Macroeconometrics
Fall 2022
Problem Set 4

- The problem set is due by **Sunday, December 11th at midnight**. Please send the material by e-mail to Mahyar Habibi, mahyar.habibi@phd.unibocconi.it.
- For points 1., 2. and 3. you are required to write the computations in the pdf file accompanying the answer. For the part that requires Matlab, you are expected to follow the same rules for problem sets 1. and 2. and 3.

You are given the following ECM

$$\Delta y_t = \begin{bmatrix} -0.6 & 0.5 \\ 0.5 & -5/12 \end{bmatrix} y_{t-1} - \begin{bmatrix} 0.4 & 0.5 \\ 0 & 0 \end{bmatrix} \Delta y_{t-1} + \varepsilon_t$$

1. How many cointegration relations characterize the vector y_t? (in the computations you will find few approximations...)
2. Find one basis of the cointegration space.
3. Write the VAR in levels.
4. Compute theoretical impulse responses. Plot them.
5. Generate 1000 samples of size $T = 250$ from the above model (assume $Cov(\varepsilon_t) = I_2$). For each of the 1000 samples:
   (a) estimate a VAR in levels and compute impulse responses. Compute the mean of the impulse responses and the 2.5th and 97.5th percentile of their empirical distribution.
   (b) estimate a (misspecified) VAR in the first differences of the variables and compute impulse responses **for the levels**. Compute the mean of the impulse responses and the 5th and 95th percentile of their empirical distribution. Plot them and compare them with those you obtained in points 4. and 5.a
   (c) Redo what you did at point b. by increasing the number of lags (go from 1 to 4 lags). What do you notice as the number of lags increases?
   (d) Implement the Johansen procedure described in the notes, estimate the ECM representation of the system above and plot impulse response functions.
   (Hint: in the data generating process shocks are orthogonal. In the estimated one, they will be slightly correlated. Do not worry and just plot the 4 impulse responses (2 shocks - 2 variables) as if shocks were uncorrelated).