# 2D uniform-mass rectangles via nested quantiles

Let $\{(u_{1,k}, u_{2,k})\}_{k=1}^{n} \subset [0,1]^2$ be the (rank-normalized) uncertainty features computed on the calibration split. Fix integers $m_1, m_2 \in \mathbb{N}$ and set $B = m_1 m_2$.

**Step 1 (equal-mass slabs in $u_1$).** Let $t_0 = 0 < t_1 < \cdots < t_{m_1} = 1$ be empirical quantile cutpoints of $\{u_{1,k}\}_{k=1}^{n}$ chosen so that each interval

$$I_a[t_{a-1}, t_a), \qquad a \in [m_1],$$

contains approximately $n/m_1$ samples (up to rounding/ties).

**Step 2 (conditional equal-mass bins in $u_2$).** For each $a \in [m_1]$, restrict to the indices

$$S_a\{k \in [n] : u_{1,k} \in I_a\}.$$

Let $s_{a,0} = 0 < s_{a,1} < \cdots < s_{a,m_2} = 1$ be empirical quantile cutpoints of $\{u_{2,k}\}_{k \in S_a}$ chosen so that each interval

$$J_{a,b}[s_{a,b-1}, s_{a,b}), \qquad b \in [m_2],$$

contains approximately $|S_a|/m_2 \approx n/B$ samples.

**Rectangles and quantizer.** Define the $B$ rectangles

$$R_{a,b} I_a \times J_{a,b}, \qquad (a,b) \in [m_1] \times [m_2],$$

and the induced quantizer $Q : [0,1]^2 \to [m_1] \times [m_2]$ by

$$Q(u_1, u_2) = (a,b) \quad \Longleftrightarrow \quad (u_1, u_2) \in R_{a,b}.$$

By construction, each $R_{a,b}$ contains approximately $n/B$ calibration samples.