Macroeconometrics
Problem Set 3

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Exercise 1

The dataset has information on three main variables: log(real GDP), log(Price deflator) and Federal Funds Rate for the US from 1960 to 2010 at quarterly frequency. After loading the data-set, we estimated a VAR(1) using the function *varm(...)* subsequently, we store the matrix coefficient $\hat{A}$. We compute the impulse responses and use the *bootstrap* method to compute confidence bands for the impulse response functions. By using a bootstrap we can avoid assuming that the shocks are Gaussian, and instead sample directly from the estimated residuals.
We proceeded according to the instructions provided in this problem set. Those can be summarized as follows.

Firstly, we estimated the VAR and saved both the coefficients, $\hat{A}$, and the residuals, $\hat{\varepsilon}$. For simplicity we assumed a VAR(1), but one can do the same proceeding also for a VAR with more lags.

Secondly, we should sample with replacement from the estimated residuals $\{\hat{\varepsilon}_1, \hat{\varepsilon}_2, ..., \hat{\varepsilon}_T\}$ in order to form a new series of residuals, say $\tilde{\varepsilon}$, equal to the realization just drawn. We do this by generating $T-1$ random integer between 1 and $T-1$ (using *randi* on MATLAB) that we collect in a vector called *PER*. Then, simply we will have

$$\tilde{\varepsilon} = \varepsilon(PER, :)$$

Now, we use the new residuals, $\tilde{\varepsilon}$, and the estimated coefficients, $\hat{A}$, to construct new series (done with a simple *for* loop on MATLAB):

$$\tilde{y}_t = \hat{A}\tilde{y}_{t-1} + \tilde{\varepsilon}_t$$

Since we have T time observations for each of our variables when we create the new vector $\tilde{y}$ this will be of dimension $((T-1) \times N)$; this is the reason we generated the random vector *PER* used for creating $\tilde{\varepsilon}$ of $T-1$ integers rather than T.

Next, we estimated a VAR on the new process $\tilde{y}_t$ and computed and stored the impulse response function, as we did above for y_t. We repeated this process 1000 times, each one saving the impulse response function. This is done on MATLAB using a *for* loop. Finally, we displayed in Figure 1 the 95% confidence interval, by plotting the 2.5% and 97.5% percentile of the empirical distribution of the 1000 impulse responses. For the sake of completeness we plotted and identified all the impulse response functions, even though we were asked only to identify the monetary policy shock. The latter can be found in the third column of Figure 1.

First, notice that in both the top and middle subplots in the third column the impulse response function starts from zero; this is because of the restriction we imposed with the lower triangular identification. As we can see from the three subplots, a monetary shock has a negative effect both on real GDP and on the Federal Funds Rate, while it has a positive effect on Price deflator. In terms of magnitude of the effect, we can say that a monetary shock has a stronger impact (positive) on Price deflator than on GDP (negative). Moreover, both the impulse response functions for GDP and Price reach a peak between the 20th and 30th period after the shock, after which they start to dissolve and slowly go back to zero. The impulse response for the Federal Funds Rate, which reaches its peak at the time of the shock, goes directly towards zero as time passes and in a faster fashion with respect to the other two variables. Finally, it is also interesting to notice that inflation (Price deflator) goes up, and this is counter-intuitive considering the standard transmission mechanism.

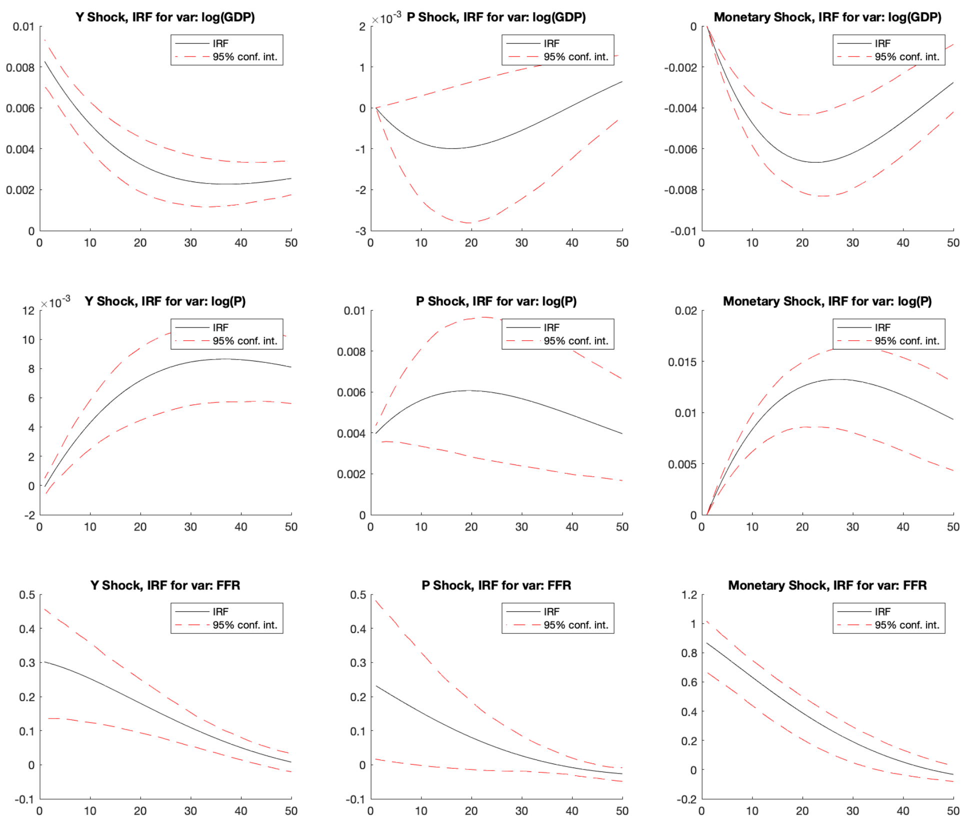

Figure 1: Estimated IRFs of a trivariate VAR(1). Bootstrap technique for Conf. Bands

Lastly, we computed the forecast error variance decomposition for the monetary policy shock. The forecast error variance decomposition (FEVD) is a part of structural analysis which "decomposes" the variance of the forecast error into the contributions from specific exogenous shocks. This kind of analysis is crucial because it helps explaining how a shock impacts the variations of specific variables over time (for example, some shocks may be responsible for long-term variations instead of short-run ones depending on the kind/setting of the shock and model).

The variance explained by the FFR shock for the one step ahead can be computed as :

$$\frac{\text{diag}(G_{FFR}G'_{FFR})}{\text{diag}(GG')}$$

where G is the Cholesky factor of Ω, the estimated variance covariance matrix of the Var model, and

G_{FFR} indicates the third column of G, the one that refers to the *Federal Fund Rate*. Note that "diag" is the MATLAB command to obtain the diagonal elements of the matrix under consideration. The fraction is an element by element division that is performed in order to get the % variance explanation of one shock in the considered variable.
For the two steps ahead, the variance explained by the monetary shock can be derived as:

$$\frac{\text{diag}(G_{FFR}G'_{FFR} + \Psi_1 G_{FFR}G'_{FFR}\Psi'_1)}{\text{diag}(GG' + \Psi_1 GG'\Psi'_1)}$$

One could go on in this fashion, as in the Lecture Notes. Ψ represents the estimated coefficients of the estimated VMA associated to the VAR model that we have estimated on the data.
In the end we get the following Table:

Forecast Error Variance Decomposition			
	Log Real GDP	Log Price deflator	Federal Funds Rate
$t+1$	0	0	0.8384
$t+2$	0	0	0.8375
$t+3$	0.0054	0.0611	0.8371
$t+5$	0.0635	0.4192	0.8368
$t+10$	0.4016	0.7692	0.8366
$t+20$	0.7100	0.8240	0.8364
$t+30$	0.7711	0.8300	0.8363
$t+50$	0.7943	0.8320	0.8363

From the table above we can notice the following. Firstly, the monetary shock does not explain in terms of variance changes in real GDP and in Price deflator for the two periods after the shock. Only starting from the third period after the shock the variance explained by the monetary shock starts to increase, and it does so at an increasing rate. After the 20-30th period the explained variances on the Log Real GDP and Log Price deflator start to stabilize around 0.83 and 0.77 respectively. This is in line with the results from Figure 1, where we see that the largest impact of the shock manifests after the 20th period. On the other hand, there is no such long run versus short run distinction in the forecast variance explained by the monetary shock for the Federal Funds Rate. It is high and stable around 0.83. Whereas, for the first two variables there is distiction between the impact of the shock in the short and long run. As time passes, the variability can be attributed more and more to the Monetary Shock.

Exercise 2

From 2 we can note the impact of a non-technology vs a technology shock. The graph is almost identical to the one proposed by the paper (slight differences due to the scale of the data), with the same ideas proposed by Gali. In fact, following the identification strategy proposed in the paper, we should have that the non-technology shock's effect on productivity should tend to zero as time elapses.

To reproduce the graphs in the paper, we have to take logs and change the scale of the data.[1] Once we are done with this process, we take differences and estimate the VAR model with the 4 lags.
In order to calculate the VAR's IRFs, we have to use the long-run identification process and to that the first step is to compute the VMA form of the model (according to the lecture notes it is of the form $Y_t = D(L)\eta_t$, where η_t is the reduced form model of the vector of shocks).

[1] We take the log of the data and then we scale the log data by 100 in order to have better view of the shocks that impact the system (otherwise the impact would result meaningless in the graph). Since the transformation is monotone and the scale is just a multiplicative factor, the path in the data is preserved.

Subsequently, we calculate D(1) and thanks to the value of the variance-covariance matrix estimated while fitting the VAR model Ω, we can get to the Cholesky decomposition, allowing us to get the expression of the form

$$SS' = D(1)\Omega D(1)'$$

Then, we are able to compute the IRFs and coefficients as we can now calculate $K = D(1)^{-1}S$ which will be fundamental in this process.

The last part of this exercise consisted in the bootstrapping part for obtaining the confidence bands and intervals, where we basically followed exercise 1's approach with some slight modifications in order to accommodate the new and updated model (of course also the size of the data is different).

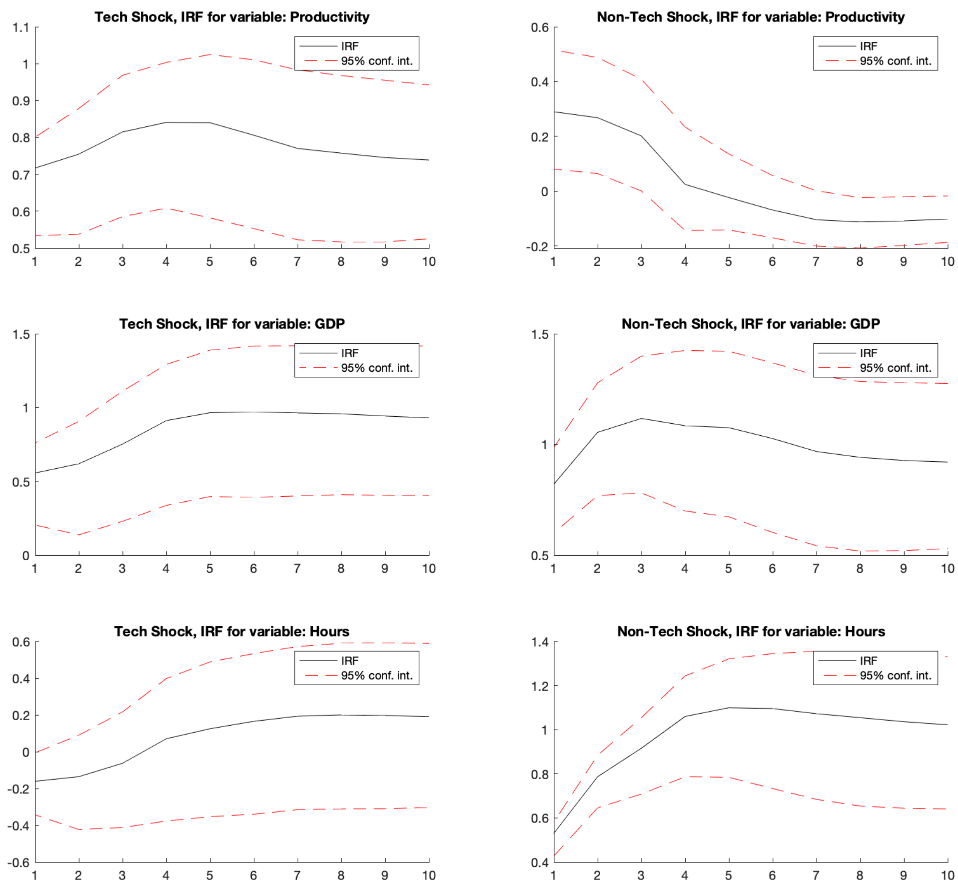

Figure 2: Estimated IRFs of a bivariate VAR(4). Bootstrap technique for Conf. Bands