Homework 8

CS 411 - Artificial Intelligence I - Fall 2019

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1 Question 1

1.1 Knowledge base

A *knowledge base* is a set of sentences (in conjunction with each other), each of which is expressed in a proper knowledge representation language, that represent some assertions that describe the current state of the world. Each sentence in a knowledge base represent a piece of information that is known to be true in the particular world we are considering.

1.2 Inference

Inference is the process of deriving a new sentence α from the ones that we already know to hold true (components of the knowledge base), through a certain procedure i. The inference operation is formally denoted as:

$$KB \vdash_i \alpha$$

It is desirable that procedure i satisfies two requirements:

- *Soundness*: an inference procedure is *sound* if, whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$ (it can entail any sentence that is derived);
- *Completeness*: an inference procedure is *complete* if, whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$ (it can derive any sentence that is entailed).

Examples of procedures that allow to perform inference include *inference by enumeration* (model checking), *modus ponens*, and *resolution*.

1.3 Model

A *model* is the formalization of the concept of a "possible world", represented by a mathematical abstraction that describes it by fixing the truth value of relevant primitive sentences.

The knowledge base KB of a knowledge-based agent may satisfy not just a single model, but instead a *set of models*, which is denoted as $M(KB)$. When the knowledge base is made up of few sentences, the cardinality of the set of models may be huge; the more knowledge the agent acquires, the more the set of models shrinks.

1.4 Entailment

The concept of *entailment* formalizes the idea of a sentence α following logically from a knowledge base KB and it is denoted with the syntax:

$$KB \models \alpha$$

Formally, $KB \models \alpha$ if and only if, in each model in which KB is true, then α is also true. In terms of sets of models, this means that:

$$KB \models \alpha \Leftrightarrow M(KB) \subseteq M(\alpha)$$

In fact, since KB represents a stronger assumption than α, this means that the models that KB satisfies are in general less than the models that α satisfies. The existence of an entailment relationship may be simply verified using the procedure known as *model checking*: this requires to enumerate all possible models for the sentences in the knowledge base and for sentence α and to verify whether the inclusion relation $M(KB) \subseteq M(\alpha)$ holds (α is true in all worlds in which KB is true).

1.5 Valid sentence

A sentence is valid (or, equivalently, *tautological*) if it holds true in all possible models; in other words, every valid sentence is inconditionally logically equivalent to true.

Valid sentences are useful for deriving entailment relationships, according to the *deduction theorem*: for any sentences α and β, $\alpha \models \beta$ if and only if the sentence $\alpha \Rightarrow \beta$ is valid.

2 Question 2

2.1 Knowledge base representation

Let us consider the following propositional symbols for the atomic sentences that appear in the knowledge base:

- X: "Sam plays baseball";
- Y: "Paul plays baseball";
- Z: "Ryan plays baseball".

According to propositional logic, the knowledge base in question may be represented by the conjunction of two sentences which hold true in the considered world:

- "Sam plays baseball or Paul plays baseball": this can be represented as a disjunction between sentences X and Y:

$$X \vee Y$$

- "Sam plays baseball or Ryan doesn't play baseball": this can be represented as a disjunction between sentences X and $\neg Z$:

$$X \vee \neg Z$$

The overall knowledge base is therefore represented by the expression:

$$KB = (X \vee Y) \wedge (X \vee \neg Z) = X \wedge (Y \vee \neg Z)$$

The truth table for such a knowledge base is shown in table 1.

X	Y	Z	$X \vee Y$	$X \vee \neg Z$	KB
false	false	false	false	true	false
false	false	true	false	false	false
false	true	false	true	true	true
false	true	true	true	false	false
true	false	false	true	true	true
true	false	true	true	true	true
true	true	false	true	true	true
true	true	true	true	true	true

Table 1: Truth table for the knowledge base

2.2 Entailment A

The sentence "Sam and Ryan both play baseball" may be represented in terms of the atomic propositions X and Z as:

$$A = X \wedge Z$$

To check whether this sentence is entailed by the knowledge base, we can use a model checking procedure, as shown in table 2.

X	Y	Z	KB	$A = X \wedge Z$
false	false	false	false	false
false	false	true	false	false
false	true	false	true	false
false	true	true	false	false
true	false	false	true	false
true	false	true	true	true
true	true	false	true	false
true	true	true	true	true

Table 2: Model checking for entailment A

Since there are worlds in which the knowledge base holds true but the presented sentence does not, we cannot state that A can be entailed from the knowledge base. For example, in a model in which Paul plays baseball but the other two do not, the knowledge base holds true but the given sentence does not (neither Sam nor Ryan play baseball).

2.3 Entailment B

The sentence "At least one among Sam, Paul and Ryan play baseball" may be represented in terms of the atomic propositions X, Y and Z as:

$$B = X \vee Y \vee Z$$

To check whether this sentence is entailed by the knowledge base, we can use a model checking procedure, as shown in table 3.

X	Y	Z	KB	$B = X \vee Y \vee Z$
false	false	false	false	false
false	false	true	false	true
false	true	false	true	true
false	true	true	false	true
true	false	false	true	true
true	false	true	true	true
true	true	false	true	true
true	true	true	true	true

Table 3: Model checking for entailment B

Since in every world in which the knowledge base is true so it is also the given sentence, we can state that B can be entailed from the knowledge base.