = \frac{125(s+1)}{(s+5)(s^2+4s+25)}$$

- Plot the Bode magnitude and phase plots for the open loop system
- Determine the gain and phase margin.

Root Locus

Q4. Properties of the Root Locus

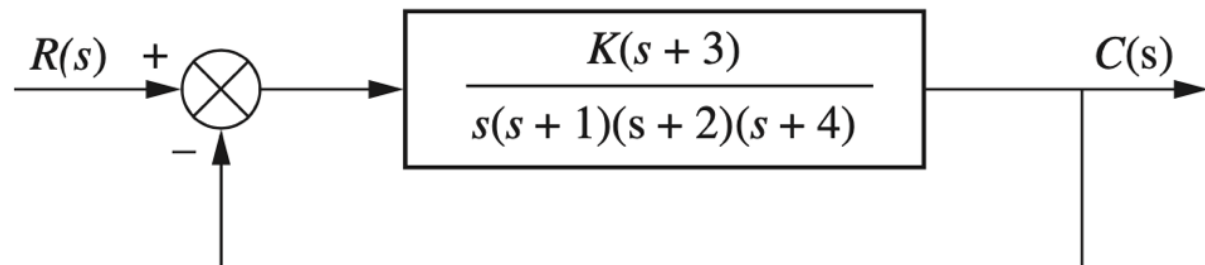
Given

$$G(s) = \frac{K(s + 2)}{(s^2 + 4s + 13)}$$

- Calculate the angle of $G(s)$ at the point $(-3 + j0)$ by finding the algebraic sum of angles of the vectors drawn from the zeros and poles of $G(s)$ to the given point.
- Determine if the point $(-3 + j0)$ is on the root locus.
- If the point specified in a is on the root locus, find the gain, K , using the lengths of the vectors.

Q5. Sketching the Root Locus

- Sketch the root locus for the system shown



Lead-Lag Compensation Design

Q6. Design a lead compensator

Given a unity feedback system where the plant

$$G(s) = \frac{K}{s(s + 50)(s + 120)}$$

Design a lag-lead compensator to have:

- 20% overshoot
- $\Phi_m > 48^\circ$
- $T_s = 0.2s$
- Steady state requirement $K_v = \lim_{s \rightarrow 0} sG(s) = 50$

Putting everything together

Consider the DC motor transfer function between of the motor-load combination which is given by:

$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s(Js + f)(L_f s + R_f)} = \frac{\frac{K_m}{JL_f}}{s(s + \frac{f}{J})(s + \frac{R_f}{L_f})}. \quad (1)$$

The system above can also be written as:

$$\frac{\theta(s)}{V_f(s)} = \frac{\frac{K_m}{JL_f}}{s(s + \frac{f}{J})(s + \frac{R_f}{L_f})} = \frac{\frac{K_m}{fR_f}}{s(\tau_f s + 1)(\tau_L s + 1)}$$

where $\tau_f = \frac{L_f}{R_f}$ and $\tau_L = \frac{J}{f}$.

When $\tau_L > \tau_f$, the field time constant τ_f can be neglected.

Let's choose the following values:

- (J) moment of inertia of the rotor 0.01 kg.m²
- (f) motor viscous friction constant 0.1 N.m.s
- (Ke) electromotive force constant 0.01 V/rad/sec
- (Kt) motor torque constant 0.01 N.m/Amp
- (R) electric resistance 1 Ohm
- (L) electric inductance 0.5 H

Note that in SI units, $K_e = K_t = K$

Q7. Close the loop with a unitary feedback and discuss the system performance.

Q8 Calculate the stability margins

Q9. Increase the inertia of the rotor and discuss what happens

- Use values $J = [0.1, 1, 2]$

Q10. Designing a Controller for the Motor

Design requirements:

- $t_r \approx 2.75s$ (90%)
- Max overshoot 20% at $5.75s$
- Settling time to $\pm 0.05\% \approx 11.15s$
- Zero steady state error
- Choose any method you prefer.
- Discuss the performance

Q11. Optional

- What happens if you add noise or load disturbances?
- How would you implement a PID controller?
- Use notebook `99_workspace` and design a controller for the pendulum attached to the DC motor. You might need to look at notebooks `90_simple_pendulum` and `91_DC_motor` to understand the model of the pendulum and of the motor used.

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