Macroeconometrics
Fall 2022
Problem Set 1

- The problem set is due by **Sunday, October 23th at midnight** (the night before the class in the computer room on Monday, October 24th). Please send the material by e-mail to Mahyar Habibi, mahyar.habibi@phd.unibocconi.it).
- Work in groups of maximum 3 people.
- You are expected to structure the problem set in two parts. The first is an explanatory note in which you explain point by point what you are doing, showing graphs to clarify the points you want to make etc. and adding references to the Matlab functions you are using or created. The second is a directory (you can zip it) in which you put all the Matlab files needed to generate your results. Please create a single Matlab file with which it is possible to generate all the results in the explanatory note. Each group must send one explanatory note and one matlab folder to Mahyar.

This problem set is a Matlab warm-up.

1. Generate 500 observations from an $AR(1)$ process Y_t with $E(Y_t) = 0$, $\phi = 0.4$ and the variance of the white noise forcing term $\sigma^2 = 0.2$ using the two methods below *(hint: the random number generator for iid normal is* `randn`*)*.
a. A `for` loop using the recursive structure of the $AR(1)$.
b. Using the function `filter`.
c. Check that when the forcing variables are the same, the output of the two approaches is the same (be careful with the starting conditions and with the random number generator).

2. Focus on the `for` approach and generate data from an AR(1) with $E(Y_t) = 3$. Pick proper values of the only free parameter to implement $E(Y_t) = 3$. Set the starting condition to 10. What happens if the starting condition you choose is far from the unconditional mean of the process? What would you do in order to make sure that the sample path is a "proper" realization of the process you want to simulate from? Do the same thing with `filter`.

3. Generate 500 observations from a $MA(1)$ process with $\theta = 0.3$ and the variance of the white noise forcing term $\sigma^2 = 0.3$, using the two methods below.
a. With a `for` loop using the recursive structure.
b. Using the function `filter`.

4. Write a function that generates T observations from an $ARMA(p, q)$ using the `for` loop approach.

The function must have the following inputs: 1) the number of observations, 2) the variance of the white noise forcing variables, 3) the coefficients of the AR and MA polynomials or the roots of the AR and MA polynomials (*hint: you may find the* `poly` *and* `roots` *functions useful*), and the realizations of the $ARMA(p,q)$ and of the white noise as outputs.

5. Compute the empirical distributions of the OLS estimator in the case of an $AR(1)$ with $\phi = 0.4$ and $T = 250$ (you are free to choose the variance of the innovation).

b. Costruct a t-test for the null hypothesis $H_0 : \phi = 0$, against a two-sided alternative $H_0 : \phi \neq 0$. How often do you reject the null at the 95% confidence level when $T = 200$?

6. Compute the empirical distributions of the OLS estimator in the case of an $AR(1)$ with $\phi = 0.9$ and $T = \{50, 100, 200, 1000\}$. How is the distribution changing with T?

7. Compute the empirical distribution of the OLS estimator in the regression $x_t = ax_{t-1} + v_t$ in the case in which the data generating process for x_t is $MA(1)$ with $\theta = 0.6$ and $T = 250$. What is the mean of the distribution? Do the same by lengthening the sample size. Does it converge to anything as $T \to \infty$? Discuss.

8. a. Compute the empirical distribution of the OLS estimator in the case of an $AR(1)$ with $\phi = 1$ and $T = 250$ (you are free to choose the variance of the innovation).

b. Construct a t-test for the null hypothesis $H_0 : \rho = \phi - 1 = 0$, in a test regression : $\Delta y_t = \alpha + \rho y_{t-1} + \varepsilon_t$, against a one-sided alternative $H_0 : \rho < 0$. Using a standard Normal distribution, how often do you reject the null hypothesis at the 95% confidence level? Is the distribution symmetric? Discuss.

c. Compute now few percentiles of the empirical distribution of the t-test you generated at point b. and check that they are close to those simulated by Dickey and Fuller. (*hint: you can find additional details in Enders*).

9. a. Perform a Monte-Carlo experiment in the case of a random walk with drift and $T = 250$ to study the performance of the Dickey-Fuller test.

b. Construct an F-test for the null hypothesis H_0 : there is unit root, against the alternative H_1 : there is no unit root using a χ^2 distibution (how many degrees of freedom?). How often do you reject H_0 at 95% confidence?

c. Generate now data from a deterministic time trend and perform a DF test using the correct distribution for the test. How often do you reject the null? (*hint: you can find additional details in Enders*).