0.1 Accuracy Measures

"As every statistical measure condenses a large number of data into a single value, it only provides one projection of the model errors emphasizing a certain aspect of the error characteristics of the model performance." (Chai and Draxler)

In the following: $(y_t)_{t=1}^{T_{tsts}}$ test set, $(\hat{y}_t)_{t=1}^{T_{tsts}}$ predicted test set, T_{tsts} test set length

0.1.1 Scale-Dependent Metrics

Mean Absolute Error (MAE)

$$MAE \stackrel{\text{def}}{=} \frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \left| y_t - \hat{y}_t \right| .$$

Mean Square Erros (MSE)

$$MSE \stackrel{\text{def}}{=} \frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \left(y_t - \hat{y}_t \right)^2 .$$

Root Mean Sqaure Error (RMSE)

$$RMSE \stackrel{\text{def}}{=} \sqrt{\frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \left(y_t - \hat{y}_t \right)^2}.$$

0.1.2 Percentage-Error Metrics

Mean Percentage Error (MPE)

$$MPE \stackrel{\text{def}}{=} \frac{100}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \left(\frac{y_t - \hat{y}_t}{y_t} \right) .$$

Mean Absolute Percentage Error (MAPE)

$$MAPE \stackrel{\text{def}}{=} \frac{100}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \left| \frac{y_t - \hat{y}_t}{y_t} \right| .$$

Symmetric Mean Absolute Percentage Error (SMAPE)

0.1.3 Relative-Error Metrics

$$SMAPE \stackrel{\text{def}}{=} \frac{100}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \frac{\left| y_t - \hat{y}_t \right|}{\left(\left| y_t \right| + \left| \hat{y}_t \right| \right)}.$$

Another formulation

$$SMAPE \stackrel{\text{def}}{=} \frac{200}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \frac{\left| y_t - \hat{y}_t \right|}{\left(\left| y_t \right| + \left| \hat{y}_t \right| \right)}.$$

SMAPE proposed by Makridakis (1993): 0%-200%. Original formulation

$$SMAPE \stackrel{\text{def}}{=} \frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} \frac{\left| y_t - \hat{y}_t \right|}{\left(y_t + \hat{y}_t \right) / 2}$$

Median Relative Absolute Error (MdRAE)

$$MdRAE \stackrel{\text{def}}{=} Median_{t=1,\dots,T_{tsts}} \left\{ \frac{|y_t - \hat{y}_t|}{|y_t - \tilde{y}_t|} \right\}, \quad \tilde{y}_t \equiv \begin{cases} y_{t-1}, & \text{Non-Seasonal ts,} \\ y_{t-P}, & \text{Seasonal ts } (P \text{ period}). \end{cases}$$

0.1.4 Scale-Free Error Metrics

Geometric Mean Relative Absolute Error (GMRAE)

$$GMRAE \stackrel{\text{def}}{=} \exp\left(\frac{1}{T_{trns}} \sum_{t=1}^{T_{trns}} \ln\left(\frac{|y_t - \hat{y}_t|}{|y_t - \tilde{y}_t|} \right) \right), \quad \tilde{y}_t \equiv \begin{cases} y_t, & \text{Non-Seasonal ts,} \\ y_{t-P}, & \text{Seasonal ts } (P \text{ period}). \end{cases}$$

Remark 1 *We have*

$$GMRAE = \sqrt[\frac{1}{T_{trns}}]{\prod_{t=1}^{T_{trns}} \frac{|y_t - \hat{y}_t|}{|y_t - \tilde{y}_t|}}, \quad \tilde{y}_t \equiv \begin{cases} y_t, & \textit{Non-Seasonal ts,} \\ y_{t-P}, & \textit{Seasonal ts } (P \textit{ period}). \end{cases}$$

In the following: $(y_t)_t^{T_{trns}}$ training set, T_{trns} training set length

Mean Absolute Scaled Error (MASE)

$$MASE \stackrel{\text{def}}{=} \begin{cases} \dfrac{\frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} |y_t - \hat{y}_t|}{\frac{1}{T_{trns}-1} \sum_{t=2}^{T_{trns}} |y_t - y_{t-1}|} & \text{Non-Seasonal ts,} \\ \dfrac{\frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} |y_t - \hat{y}_t|}{\frac{1}{T_{trns}-P} \sum_{t=P+1}^{T_{trns}} |y_t - y_{t-P}|} & \text{Seasonal ts } (P \text{ period}). \end{cases}$$

Root Mean Squared Scaled Error (RMSSE)

$$RMSSE \stackrel{\text{def}}{=} \begin{cases} \sqrt{\dfrac{\frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} (y_t - \hat{y}_t)^2}{\frac{1}{T_{trns}-1} \sum_{t=2}^{T_{trns}} (y_t - y_{t-1})^2}} & \text{Non-Seasonal ts,} \\ \sqrt{\dfrac{\frac{1}{T_{tsts}} \sum_{t=1}^{T_{tsts}} (y_t - \hat{y}_t)^2}{\frac{1}{T_{trns}-P} \sum_{t=P+1}^{T_{trns}} (y_t - y_{t-P})^2}} & \text{Seasonal ts } (P \text{ period}). \end{cases}$$

https://otexts.com/fpp3/accuracy.html