05c_linear_systems_direct_2

October 16, 2019

1 Direct methods for solving linear systems (homework)

Exercise 1. Let us consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} \epsilon & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}.$$

1. Find the range of values of $\epsilon \in \mathbb{R}$ such that the matrix A is symmetric and positive definite. **Suggestion**: use the *Sylvester's criterion* which states that a symmetric matrix $A \in \mathbb{R}^{n\times n}$ is positive definite if and only if all the main minors (The main minors of $A \in \mathbb{R}^{n\times n}$ are the determinants of the submatrices $A_p = (a_{i,j})_{1\leq i,j\leq p}$, $p = 1, ..., n$). of A are positive.
2. What factorization is more suitable for solving the linear system $A\mathbf{x} = \mathbf{b}$ for the case $\epsilon = 0$? Motivate the answer.
3. Compute the Cholesky factorization $A = R^T R$ for the case $\epsilon = 2$.
4. Given $\mathbf{b} = (1, 1, 1)^T$, solve the linear system by using the Cholesky factorization computed at the previous point.

Exercise 2. Let us consider the following matrix $A \in \mathbb{R}^{3\times 3}$ depending on the parameter $\epsilon \in \mathbb{R}$:

$$A = \begin{bmatrix} 1 & \epsilon & -1 \\ \epsilon & \frac{35}{3} & 1 \\ -1 & \epsilon & 2 \end{bmatrix}.$$

1. Calculate the values of the parameter $\epsilon \in \mathbb{R}$ for which the matrix A is invertible (non singular).
2. Calculate the Gauss factorization LU of the matrix A (when non singular) for a generic value of the parameter $\epsilon \in \mathbb{R}$.
3. Calculate the values of the parameter $\epsilon \in \mathbb{R}$ for which the Gauss factorization LU of the matrix A (when non singular) exists and is unique.
4. Set $\epsilon = \sqrt{\frac{35}{3}}$ and use the pivoting technique to calculate the Gauss factorization LU of the matrix A.
5. For $\epsilon = 1$, the matrix A is symmetric and positive definite. Calculate the corresponding Cholesky factorization of the matrix A, i.e. the upper triangular matrix with positive elements on the diagonal, say R, for which $A = R^T R$.