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IOT Homework - Exercise 3

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Contents

Contents	i
1 Exercise 1	1
1.1 1	1
1.1.1 Computation of L_4	1
1.1.2 Efficiency for $r_1 = 1..6$	2
1.2 2	2
1.3 3	3

1 | Exercise 1

1.1. 1

1.1.1. Computation of L_4

Knowing that $L_N(r)$ is computed via the recursive formula:

$$L_N = r_i + \sum_{i=0}^{N-1} P(S=i) L_{N-i},$$

we can extract the following formula:

$$L_N = \frac{r_i + \sum_{i=1}^{N-1} P(S=i) L_{N-i}}{1 - P(S=0)}.$$

When $N = 4$ and $r_i = 4$, numerically we found:

$$P(S=0) = 0.15625$$

$$P(S=1) = 0.1875$$

$$P(S=2) = 0.5625$$

$$P(S=3) = 0$$

$$P(S=4) = 0.09375$$

Substituting into the previous formula:

$$\begin{aligned} L_4 &= \frac{4 + 0.1875 L_3 + 0.5625 L_2 + 0 L_1 + 0.09375 L_0}{1 - 0.15625} \\ &= \frac{4 + 0.1875 \times 6.375 + 0.5625 \times 4}{0.84375} = \frac{7.4453125}{0.84375} \approx 8.8241. \end{aligned}$$

1.1.2. Efficiency for $r_1 = 1 \dots 6$

For each initial frame r_1 , the total mean resolution time is

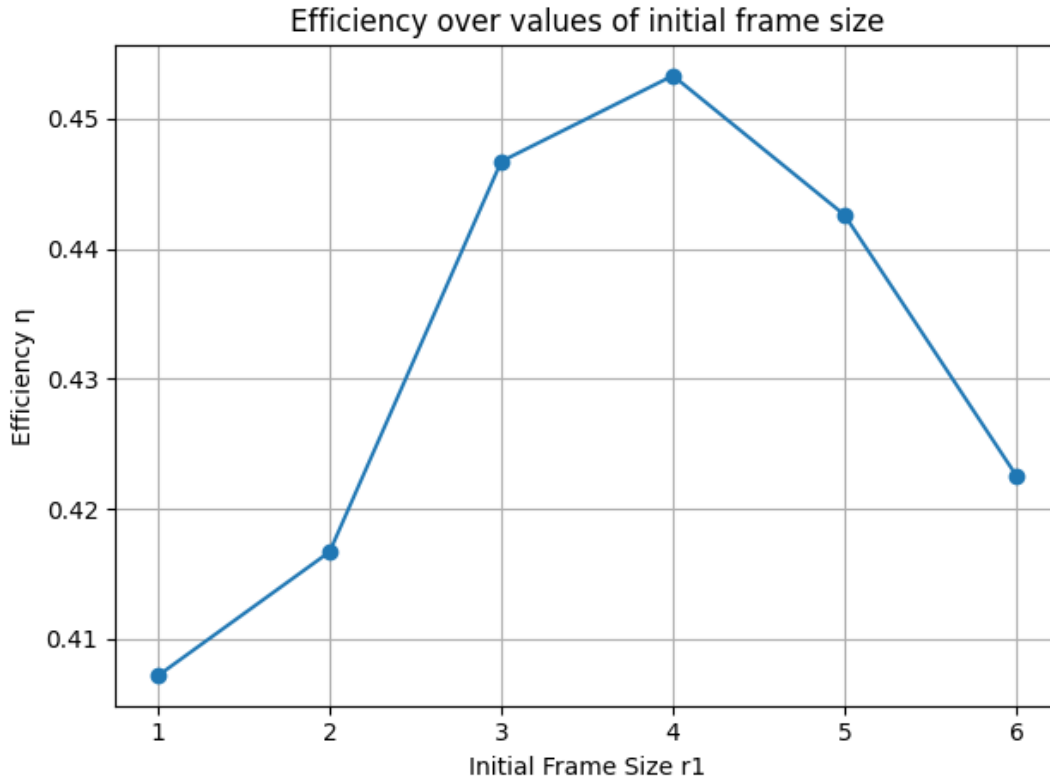
$$\mathbb{E}[T] = r_1 + \sum_{i=0}^3 P(S = i) L_{4-i},$$

where $P(S = i)$ is recomputed for each r_1 . The *efficiency* is $\eta = 4/\mathbb{E}[T]$, after all the computations we get:

r_1	$\mathbb{E}[T]$	η
1	9.8241	0.4072
2	9.5995	0.4167
3	8.9544	0.4467
4	8.8241	0.4533
5	9.0377	0.4426
6	9.4661	0.4226

We observe the maximum efficiency $\eta \approx 0.4533$ at $r_1 = 4$.

1.2. 2



1.3. 3

From the results, we observe that the efficiency η reaches its maximum value of approximately 0.4533 when the initial frame size is $r_1=4$, which matches the number of tags $N=4$.

This happens because when the frame size matches the backlog size, the probability of having exactly one tag per slot is maximized. This reduces both collisions and slots that are not chosen, which are both sources of inefficiency.