Translating Extensive Form Games to Open Games with Agency

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Introduction

Game theory is the mathematical study of interaction
among independent, self-interested agents.
– Essentials of Game Theory, Leyton-Brown and Shoham 2008

Examples: economic and ecological games, machine learning, etc. $\rightsquigarrow$ *cybernetics.*

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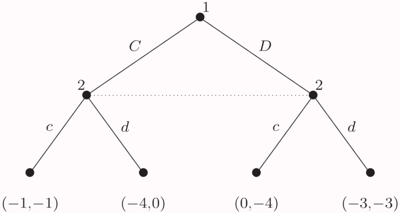
Classical game theory: **normal form** and **extensive form**

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$



Drawbacks: too little information (NF), too much information (EF), unclear causal relationships (both). Most importantly: non-compositional! ($\rightsquigarrow$ small scale)

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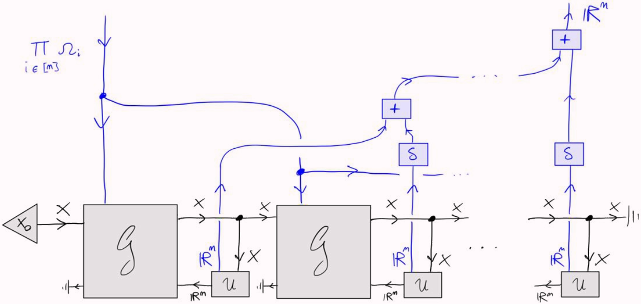
Open games as a framework for compositional game theory



Shows causality without being too unwieldy, compositional (⇝ large scale).

Also: string diagrams are nice to work with!

Introduction

Translating NF to open games is 'trivial' (there's only a utility function)...

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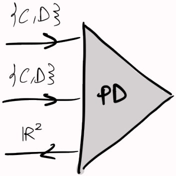

Introduction

Translating EF to open games is non-trivial:

1. We need to seriously consider the problem of agency:
    1. How do we represent correlated interests across a game?
    2. How do we represent player-dependent observational constraints (imperfect information)?
2. We need operators to reflect the tree structure of the game.

[1] Capucci, Gavranović, Hedges, and Rischel 2021

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To do so, we introduce:

1. **Open games with agency**, an improved compositional (and conceptual) framework for games, drawing from/inspiring 'open cybernetic systems'[1].
2. An operator calculus for games, in particular new **choice operators**.

[1] Capucci, Gavranović, Hedges, and Rischel 2021

Open games with agency

What is a game?

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A game factors in two parts:

1. An **arena**, which models the **dynamics** of the game.
2. **Players**, which intervene in the arena by making **decisions** at different points.

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2. **Players**, which intervene in the arena by making **decisions** at different points.

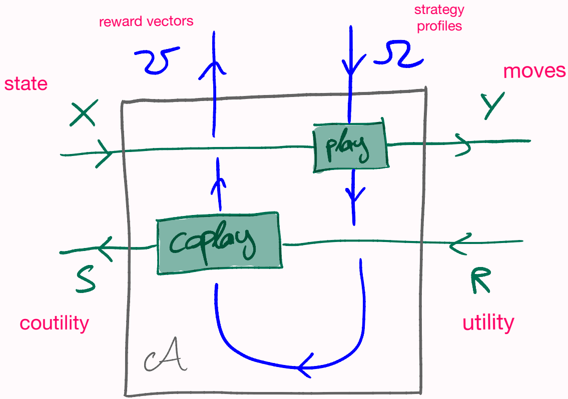
A strategy $\omega \in \Omega_p$ for a player $1 \leq p \leq N$ is a policy p uses to make their decisions (e.g. a choice of move for each of p's rounds).

A strategy profile is a strategy for each player $\Omega = \Omega_1 \times \cdots \times \Omega_N$.

What is an arena?

An arena is an open system with three boundaries:

change strategies to profiles



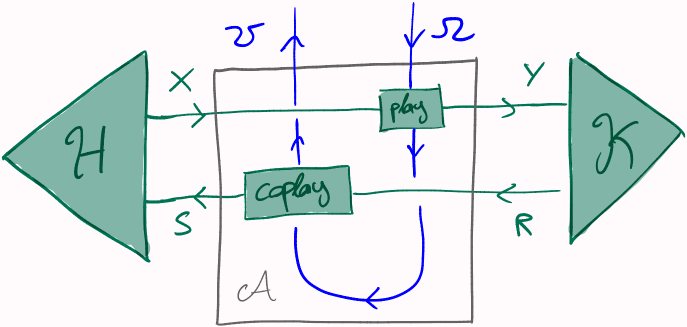
This is a **parametrised lens** $\mathcal{A} : (X, S) \xrightarrow{(\Omega,\mho)} (Y, R)$ specified by two maps

$$\text{play} : \Omega \times X \to Y,$$

$$\text{coplay} : \Omega \times X \times R \to \mho \times S$$

What is an arena?

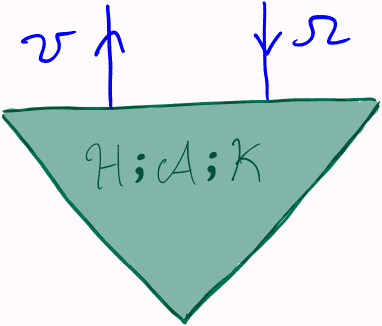
It can be 'closed' by specifying an **initial state** and a **utility function**:



Observe that $\mathrm{Lens}(1,1)(X,S) \cong X$ and $\mathrm{Lens}(Y,R)(1,1) \cong Y \to R$.

What is an arena?

A **closed arena** amounts to an evaluation of strategies with rewards:



$\cong \Omega \longrightarrow \mho$

What are players?

Players are a *distinct part* of the game (seen as a **system**) which expresses preferences by means of a **selection function**:

$$\varepsilon : (\Omega \to \mho) \longrightarrow \mathsf{P}\Omega$$

In a single-player game, typically Ω is 'finite', $\mho = \mathbb{R}$ and ε is argmax:

$$\mathrm{argmax}(u : \Omega \to \mathbb{R}) = \{\omega \in \Omega \,|\, \omega \text{ maximises } u\} \subseteq \Omega.$$

What is a Nash equilibrium?

A strategy profile is a **Nash equilibrium**
if no player has incentive to *unilaterally* change strategy.

Traditionally 'the' goal of game theory is determining Nash equilibria of games, though this is not necessarily the case anymore (see: Fudenberg and Levine 1998).

What is a Nash equilibrium?

The assignment $\mathbb{S}(\Omega, \mho) := (\Omega \to \mho) \longrightarrow \mathrm{P}\Omega$ is functorial on Lens.

Crucially, $\mathbb{S}$ admits a lax monoidal structure we call **Nash product**:

$$- \boxtimes - : \mathbb{S}(\Omega_1, \mho_1) \times \mathbb{S}(\Omega_2, \mho_2) \longrightarrow \mathbb{S}(\Omega_1 \times \Omega_2, \mho_1 \times \mho_2)$$

$$(\varepsilon \boxtimes \eta)(u) = \{(\omega_1, \omega_2) \mid \omega_1 \in \varepsilon(u_1(-, \omega_2)) \text{ and } \omega_2 \in \eta(u_2(\omega_1, -))\}$$

where $u = (u_1, u_2) : \Omega_1 \times \Omega_2 \to \mho_1 \times \mho_2$.

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Hence if we have players $P = \{1, \ldots, N\}$, each with preferences $\varepsilon_i \in \mathbb{S}(\Omega_i, \mho_i)$, we get:

$$\varepsilon_1 \boxtimes \cdots \boxtimes \varepsilon_N : (\Omega_1 \times \cdots \times \Omega_N \longrightarrow \mho_1 \times \cdots \times \mho_N) \longrightarrow \mathrm{P}(\Omega_1 \times \cdots \times \Omega_N)$$

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The definition

Definition

An **open game with agency** is a pair

$$\mathsf{G} \quad = \quad \left(\mathcal{A} : (X, S) \xrightarrow{(\Omega, \mho)} (Y, R), \quad \varepsilon : \mathbb{S}(\Omega, \mho) \right)$$

whose equilibria are given by

$$\mathrm{eq}_{\mathsf{G}}(x, k) = \varepsilon(x \mathbin{⨾} \mathcal{A} \mathbin{⨾} k).$$

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Definition

G **has set of players** P when

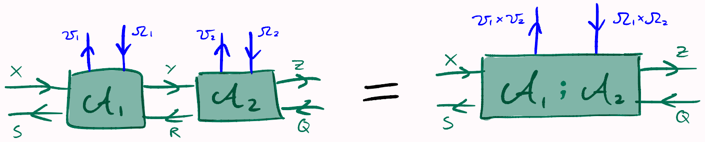
$$\Omega = \prod_{p \in P} \Omega_p, \quad \mho = \prod_{p \in P} \mho_p, \quad \varepsilon = \boxtimes_{p \in P} \varepsilon_p$$

in which case

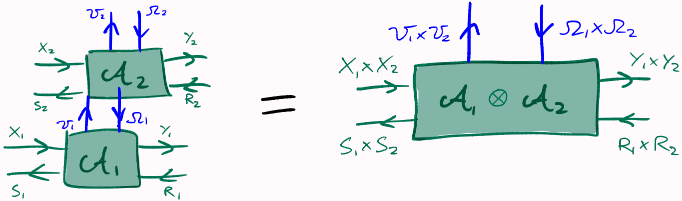
$$\mathsf{eq}_{\mathsf{G}}(x, k) = (\boxtimes_{p \in P} \varepsilon_p)(x \mathbin{⨟} \mathcal{A} \mathbin{⨟} k).$$

Composing games

Sequential composition



Parallel composition

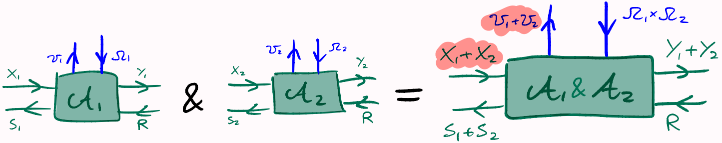


Notice: these operators <u>can</u> be extended to games:

$$(\mathcal{A}_1, \varepsilon) \fatsemi (\mathcal{A}_2, \eta) := (\mathcal{A}_1 \fatsemi \mathcal{A}_2, \varepsilon \boxtimes \eta), \quad (\mathcal{A}_1, \varepsilon) \otimes (\mathcal{A}_2, \eta) := (\mathcal{A}_1 \otimes \mathcal{A}_2, \varepsilon \boxtimes \eta)$$

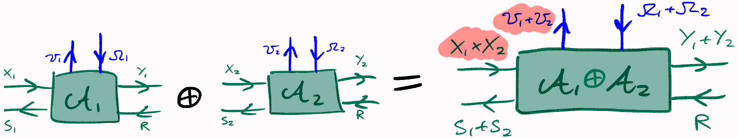
Composing arenas

External choice



The ‘environment’ chooses which game to play, agents are prepared to play both.

Internal choice



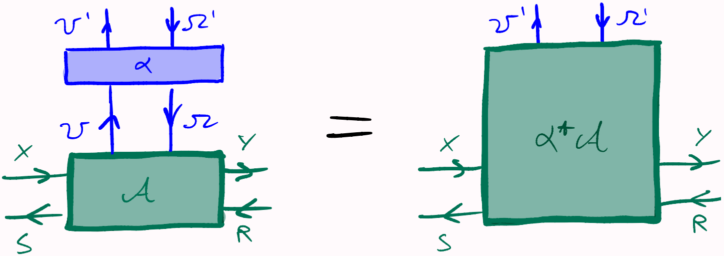
The ‘environment’ can play either game, agents choose which one.

Notice: these operators <u>can’t</u> be extended to selection functions in a canonical way!

(Actually $\oplus$ <u>can</u> if we refine our typing judgments)

Reparametrisation

Most importantly arenas form a (locally fibred) bicategory: one can **reparametrise** along a lens $\alpha : (\Omega', \mho') \to (\Omega, \mho)$.



This is crucial for introducing agency!

Lenses in the blue direction represent 'players dynamics', e.g. voting, observational constraints, reward distributions (imputation), ...

Regrouping

If $\mathcal{A}$ has set of players P and $r : P \to Q$ is a function, we can turn $\mathcal{A}$ into an arena with players Q by reparametrising along the permutation of $\prod_{p \in P} \Omega_P$ induced by r:

$$\mathrm{regroup}_r : (\textstyle\prod_{q \in Q} (r^* \Omega)_q, \prod_{q \in Q} (r^* \mho)_q) \longrightarrow (\prod_{p \in P} \Omega_p, \prod_{p \in P} \mho_p)$$

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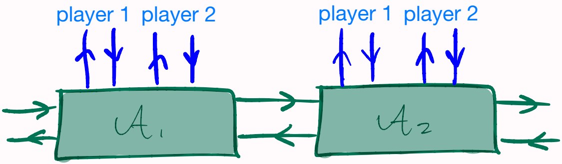
Example

If $\mathcal{A}_1$ and $\mathcal{A}_2$ have the same players $P = \{1, 2\}$, $\mathcal{A}_1 \mathbin{⨟} \mathcal{A}_2$ has players $P + P$.
Regrouping along $\nabla : P + P \to P$ restores the correct set of players.



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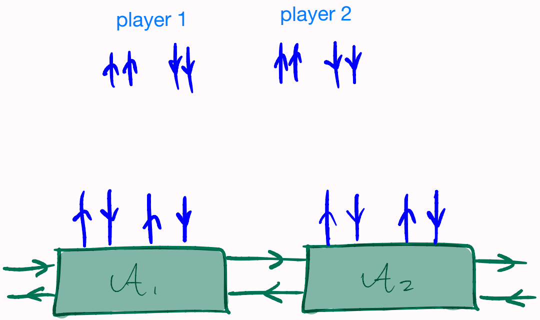

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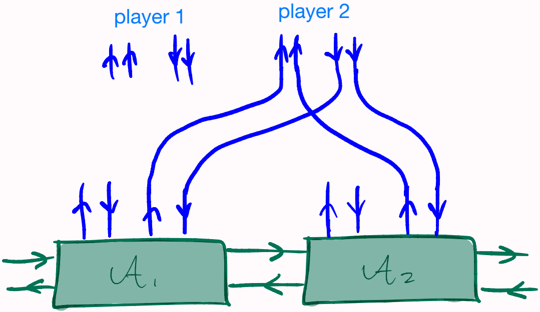

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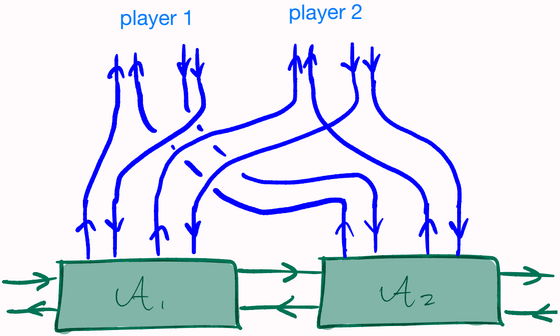

Tying

By reparametrising along $(\Delta, \mathsf{combine})$ (where $\mathsf{combine} = +, \pi_2, \mathsf{max}, \dots$), we can enforce the same strategies to be played at different points of a game.

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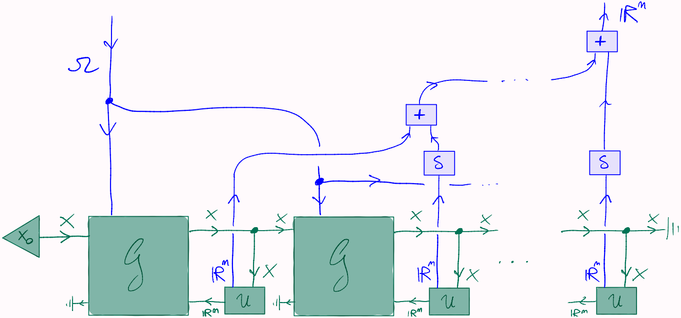
Repeated games: play the same strategies at every round



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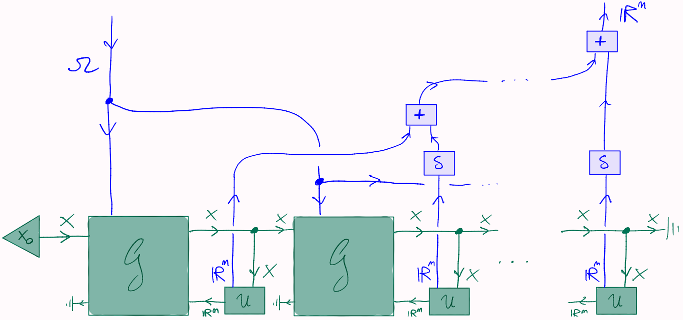
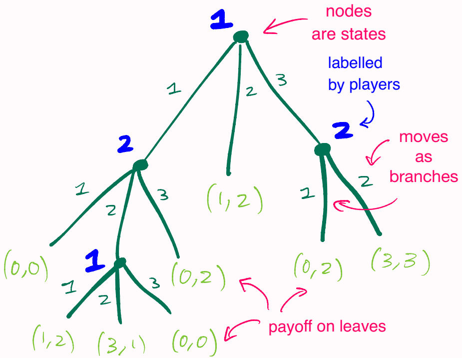


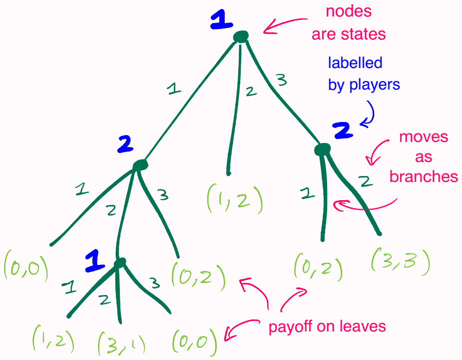
Notice: $(\Delta, \mathsf{combine})^*(\mathcal{A}_1 \mathbin{;} \mathcal{A}_2)$ lies outside the image of $\mathbin{;}/\otimes/\oplus/\&$, hence introduces 'non-compositional' effects $\rightsquigarrow$ **'agency is non-local'.**

Extensive form games & their translation

Extensive form



Extensive form



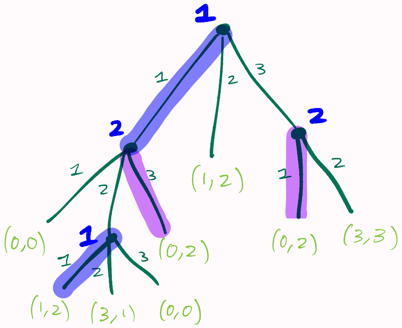
Definition

A **(perfect information)** **extensive-form tree** is a term of

$$\text{data PETree} = \text{Leaf} \;\; R^P \quad | \quad \text{Node} \;\; P \;\; (n : \mathbb{N}^+) \;\; ([n] \to \text{PETree})$$

where P = set of **players**, R = type of **rewards** (usually $\mathbb{R}$).

Extensive form: strategies



Definition

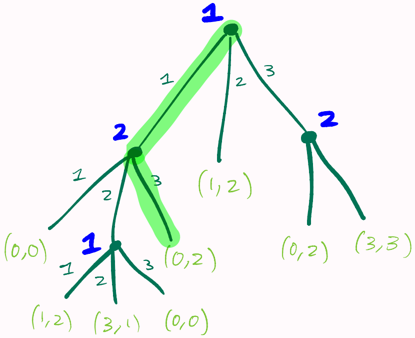
$\mathrm{strat_{PET}} : \mathrm{PETree} \to P \to \mathrm{Set}$

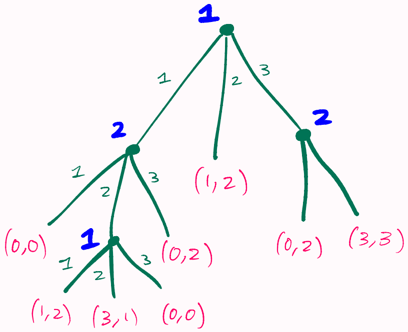
$\mathrm{strat_{PET}}\ (\mathrm{Leaf}\ v)\ p = 1$

$\mathrm{strat_{PET}}\ (\mathrm{Node}\ q\ n\ f)\ p$

$= (\text{if } p \equiv q \text{ then } [n] \text{ else } 1) \times (\Pi\, m \in [n])\ \mathrm{strat_{PET}}\ (f\ m)\ p$

Extensive form: moves





Definition

$$\mathrm{path}_{\mathrm{PET}} : \mathrm{PETree} \to \mathrm{Set}$$

$$\mathrm{path}_{\mathrm{PET}}\ (\mathrm{Leaf}\ v) = 1$$

$$\mathrm{path}_{\mathrm{PET}}\ (\mathrm{Node}\ p\ n\ f) =$$

$$(\Sigma\, m \in [n])\ \mathrm{path}_{\mathrm{PET}}\ (f\ \ m)$$

Definition

$$\mathrm{payoff}_{\mathrm{PET}} : (T : \mathrm{PETree}) \to (\mathrm{path}_{\mathrm{PET}}\ \ T) \to R^P$$

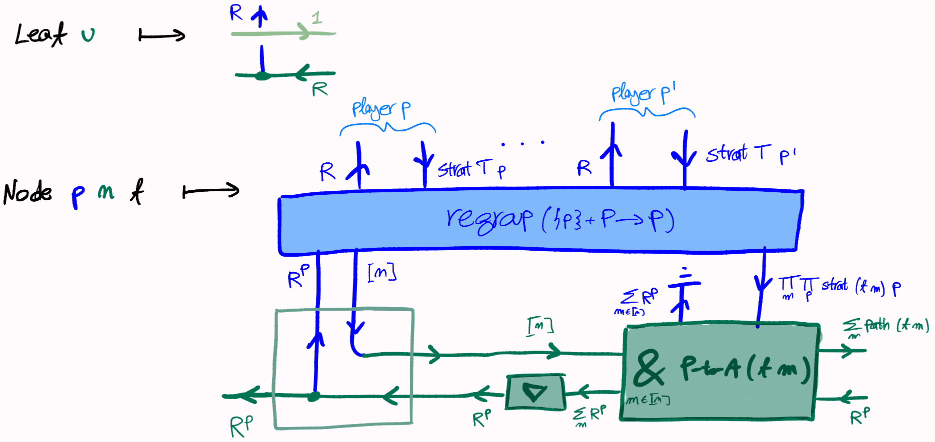
$$\mathrm{payoff}_{\mathrm{PET}}\ (\mathrm{Leaf}\ v)\ \bullet = v$$

$$\mathrm{payoff}_{\mathrm{PET}}\ (\mathrm{Node}\ p\ n\ f)\ (m, \pi)$$

$$= \mathrm{payoff}_{\mathrm{PET}}\ (f\ \ m)\ \pi$$

Translation

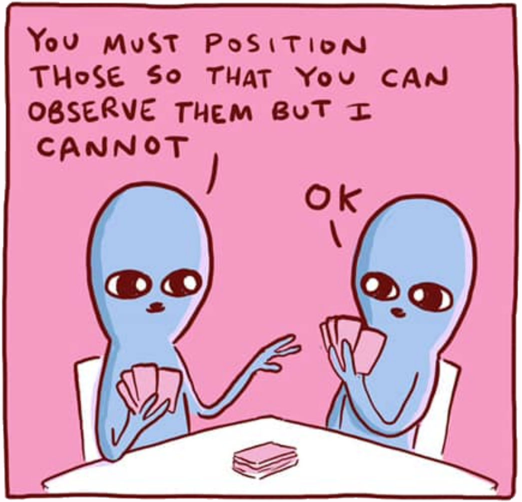
Finally, we can translate any PETree to an open game with agency:



and we equip this arena with $\boxtimes_{p \in P} \operatorname{argmax}(- \fatsemi \pi_p)$.

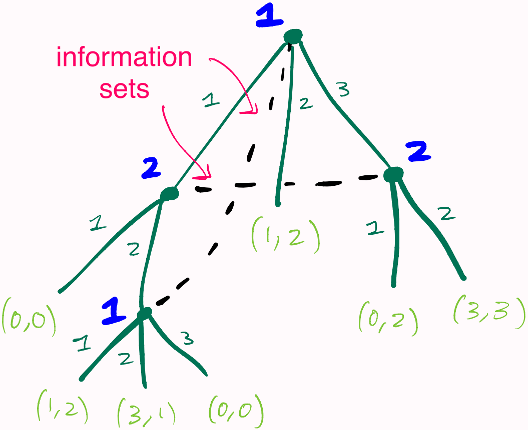
Imperfect information

Sometimes players can't access the whole state of the game (i.e. history of play)



Imperfect information

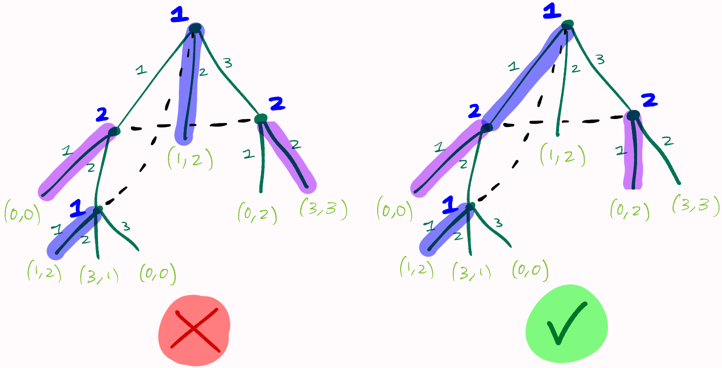
Access to information (or better, lack thereof) is represented by sets of 'indistinguishable states' called **information sets**:



Limit case: all information sets are singletons $\equiv$ perfect information.

Imperfect information

Strategies need to respect information sets: players can't distinguish between states in the same set:



It gets a bit more complicated with mixed strategies.

Adding imperfect information

Definition

An **imperfect information extensive-form tree** is a term of

$$\text{data IETree} = \text{Leaf } R^P \mid \text{Node } (i : I) \; ([n\ i] \to \text{IETree})$$

where

1. $I : \text{Set}$ is a set of **information labels**,
2. $n : I \to \mathbb{N}^+$ assigns moves to nodes of the same information set and
3. there is a (surjective) map $\text{belongs} : I \to P$.

Translation

Idea: information sets are instances of 'tying':

1. Forget about information sets and recover a perfect information game:

$$\text{IET-to-PET} : \text{IETree} \to \text{PETree}$$

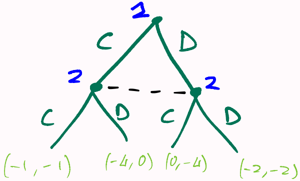
$$\text{IET-to-PET}\ (\text{Leaf}\ v) = \text{Leaf}\ v$$

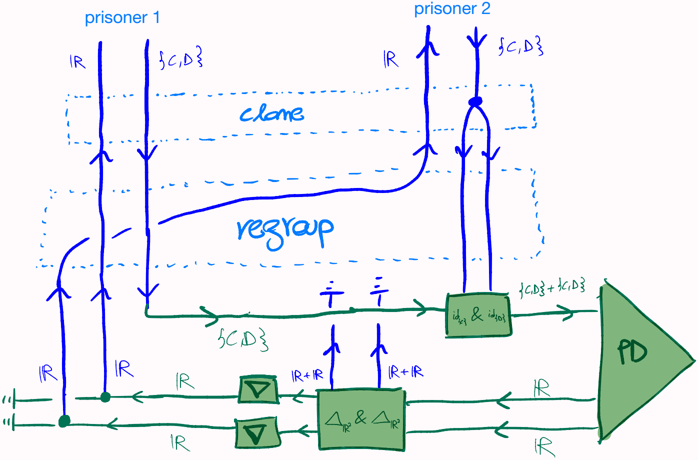
$$\text{IET-to-PET}\ (\text{Node}\ i\ f) = \text{Node}\ (\text{belongs}\ i)\ (n\ i)\ (\lambda m\,.\,\text{IET-to-PET}\ (f\ m))$$

2. Translate it using PET-to-Arena.
3. Reparametrise along clone which ties strategies in the same information sets.

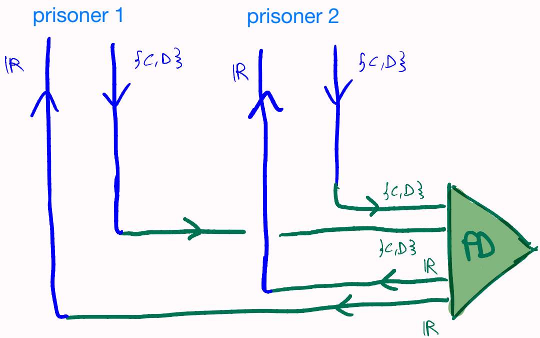
$$\text{IET-to-Arena}(T) = \text{clone}^*(\text{PET-to-Arena}\ (\text{IET-to-PET}\ T))$$

Example: Prisoner's Dilemma





Example: Prisoner's Dilemma



Conclusions

In this work:

1. We have seen how games naturally decompose in *arenas* and *selection functions*, with *reparametrisations* playing a key role
2. This allows to handle **long-range correlations** in players' behaviour
3. Hence we can easily map extensive form trees to open games with agency

Future directions

There remain some open questions related to the EF$\to$OG translation:

1. Can we simplify the resulting game using topological moves?
   e.g. translating IEF often yields OG which are $\otimes$-decomposable
2. Can we treat subgame-perfect equilibrium?
   Can Escardó-Oliva product of selections[2] be a lax monoidal structure on $\mathbb{S}$?

[2] Escardó and Oliva 2015

Future directions

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1. Can we simplify the resulting game using topological moves?
   e.g. translating IEF often yields OG which are $\otimes$-decomposable
2. Can we treat subgame-perfect equilibrium?
   Can Escardó-Oliva product of selections[2] be a lax monoidal structure on $\mathbb{S}$?

Moreover, the framework of open games (with agency) is still incomplete.

1. Selection functions do not interact much with arenas
   Is there a better way to equip arenas with equilibrium predicates?
2. What kind of assignment is '$\mathrm{argmax} : \mathrm{Arenas} \to \mathrm{Games}$'?
   An oplax Para coalgebra?

[2] Escardó and Oliva 2015

Thanks for your attention!

Questions?

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