es1) c_i prezzo quota i

r_i rendimento quota i (%)

f_i rischio quota i

- $x_i \in \mathbb{N}$ quante quota i compra

$200'000 = $ soldi

---

sogg.

- diff. $x_j c_j \leq 0,3 \sum_{i=1}^{5} x_i \cdot c_i \quad \forall j \in \{1,2,3,4,5\}$

bo su poco rischio $x_1 c_1 + x_2 c_2 + x_4 c_4 \geq 0,4 \sum_{i=1}^{5} x_i c_i$

riduzione rischio $\sum_{i=1}^{5} (x_i \cdot c_i) f_i \leq 2,8 \sum_{i=1}^{5} (x_i c_i)$

non soldi infiniti $\sum_{i=1}^{5} x_i c_i \leq 200000$

- $\max z = \sum_{i=1}^{5} (x_i \cdot c_i) \cdot r_i$

Modifiche

y_i se ho scelto i come quota da usare

$\in \{0,1\}$

- $\sum_{i=1}^{5} y_i \leq 3$

- $x_i \le y_i N$ $\quad \begin{cases} y_i = 0 & \Rightarrow 0 \le x_i \le 0 \quad x_i = 0 \\ y_i = 1 & \cancel{x_1 \le N} \end{cases}$

$$N = \max \left\{ \frac{200000}{c_i} \right\}$$

$\boxed{\text{es } i}$

$$\max z = -(x_1 + \cancel{x_2} + \cancel{x_3}) \qquad -\left(\frac{4+t}{2}\right)$$

$$x_1 = \frac{4+t}{2} \ge 0 \qquad \boxed{t \ge -4}$$

sogg a. $\quad 2x_1 + \cancel{x_2} - 3\cancel{x_3} - \cancel{x_4} = 4 + t$

$$-x_1 + 3\cancel{x_2} + \cancel{x_3} + x_5 = 8 - \frac{1}{2}t$$

$$x_3 = 8 - \cancel{\tfrac{1}{2}t} + 2 + \cancel{\tfrac{1}{2}t} = 10$$

$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

Prima fase

$$\begin{array}{c|c} & \begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & s_1 & \end{array} \\ \begin{array}{c} s_1 \\ x_5 \end{array} & \left(\begin{array}{cccccc|c} 2 & 1 & -3 & -1 & & 1 & 4 \\ -1 & \underline{3} & 1 & & 1 & & 8 \end{array}\right) \end{array}$$

$t_2 \leftarrow t_2 - \frac{1}{2} t_1$

$t_1 \leftarrow \frac{1}{2} t_1$

$$\max \; -s_1 = -4 + 2x_1 + x_2 - 3x_3 - x_4$$

$$\begin{array}{c|c} & \begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & s_1 & \end{array} \\ \begin{array}{c} x_1 \\ x_5 \end{array} & \left(\begin{array}{cccccc|c} 1 & 1/2 & -3/2 & -1/2 & & \cancel{1/2} & 2 \\ 0 & 7/2 & -1/2 & -1/2 & 1 & \cancel{1/2} & 10 \end{array}\right) \end{array}$$

Seconda fase

$$\max z = -(x_1 + x_2 + x_3)$$

$$= -\left(\left(2 - \frac{1}{2}x_2 + \frac{3}{2}x_3 + \frac{1}{2}x_4\right) + x_2 + x_3\right)$$

$$= -2 + \frac{1}{2}x_2 - \frac{3}{2}x_3 - \frac{1}{2}x_4 - x_2 - x_3$$

$$= -2 - \frac{1}{2}x_2 - \frac{5}{2}x_3 - \frac{1}{2}x_4$$

↳ soluz. ott.

$\{x_1, x_5\}$

Soluz

$$x = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 10 \end{pmatrix}$$

Scrivo sol

6

$$\left(\begin{array}{cccccc|c} 2 & 1 & -3 & -1 & & -1 & 4 \\ -1 & 3 & 1 & & 1 & +\frac{1}{2} & 8 \end{array}\right)$$

$$\left(\begin{array}{cccccc|c} 1 & 1/2 & -3/2 & -1/2 & & -1/2 & 2 + \frac{1}{2}t \\ 0 & 7/2 & -1/2 & -1/2 & 1 & 1/4 & 18 \quad 0 \end{array}\right)$$

$$\max z = -x_1 - x_2 - x_3$$

$$-2 - \frac{1}{2}t + \frac{1}{2}x_2 - \frac{3}{2}x_3 - \frac{1}{2}x_4 - x_2 - x$$

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -1/2 & -1/2 & 0 & -1 \\ 2 & 1 & 1 & 3 \end{pmatrix}$$

$$\begin{array}{c} \\ -1 \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ -1/2 & -1/2 & 0 & -1 & 2 \\ 1 & 1 & 0 & 2 & -2 \end{array}\right) \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right) \end{array}$$

$0x_1 + 0x_2 \ldots = 1$

$\Rightarrow v \notin L(B)$

Base $\{x_2, x_3\} = B$

$\rho(A) = 2$

es 5

(1) $x_1 = \frac{2}{3} x_2$ $\qquad \left(\frac{2}{3} x_2, x_2\right)$

$$\left(\frac{2}{3} t, t\right) = \begin{pmatrix} 2/3 \\ 1 \end{pmatrix} t$$

$$v'' = \underbrace{\begin{pmatrix} 2/3 \\ 1 \end{pmatrix} t}_{v} + \underbrace{\begin{pmatrix} 2/3 \\ 1 \end{pmatrix} t'}_{v'} = \begin{pmatrix} 2/3 \\ 1 \end{pmatrix} (t + t') \in S_1$$

$$\Rightarrow v + v' \in S_1$$

somma e' interna a sotto

$$v' = h \cdot v = h \begin{pmatrix} 2/3 \\ 1 \end{pmatrix} t = \begin{pmatrix} 2/3 \\ 1 \end{pmatrix} (ht)$$

$$v' = h \cdot v \in S_1$$ prodotto x scal. è interno

(2)

$$X = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -3 \end{pmatrix} t$$

$$x_1 = -1 + 3/2\, t$$

$$t = \frac{(x_1 + 1)2}{3}$$

$$x_2 = \cancel{2} - 2(x_1 + \cancel{1}) \qquad x_2 = -2x_1$$

$$X'' = X + X' = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -3 \end{pmatrix} t + \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -3 \end{pmatrix} t'$$

$$= \begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -3 \end{pmatrix} (t + t')$$

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -\frac{2}{3} \end{pmatrix} + \ldots$$

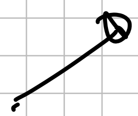
$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -3 \end{pmatrix} (t + t' + 1)$$

vale per somma

$$hV = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \cdot 1 \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix} (h-1) \quad \ldots$$

$x_2 = 2x_1$

$(1 \ , \ 2)\, t$



$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2/3 \\ 1 \end{pmatrix} = \begin{pmatrix} 5/3 \\ 3 \end{pmatrix}$$

$\notin S_2 \qquad \notin S_1$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} t = \begin{pmatrix} 2/3 \\ 1 \end{pmatrix} t'$$

$$\begin{cases} t - \frac{2}{3}t' = 0 \\ 2t - t' = 0 \end{cases} \qquad t = \frac{2}{3}t'$$

$\cancel{2t}\ \frac{4}{3}t' - t' = 0 \qquad t' = 0$

$\Rightarrow t = 0$

$S_4 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ e un sot(espace)

$\vec{0} + \vec{0} = \vec{0}$

$\vec{0}\, \lambda = \vec{0}$

$= S$ di n elementi

V_i valore

C_i costo

$\}$ 1 a n

- $S_i \begin{cases} 1 & \text{se } i \text{ scelto} \\ 0 & \text{altri} \end{cases}$

B_J budget $\}$ da 1 a l

- $M_{iJ} \begin{cases} 1 & \text{l'elem. } i \text{ è in lista } J \\ 0 & \text{altri} \end{cases}$

- $\max z = \sum_{i=1}^{n} \sum_{J=1}^{l} M_{iJ} V_i$

- $$S_i = \sum_{J=1}^{l} M_{iJ} \leq 1$$

$$\sum_{I=1}^{n} M_{iJ} \cdot C_i \leq B_J$$

Modifica

V_J

$\max z = y$

$y \leq V_J \quad \forall J$

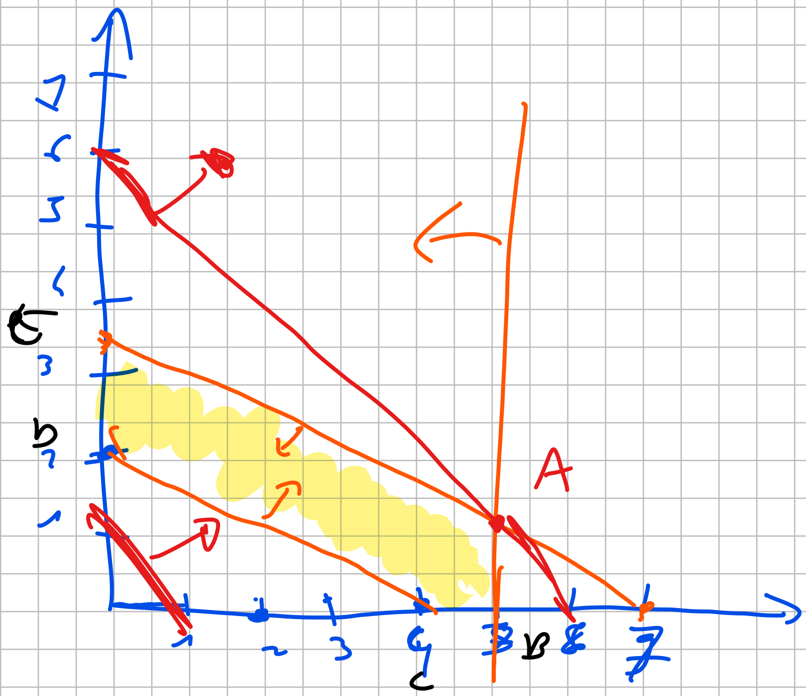
$$V_J = \sum_{I=1}^{n} M_{iJ} \cdot V_i \qquad \forall J$$

eset 2

$$\max z = x_1 + x_2$$

sogg. a

$$x_1 + 2x_2 \geq 4$$
$$x_1 + 2x_2 \leq 7$$
$$x_1 \leq 5$$
$$x_1 + x_2 \geq 0$$



$$A = (5, 1)$$

max

bas. amm. sono i vertici

$$\max z = x_1 \quad x_2$$

sog.

$$x_1 + 2x_2 - x_3 = 4$$
$$x_1 + 2x_2 + x_4 = 7$$
$$x_1 \qquad + x_5 = 5$$

$$x_1 \ldots x_5 \geq 0$$

$* \quad \underline{A} = (\underset{x_1}{5}, \underset{x_2}{1}, \underset{x_3}{3}, 0, 0)$ $B = \{x_1, x_2, x_3\}$

$\underline{B}$ è ist $x_1 = 5$ $x_2 = 0$

$(5, 0)$ coord in R^2

$$\begin{cases} 5 + 2\cdot 0 - x_3 = 4 \\ 5 + 2\cdot 0 + x_4 = 7 \\ \cancel{x_5} = 5 + x_5 \\ x_1 \quad \_\_\_ \quad x_3 \geq 0 \end{cases}$$

$x_3 = 1$
$x_4 = 2$
$x_5 = 0$

$\Rightarrow B = (5, 0, 1, 2, 0)$ $B = \{x$

$C = (4, 0, 0, 3, 1)$

$D = (0, 2, 0, 3, 3)$

$E = (0, \frac{7}{2}, 3, 0, 5)$

$$\left(\begin{array}{ccccc|c} 1 & 2 & -1 & & & 4 \\ 1 & 2 & & 1 & & 7 \\ 1 & & & & 1 & 5 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1/2 & 1 & -1/2 & & & 2 \\ & & \boxed{1} & 1 & & 3 \\ 1 & & & & 1 & 5 \end{array}\right)$$

$$\max \quad z = x_1 + x_2 = x_1 + 2 - \frac{1}{2} x_1 + \frac{1}{2} x_3$$

$$\left(\begin{array}{ccccc|c} 1/2 & 1 & 0 & 1/2 & & 7/2 \\ & & 1 & 1 & & 3 \\ 1 & & & & 1 & 5 \end{array}\right)$$

$$\max \quad x_1 + x_2 = x_1 + \frac{7}{2} - \frac{1}{2} x_1 - \frac{1}{2} x_4$$

$$\left(\begin{array}{ccccc|c} & 1 & & 1/2 & -1/2 & 1 \\ & & 1 & 1 & & 3 \\ 1 & & & & 1 & 5 \end{array}\right)$$

$$\max \quad x_1 + x_2 = 1 - \frac{1}{2} x_4 + \frac{1}{2} x_5 + 5 - x_5$$

$$6 - \frac{1}{2} x_4 - \frac{1}{2} x_5$$

ottimo

$$\max \; 2x_1 - x_2 = 2\left(1 - \frac{1}{2}x_1 + \frac{1}{2}x_3\right) - \left(5 - x_3\right)$$

$$= 2 - 5 - x_1 + x_3 + x_3$$

non più ottimo

es 3

$$\left(\begin{array}{cccc|c} 2 & -1 & & -2 & 1 \\ & 2 & 1 & 2 & 3 \\ 3 & & 1 & -1 & 2 \end{array}\right)$$

$E_3 \leftarrow E_3 - E_2$
$E_2 \leftarrow E_2 \cdot 1$
$E_1 \leftrightarrow E_2$

$$\left(\begin{array}{cccc|c} & 2 & 1 & 2 & 3 \\ 2 & -1 & & -2 & 1 \\ 3 & -2 & & -3 & -1 \end{array}\right)$$

$E_1 \leftarrow E_1 + 2E_2$
$E_3 \leftarrow E_3 - 2E_2$
$E_2 \leftarrow -E_2$

$$\left(\begin{array}{cccc|c} 4 & & 1 & -2 & 5 \\ -2 & 1 & & 2 & -1 \\ -1 & & & 1 & -3 \end{array}\right)$$

$E_1 \leftarrow E_1 + 2E_3$
$E_2 \leftarrow E_2 - 2E_3$

$$\left(\begin{array}{cccc|c} 2 & & 1 & & -1 \\ & 1 & & & +3 \\ -1 & & & 1 & -3 \end{array}\right)$$

$$\begin{cases} 2x_1 \quad x_3 = -1 \\ x_2 = 3 \\ -x_1 \quad x_{\cancel{4}} = -3 \end{cases}$$

$$\begin{cases} x_3 = -1 - 6 - 2x_4 = -7 - 2x_4 \\ x_2 = 5 \\ x_1 = +3 + x_4 \end{cases}$$

$$\begin{cases} x_1 = 3 + x_4 \\ x_2 = 5 \\ x_3 = -7 - 2x_4 \end{cases}$$

$\underline{x_4 \text{ libera}}$

$\boxed{5}$

① $A_B^{-1} A_k \leq 0 \quad \forall k > 0 \Rightarrow$ prob. illimitato

per x_k fuori base

⑪ ⓐ $S = \{v_1, \dots v_k\}$ $\quad x_1 v_1 + \dots x_k v_k = 0$

$\vec{x} = \vec{0}$

ⓑ $\Longleftrightarrow \neg \exists v_j \left(v_j \in L\left(S / \{v_j\}\right)\right)$

se vale ⓐ ma per as. non ⓑ

allora esista v_j (riordino in modo che v_j sia all'inizio quindi di v_1)

$$v_1 = \lambda_2 v_2 + \dots + x_k v_k$$

$$v_1 - x_2 v_2 - \dots - x_k v_k = 0$$

$$x_1 = 1 \Rightarrow \vec{x} \neq \vec{0}$$

assurdo per ipotesi

se vale ⓑ ma non ⓐ

$$x_1 v_1 + \dots + x_k v_k = 0$$

$$\vec{x} \neq \vec{0}$$

$$\exists x_i \neq 0 \quad \text{(riordino in modo che sia } x_1\text{)}$$

$$\frac{\cancel{x_1} v_1}{\cancel{x_1}} = -\frac{(x_2 v_2 + \dots + x_k v_k)}{x_1}$$

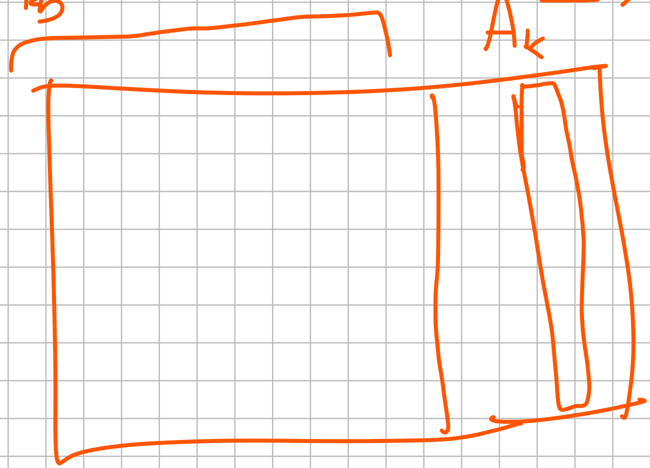
$$v_1 = -\frac{x_2}{x_1} v_2 - \dots - \frac{x_k}{x_1} v_k$$

$$\Rightarrow v_1 \in \mathcal{L}(S / \{v_1\})$$

Assurdo ~~x hp~~

$A_B^{-1} A_k \leq 0 \quad r_k > 0$

A_B $A_k \Rightarrow$ $\parallel$



r_k

$A_B \cdot A_B^{-1} A_k \leq 0 \cancel{A_B}$

---

es 4 penultimi secondo x_i fuori base

dopo riformulazione ottengo

"max attuale" $= z(B)$

$$\max \ z = \underbrace{z(B)}_{cost} + \sum_{j \notin B} x_j r_j$$

con $r_j \leq 0$

se lo porto una qualunque x_{J_1} in base

il max diventa

$$\max z = z(B) + x_{J_1} r_{J_1} + \sum_{J \notin B'} x_J r_J$$

pongo fuori base $= 0$

$$z = z(B) + x_{J_1} r_{J_1}$$

dove $x_{J_1} \geq 0$ in quanto in base (altrimenti non accettabile)

$$r_{J_1} \leq 0$$

$$\Rightarrow z(B) + x_{J_1} r_{J_1} \leq z(B)$$

es 1) 3 tipi di pavimento

R_i richiesta di m^2 di pavimento i

Pacco = 0,25 mq

C_{ij} costo pacco di pavimento i dal fornitore j

tot ordine per fornitore ≤ 20% ordi totale

scanti

F(1) ogni 4 pl. acce $\rightsquigarrow$ 12 € (-3)

F(2) ogni 3 pachi $\overset{\text{plastmade}}{\rightsquigarrow}$ 7 € (-1)

F(3) ogni 3 pacchi $\overset{\text{P. grand}}{\rightsquigarrow}$ 10 € (-1)

- $X_{ij} \in \mathbb{N}$ numero pacchi $\overset{\text{part}}{\vee}$ i da fornitore j

- min $z = \sum_{i=1}^{3} \sum_{j=1}^{3} C_{ij} \cdot X_{ij} - \sum_{i=1}^{3} \underline{y_i} \cdot S_i$

Sogg. a.

- $\forall_i \in \{1, 2, 3\}$ $R_i \leq \sum_{j=1}^{3} X_{ij} \cdot 0,25$

$$\forall j \in \{1,2,3\} \quad \sum_{i=1}^{3} x_{ij} \geq 0,2 \cdot \sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij}$$

$s_i \in \mathbb{N} \qquad r_i \in \mathbb{N}$

$S_i = [3, 1, 1]$ quanto risparmio

$O_i = [3, 4, 4]$ ogni quanto

dati

div. euclid.

$$X_{ii} = s_i \cdot O_i + r_i \qquad \forall i \in \{1,2,3\}$$

$$0 \leq r_i < O_i \qquad \forall i \in \{1,2,3\}$$

---

b) $\cdot F_j \begin{cases} 1 & \text{se compro tutto da } j \\ 0 & \text{altrimenti} \end{cases}$

$$\sum_{i=1}^{3} X_{ij} \leq F_j \cdot N$$

dove N arbitrar. grande

$$\sum_{j=1}^{3} F_j = 1$$

---

es 2

$\max z = -x_1 + 3x_2$

sogg. a

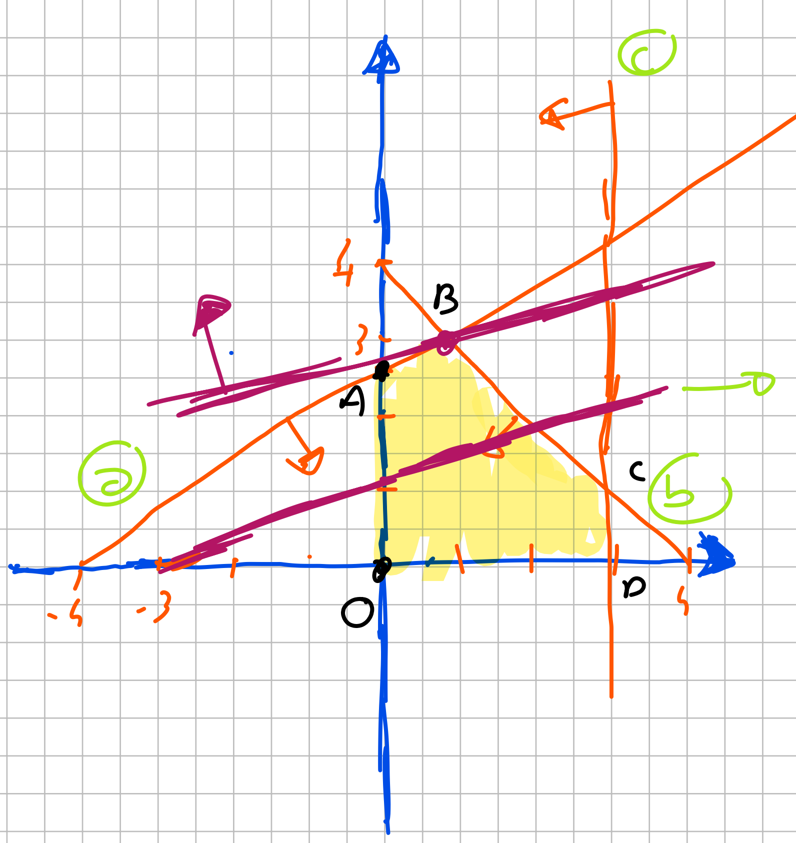
$-2x_1 + 3x_2 + x_3 = 8$ (a)

$x_1 + x_2 + x_4 = 4$ (b)

$x_1 + x_5 = 3$ (c)

$x_1, \ldots, x_5 \geq 0$

j



$A = \left(0, \frac{8}{3}\right)$

$B = \left(\frac{3}{5}, \frac{16}{3}\right)$

$C = (3, 1)$

$D = (3, 0)$

$O = (0, 0)$

Interseco 2 rette e trovo cordinata B

$$\begin{cases} -2x_1 + 3x_2 = 8 \\ x_1 + x_2 = 4 \end{cases}$$

$$5x_2 = = 16$$

$x_2 = \frac{16}{5}$ $\qquad$ $x_1 = +\frac{4}{5}$

sost. x_1 e x_2 di B in vincoli

$$\begin{cases} -2 \cdot \frac{4}{5} + 3 \cdot \frac{16}{5} + x_3 = 8 \\ \frac{4}{5} + \frac{16}{5} + x_4 = 4 \\ \cancel{x_1}\frac{4}{5} + x_5 = 3 \end{cases}$$

$x_3 = 0$

$x_4 = 0$

$x_5 = \frac{11}{5}$

quindi $B = \left(\frac{4}{5}, \frac{16}{5}, 0, 0, \frac{11}{5} \right)$

e quindi la base $B_B = \{x_1, x_2, x_5\}$

$A = \left(0, \frac{8}{3}, 0, \frac{4}{3}, 3\right)$ $\quad B_A = \{x_2, x_4, x_5$

$C = (3, 1, 11, 0, 0)$ $\quad B_C = \{x_1, x_2, x_3\}$

$D = (3, 0, 14, 1, 0)$ $\quad B_D = \{x_1, x_3, x_4\}$

$O = (0, 0, 8, 4, 3)$ $\quad B_O = \{x_3, x_4, x_5\}$

Simplesso IV

$$\begin{array}{c} x_3 \\ x_4 \\ x_5 \end{array} \left(\begin{array}{ccccc|c} -2 & 3 & 1 & & & 8 \\ 1 & 1 & & 1 & & \cancel{6}\,4 \\ 1 & & & & 1 & 3 \end{array} \right)$$

$$\max \ z = -x_1 + 3x_2$$

$$\left(\begin{array}{ccccc|c} -\frac{2}{3} & 1 & \frac{1}{3} & & & 8/3 \\ 5/3 & & -1/3 & 1 & & 4/3 \\ 1 & & & & 1 & 3 \end{array} \right)$$

$$\max \ z = -x_1 + 3x_2 = .$$

$$-x_1 + 3\left(\frac{8}{3} + \frac{2}{3}x_1 - \frac{1}{3}x_3\right.$$

$$-x_1 + 8 + \underline{2x_1} - x_3$$

$$\left(\begin{array}{ccccc|c} & 1 & 1/3 & 2/3 & & 16/13 \\ 1 & & -\frac{1}{5} & \frac{3}{3} & & 9/5 \\ & & +1/5 & -3/5 & 1 & 11/5 \end{array}\right)$$

$$\max \quad z = -\left(\frac{9}{5} + \frac{1}{5}x_3 - \frac{2}{5}x_4\right)$$

$$+2\left(\frac{16}{13} - \frac{1}{5}x_3 - \frac{2}{5}x_3\right)$$

ottimo

$$\max \quad z = -2x_1 + x_2$$

$$-2\left(\frac{9}{5} + \frac{1}{5}x_3 - \frac{2}{5}x_4\right)$$

$$+\left(\frac{16}{13} - \frac{1}{5}x_3 - \frac{2}{5}x_4\right)$$

$$= \text{numero} \underline{-\frac{3}{5}}x_3 + \underline{\frac{2}{5}}_{\text{non}}x_4 \Rightarrow \text{ott!}$$

esercizio 3

$$\left(\begin{array}{cccc|c} 2 & 3 & -1 & -1 & 5 \\ -1 & 1 & 2 & -1 & 6 \\ 1/2 & 2 & 1/2 & -1 & 3 \end{array}\right)$$

$E_1 \leftarrow E_1 - 3E_2$
$E_3 \leftarrow E_3 - 2E_2$
$E_1 \leftrightarrow E_2$

$$\left(\begin{array}{cccc|c} -1 & 1 & 2 & -1 & 6 \\ 5 & 0 & -7 & 5 & -13 \\ \frac{5}{2} & 0 & -\frac{7}{2} & 1 & -9 \end{array}\right)$$

$E_1 \leftarrow E_1 + E_3$
$E_2 \leftarrow E_2 - 5E_3$
$E_2 \leftrightarrow E_3$

$$\left(\begin{array}{cccc|c} 3/2 & 1 & -3/2 & 0 & -3 \\ +5/2 & 0 & -7/2 & 1 & -9 \\ -15/2 & 0 & +21/2 & 0 & 32 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & 3 & -1 & -1 & 5 \\ -1 & 1 & 2 & -1 & 6 \\ 1/2 & 2 & 1/2 & -1 & 3 \end{array}\right)$$

$$\cdot 2 \quad \left(\begin{array}{cccc|c} -2 & -3 & 1 & 1 & -5 \\ -3 & -2 & 3 & 0 & 1 \\ -3 & -2 & 3 & 0 & -4 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} -2 & -3 & 1 & 1 & -5 \\ -3 & -2 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & -5 \end{array}\right)$$

↳ $\emptyset$ Lsg