01c_interpolation_error

October 16, 2019

1 Task for today:

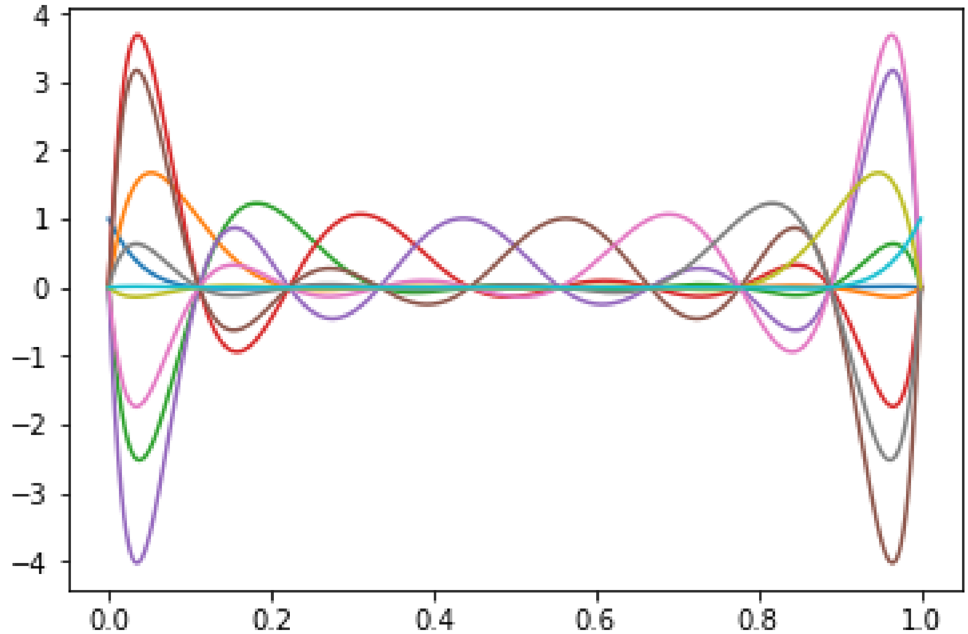
- compute the error between Lagrange interpolation for equispaced points (in "approximate Linfty") and a given function when the degree increases
- compute the error between Lagrange interpolation for Chebyshev (in "approximate Linfty") and a given function when the degree increases
- compute the error between Bernstein approximation (in "approximate Linfty") and a given function when the degree increases
- compute the L2 projection and compute the error ("in approximate Linfty") norm and compare with previous results

[1]:
```
%pylab inline
```

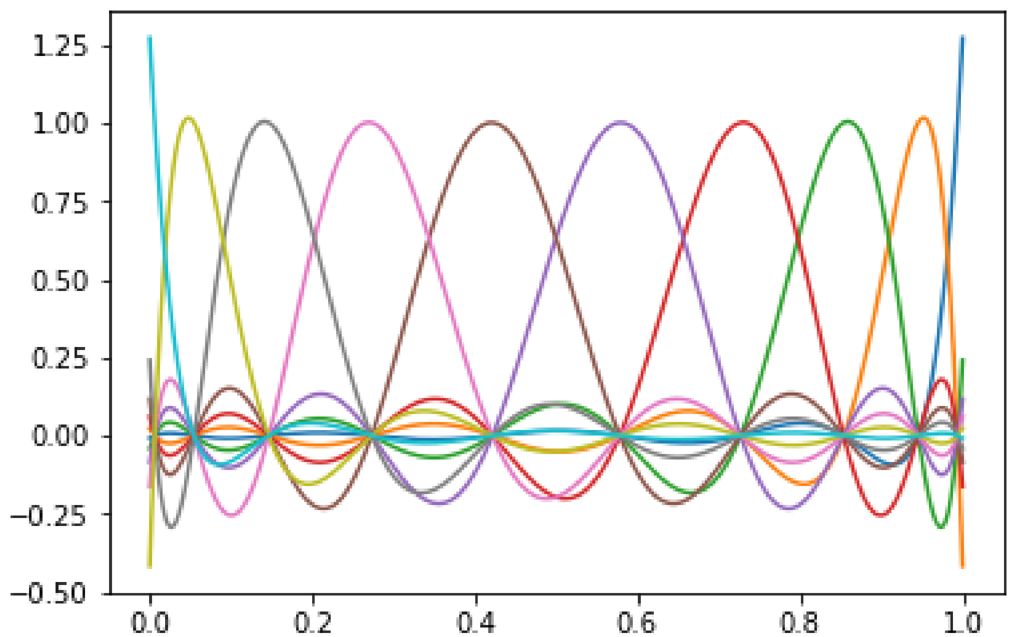
```
Populating the interactive namespace from numpy and matplotlib
```

[2]:
```
import scipy.special

def lagrange(i, q, x):
    return product([(x-qj)/(q[i]-qj) for qj in q if qj != q[i]], axis=0)

def bernstein(i, q, x):
    n = len(q)-1
    return scipy.special.binom(n,i)*(1-x)**(n-i)*x**i

def cheb(n):
    return numpy.polynomial.chebyshev.chebgauss(n)[0]*.5+.5
```

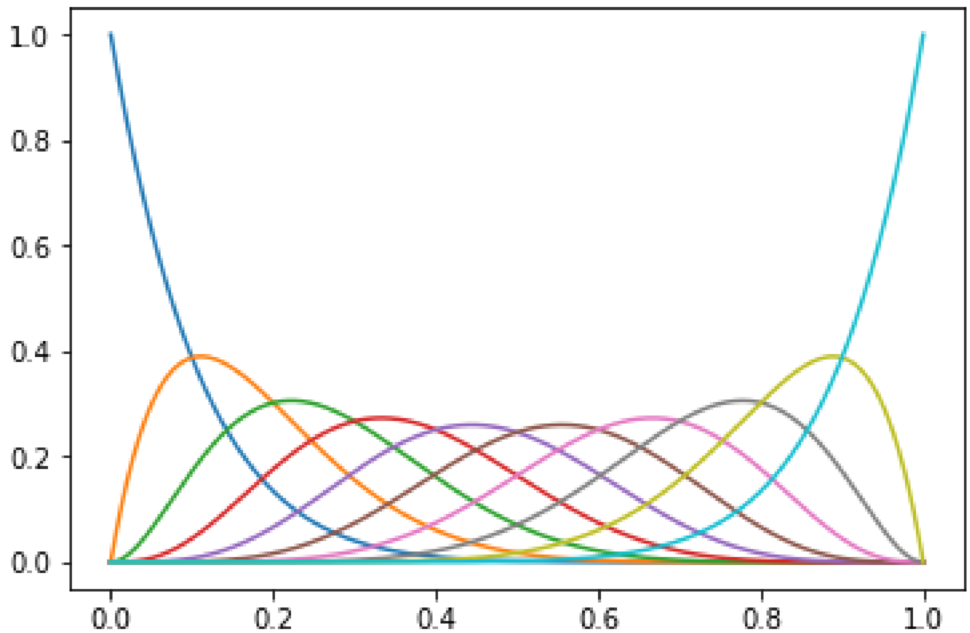
[3]:
```
x = linspace(0,1,1025)
q = linspace(0,1,10)
y = array([lagrange(i,q,x) for i in range(len(q))])
_ = plot(x, y.T)
```



[4]:
```
q = cheb(10)
y = array([lagrange(i,q,x) for i in range(len(q))])
_ = plot(x, y.T)
```

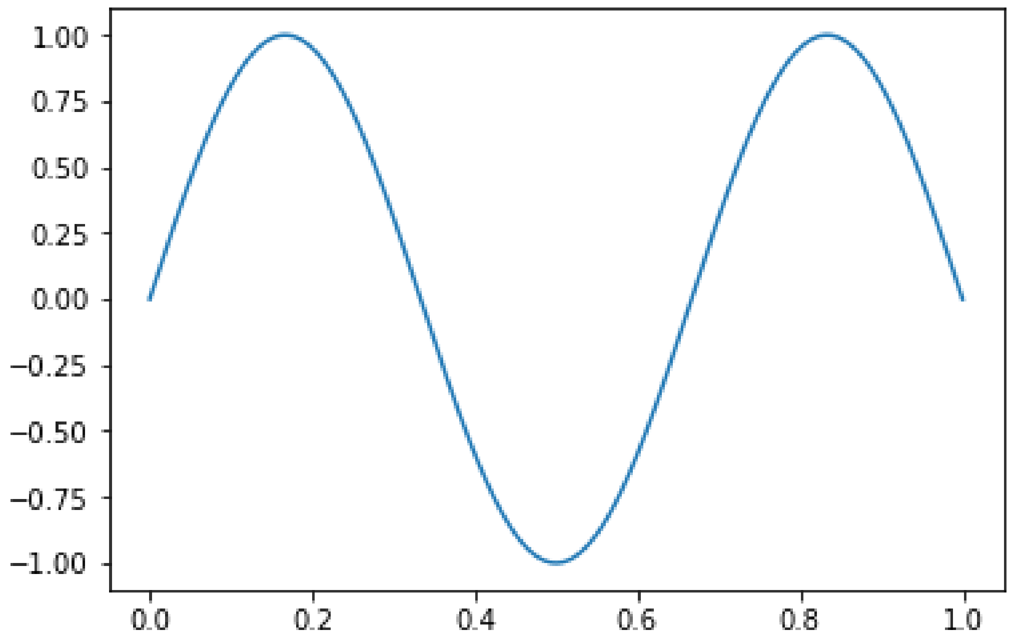


[5]:
```
q = linspace(0,1,10)
y = array([bernstein(i,q,x) for i in range(len(q))])
_ = plot(x, y.T)
```



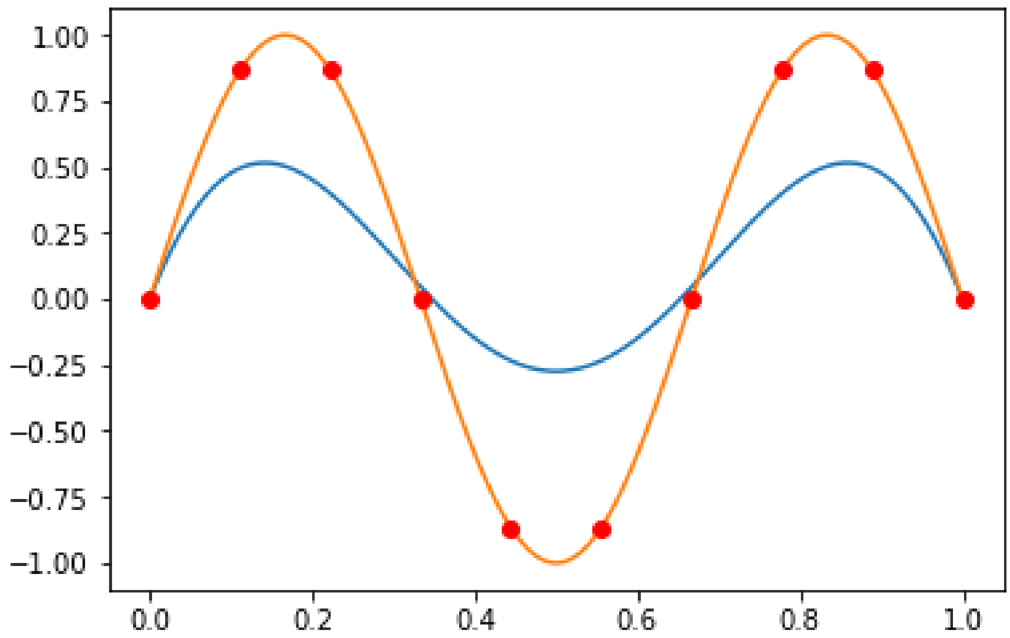
[6]:
```
def myfun(x):
    return 1/(1+100*(x-.5)**2)

def myfun(x):
    return sin(3*numpy.pi*x)


plot(x, myfun(x))
```

[6]: [<matplotlib.lines.Line2D at 0x120e37a20>]



[7]:
```
p = y.T.dot(myfun(q))
f = myfun(x)

plot(x,p)
plot(x,f)
plot(q,myfun(q), 'or')
```

[7]:
```
[<matplotlib.lines.Line2D at 0x120f318d0>]
```



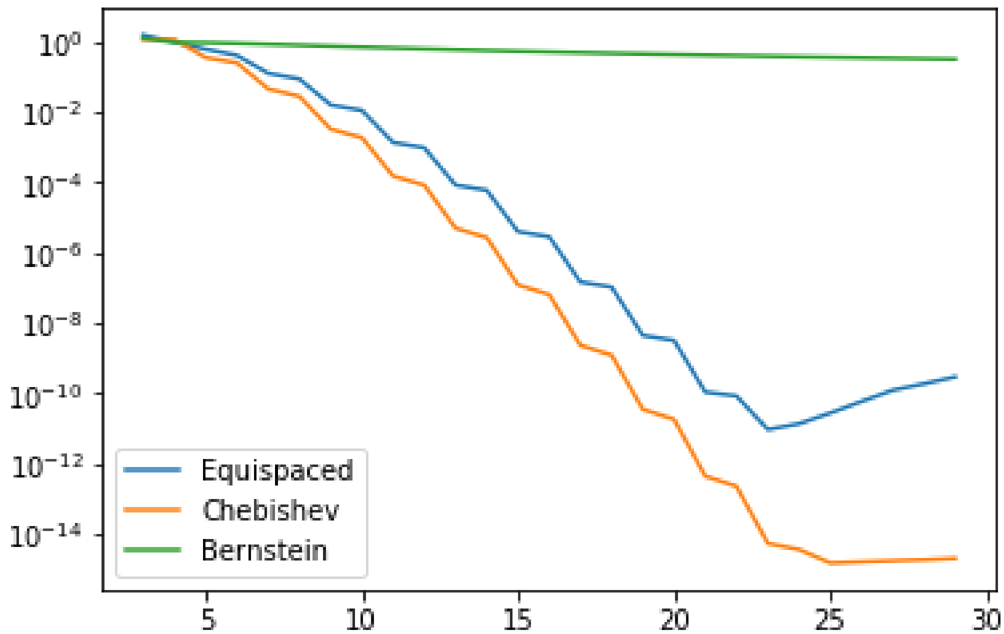
[8]:
```
linfty = max(abs(f-p))
linfty
```

[8]: 0.72598414958382995

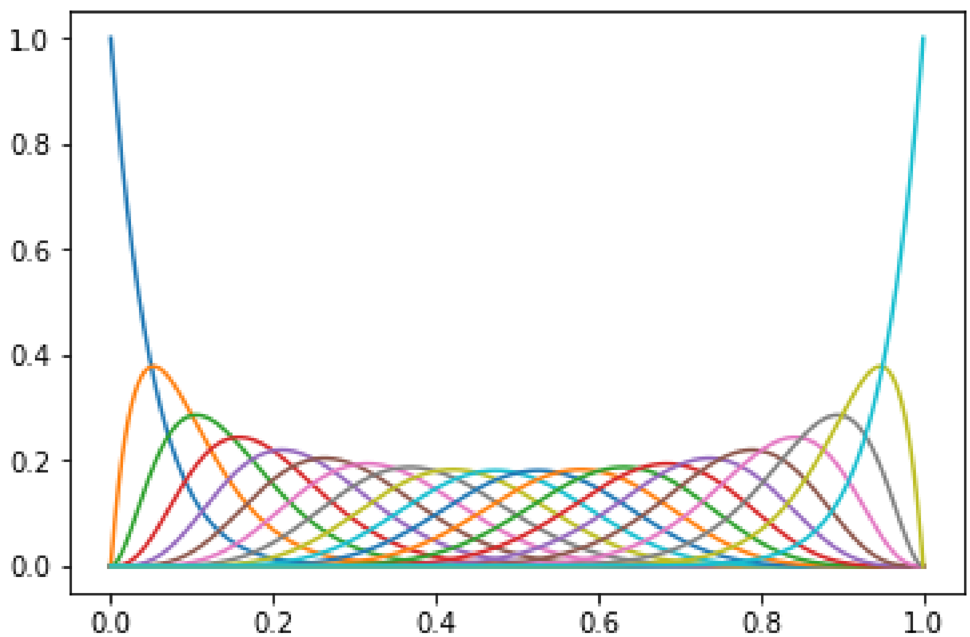
[9]:
```
def error(q, myfun, interpolation=lagrange) :
    y = array([interpolation(i,q,x) for i in range(len(q))])
    p = y.T.dot(myfun(q))
    f = myfun(x)
    return (max(abs(f-p)))
```

[10]:
```
N = range(3, 30)
error_equispaced = []
error_cheb = []
error_bernstein = []
for n in N:
    error_cheb.append(error(cheb(n), myfun))
    error_equispaced.append(error(linspace(0,1,n), myfun))
    error_bernstein.append(error(linspace(0,1,n), myfun, bernstein))
```

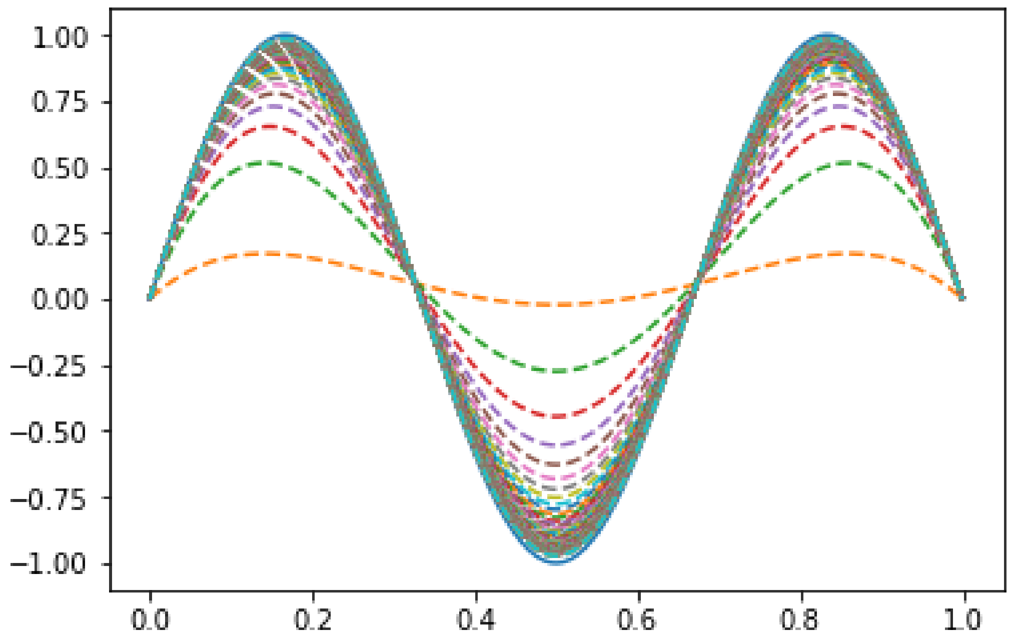
[11]:
```
semilogy(N, error_equispaced)
semilogy(N, error_cheb)
semilogy(N, error_bernstein)
_ = legend(['Equispaced','Chebishev','Bernstein'])
```



[12]:
```
q = linspace(0,1,20)
y = array([bernstein(i,q,x) for i in range(len(q))])
_ = plot(x, y.T)
```



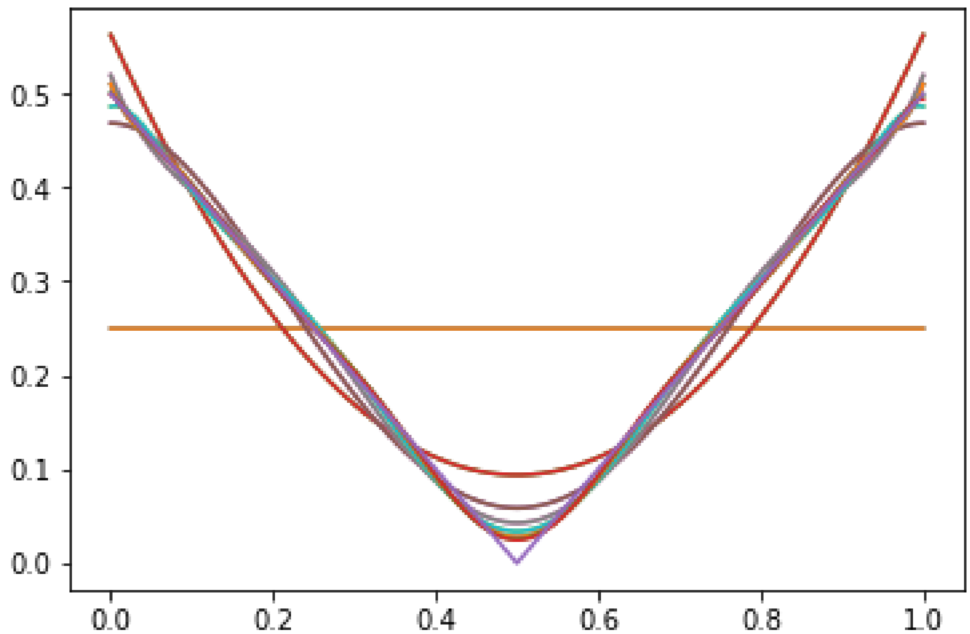
[13]:
```
N = range(5,400,5)
plot(x,myfun(x))

for n in N:
    q = linspace(0,1,n)
    y = array([bernstein(i,q,x) for i in range(len(q))])
    p = y.T.dot(myfun(q))
    _ = plot(x, p, '--')
```



[14]:
```
def myfun(x):
    return abs(x-.5)

import scipy
from scipy.integrate import quad as integrate

N = range(1,15)

for n in N:
    M = zeros((n,n))

    for i in range(n):
        for j in range(n):
            M[i,j] = 1.0/(i+j+1)

    F = array([integrate(lambda x: myfun(x)*x**i, 0,1)[0] for i in range(n)])
```

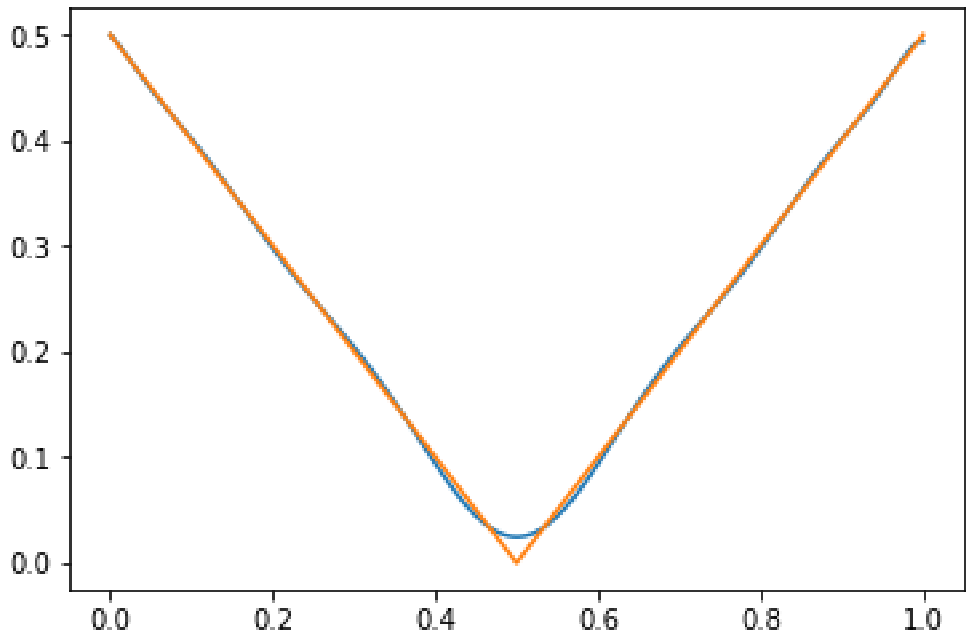
```
    pi = linalg.solve(M, F)
    p = sum([x**i*pi[i] for i in range(n)], axis=0)
    plot(x,p)
plot(x,myfun(x))
```

[14]: [<matplotlib.lines.Line2D at 0x120c069e8>]



[15]:
```
plot(x,p)
plot(x,myfun(x))
```

[15]: [<matplotlib.lines.Line2D at 0x120a1ca90>]



```
[16]: max(abs(p-myfun(x)))
```

```
[16]: 0.02462443416821003
```

Why do we get these errors in the L2 projection? The matrix M is not well conditioned...

```
[17]: linalg.cond(M)
```

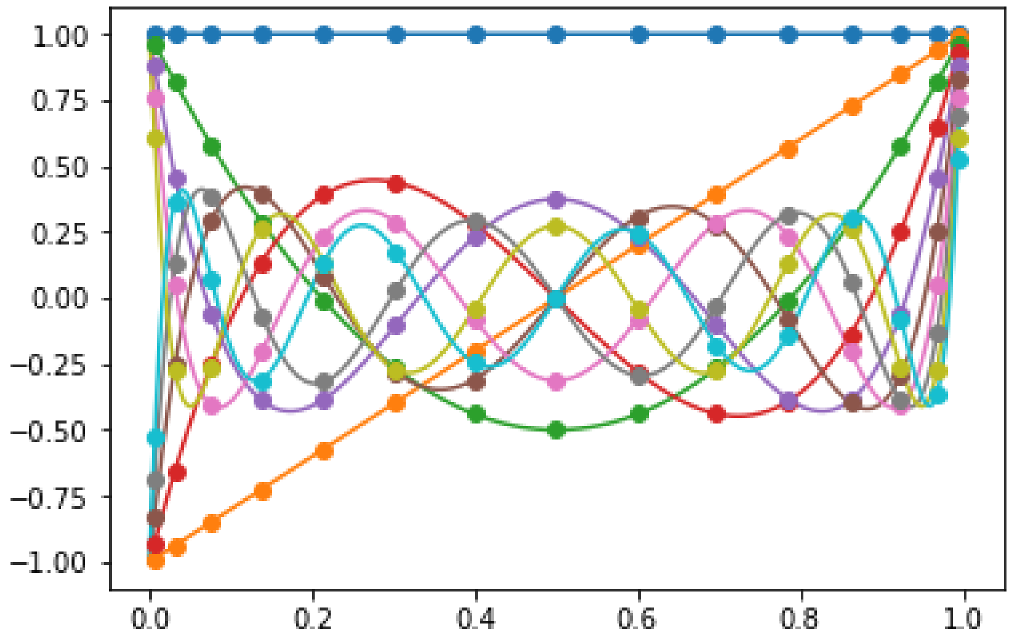
```
[17]: 6.2007862631614438e+17
```

Let's turn to numerical quadrature, and Legendre polynomials (for which M is the identity by construction...)

```
[18]: from numpy.polynomial.legendre import leggauss
      from numpy.polynomial.legendre import legval
      from numpy.polynomial.legendre import Legendre
```

```
[19]: n = 10
      N = n+5



      q,w = leggauss(N)
      w *= .5
      q +=1
      q /=2
```

```
[20]: v = array([Legendre.basis(i, domain=[0,1])(x) for i in range(n)])
      vq = array([Legendre.basis(i, domain=[0,1])(q) for i in range(n)])
      _ = plot(x,v.T)
      _ = plot(q, vq.T, 'o')
```



Check that we get a diagonal matrix as M:

```
[21]: vq.shape
```

```
[21]: (10, 15)
```

```
[22]: M = einsum('iq, jq, q', vq, vq, w)
```

```
[23]: diag = array([M[i,i] for i in range(n)])
```

```
[24]: diag
```

```
[24]: array([ 1.        ,  0.33333333,  0.2       ,  0.14285714,  0.11111111,
              0.09090909,  0.07692308,  0.06666667,  0.05882353,  0.05263158])
```

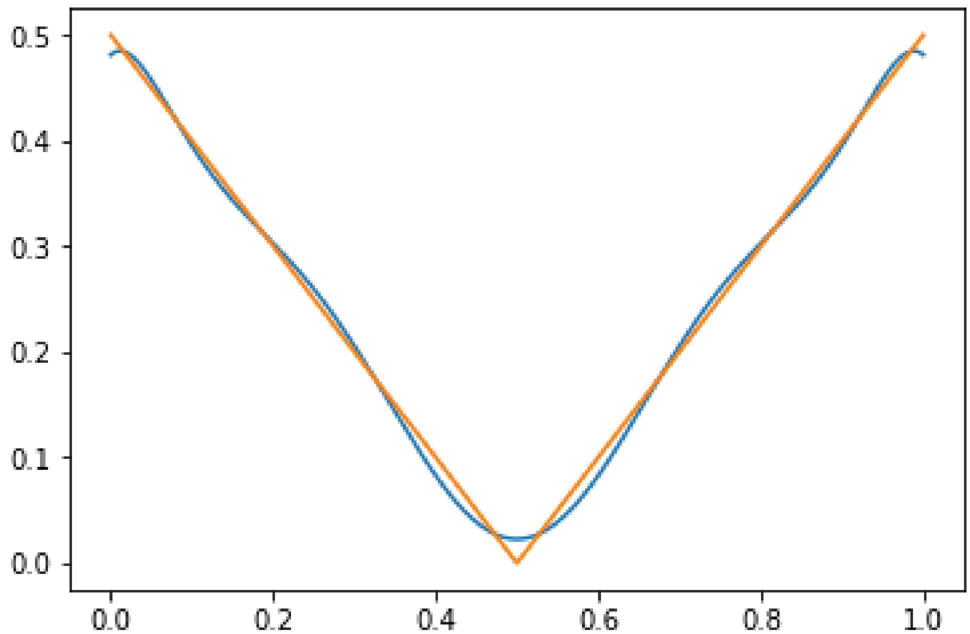
Now perform the integral

```
[25]: pi = sum(vq*myfun(q)*w, axis=1)
```

And plot the function, and its interpolation

[26]:
```
p = (pi/diag).dot(v)
```

[27]:
```
plot(x, p)
_ = plot(x, myfun(x))
```



[ ]:
```
```