Fisica

sabato 1 febbraio 2025 21:34

SIMPLE MODEL

Inputs: $[v, \varphi]$ v = velocity, φ = steering rate

State: $[x, y, \theta, \delta]$
x = horizontal position with respect to road reference system
y = vertical position with respect to road reference system
θ = heading angle with respect to road reference system
δ = steering angle with respect to center of car axis

Reference point: center of gravity (x_c, y_c)

REAR AXLE

State change rate: $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\delta}] = [v\cos(\theta), v\sin(\theta), \frac{v}{L}\tan(\delta), \varphi]$

$\dot{x} = v\cos(\theta)$

$\dot{y} = v\sin(\theta)$

$\dot{\theta} = \omega = \frac{v}{L}\tan(\delta)$

$\omega = \frac{v}{R} = \frac{v}{L}\tan(\delta)$

$R = \frac{L}{\tan(\delta)}$

$\dot{\delta} = \varphi$

FRONT AXLE

State change rate: $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\delta}] = [v\cos(\delta+\theta), v\sin(\delta+\theta), \frac{v}{L}\sin(\delta), \varphi]$

$R = \frac{L}{\sin(\delta)}$

$\dot{x} = v\cos(\delta+\theta)$

$\dot{y} = v\sin(\delta+\theta)$

$\dot{\theta} = \frac{v}{R} = \frac{v}{L}\sin(\delta)$

$\dot{\delta} = \varphi$

CENTER OF GRAVITY

State change rate: $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\delta}] = [v\cos(\beta+\theta), v\sin(\beta+\theta), \frac{v}{L}\tan(\delta)\cos(\beta), \varphi]$

$\beta = \arctan(\frac{l_r}{L}\tan(\delta)) = \arctan(\frac{1}{2}\tan(\delta))$ slip angle

$\dot{x} = v\cos(\beta+\theta)$

$\dot{y} = v\sin(\beta+\theta)$

$S = \frac{L}{\tan(\delta)}$

$R = \frac{S}{\cos(\beta)} = \frac{L}{\cos(\beta)\tan(\delta)}$

$\dot{\theta} = \frac{v}{R} = \frac{v}{L}\cos(\beta)\tan(\delta)$

$l_r = \frac{L}{2}$ distance between rear wheel and center of gravity

$\tan(\beta) = \frac{l_r}{S} = \frac{l_r}{L}\tan(\delta)$

ROAD CURVES

Geometric random variable to represent when a curve is encountered:

$C_1 \rightarrow P(C_1 = K) = (1-p_1)^{K-1} \cdot p_1$

Gaussian random variable to represent rotation rate:

$\omega_c \rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Introduce a new state S_C different from the previously discussed S_R with transition probabilities:

$$P = \begin{bmatrix} P(R\to R) & P(R\to C) \\ P(C\to R) & P(C\to C) \end{bmatrix} = \begin{bmatrix} 1-p_1 & p_1 \\ p_1 & 1-p_1 \end{bmatrix}$$

In this state we introduce a road rotation rate ω_c that is used to compute the angle ϕ of the road reference system:

State: $[x, y, \theta, \delta, \phi]$

State change rate: $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\delta}, \dot{\phi}]$

Most of the equation remain the same, except:

$\dot{\phi} = \omega_c$

$\dot{\theta} = \frac{v}{R} - \omega_c$

VEHICLE INSERTION

Geometric random variable to represent when a car enters/exit the road in front of the agent:

$C_2 \rightarrow P(C_2 = K) = (1-p_2)^{K-1} \cdot p_2$

Always inserted at a distance d_0 sufficiently long.
Gaussian random variable to represent car speed:

$v_1 \rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

With $\mu = l$ where l is the road speed limit.

Introduce a new state S_F different from the previously discussed S_R with transition probabilities:

$$P = \begin{bmatrix} P(R\to R) & P(R\to F) \\ P(F\to R) & P(F\to F) \end{bmatrix} = \begin{bmatrix} 1-p_2 & p_2 \\ p_2 & 1-p_2 \end{bmatrix}$$

In this state we introduce the next vehicle speed v_f and a distance from it d:

State: $[x, y, \theta, \delta, d]$

State change rate: $[\dot{x}, \dot{y}, \dot{\theta}, \dot{\delta}, \dot{d}]$

Most of the equation remain the same, except:

$\dot{d} = v_f - v_x$

REWARD FUNCTION

The reward function must include:

- A term that rewards the agent for being in the center of the road:

  $r_1 = A e^{-|x|}$

- A term that rewards the agent for going close to the speed limit:

  $r_2 = -B|l - v_x|$

- A term that rewards the agent for respecting the safety distance from the next vehicle:

  $g(v) = \left(\frac{v}{10}\right)^2$

  $r_3 = -C|d - g(v_x)| = -C\left|d - \left(\frac{v_x}{10}\right)^2\right|$

The reward function is:

$r = r_1 + r_2 + r_3$

COST FUNCTION

We introduce 2 cost functions:

- A cost function that is used to mantain the car inside the limit:

  $c_1 = \begin{cases} 0 & \text{If } -L_r \le y \le L_r \\ 1 & \text{otherwise} \end{cases}$

- A cost function that is used to mantain the car outside the minimum safety distance:

  $c_2 = \begin{cases} 0 & \text{If } d > g(v_x) \\ 1 & \text{otherwise} \end{cases}$

S_R = rettilineo in strada
S_C = curva in strada
S_X = fuori strada
S_{FR} = macchina davanti (come rettilineo)
S_{FC} = macchina davanti (come curva)

S_I = rischio incidente

R→R	R→C	R→X	R→FR	R→FC	R→I
C→R	C→C	C→X	C→FR	C→FC	C→I
X→R	X→C	X→X	X→FR	X→FC	X→I
FR→R	FR→C	FR→X	FR→FR	FR→FC	FR→I
FC→R	FC→C	FC→X	FC→FR	FC→FC	FC→I
I→R	I→C	I→X	I→FR	I→FC	I→I

p_1 = probabilità di curvare
p_2 = probabilità di smettere di curvare
p_3 = probabilità di immissione/emissione

$$\begin{bmatrix} (1-p_1)(1-p_3) & p_1 & ? & p_3 & 0 & 0 \\ p_2 (1-p_1)? & (1-p_2) & ? & 0 & 0 & 0 \\ p_3 & p_1 ? & ? & 0 & 0 & ? \\ 0 & 0 & ? & (1-p_1)(1-p_3) & p_1 & ? \\ 0 & 0 & ? & p_2 ? & (1-p_2) ? & ? \\ 0 & 0 & ? & & & ? \end{bmatrix}$$

Valore di y_{t+1}

$y_{t+1} = y_t + v_t\sin(\delta_t + \theta_t)\Delta t$

Introduciamo rumore:

$\delta_t' \sim \mathcal{N}(\mu, \sigma) \leftarrow$ variabile aleatoria normale

Probabilità di uscire di strada date:

$a_t = [v_t, \delta_t]$

$s_t \in \{S_R, S_C, S_{FR}, S_{FC}, S_X\}$

$P(|y_{t+1}| > L_{max}) = P(y_{t+1} > L_{max}) + P(y_{t+1} < -L_{max})$

$P(y_{t+1} > L_{max}) = P(y_t + v_t\sin(\delta_t' + \theta_t)\Delta t > L_{max}) =$

$= P\left(\sin(\delta_t' + \theta_t) > \frac{L_{max} - y_t}{v_t\Delta t}\right) =$

$= P\left(\delta_t' + \theta_t > \arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right)\right) =$

$= P\left(\delta_t' > \arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \theta_t\right) =$

normalizza

$= P\left(\delta_t' > \frac{\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \theta_t - \delta_t}{\frac{v_t}{K}}\right)$

Valore di d_{t+1}:

$d_{t+1} = d_t + [v_f - v_t\cos(\delta_t' + \theta_t)]\Delta t$

~~Introduciamo rumore:~~

$\cancel{v_f \sim \mathcal{N}(l, \mu)}$

Probabilità di entrare nella zona rischio incidente:

$a_t = [v_t, \delta_t]$

$s_t \in \{S_{FR}, S_{FC}\}$ si incula tutto perché ci sono 2 variabili aleatorie

$P(d_{t+1} < g(v_t)) = P\left([v_f + v_t\cos(\delta_t' + \theta_t)]\Delta t > \left(\frac{v_t}{10}\right)^2\right) =$

$l = 90$ Km/h $= v_f$
$L_{max} = 2$ m
$K = 10$

$p_1 = 5\%$
$p_2 = 90\%$
$p_3 = 10\%$

$C \sim \mathcal{N}(15, 10)$

$\Delta t = 1$ s

$\delta_{rt} \sim \mathcal{N}(0, v_t)$

Suppongo:

$v_t = l = 90$ Km/h $= 25$ m/s
$y_t = 0$ m
$\theta_t = 0°$
$\delta_t = 30°$
$d_t = 40$ m

$a = [v_t, \delta_t] \mid v_t \in \{50, 90, 130\}, \delta_t \in \{30, 60, 90\}$

$y_{t+1} = y_t + v_t\sin(\delta_t + \delta_{rt} + \theta_t)\Delta t$

$|y_{t+1}| > L_{max}$

$y_{t+1} > L_{max}$

$y_t + v_t\sin(\delta_t + \delta_{rt} + \theta_t)\Delta t > L_{max}$

$\delta_t + \delta_{rt} + \theta_t > \arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right)$

$\delta_{rt} > \arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t = -25.411$

Normalizzando: $\frac{-25.411}{v_t} = -1.01644$

$y_{t+1} < -L_{max}$

$y_t + v_t\sin(\delta_t + \delta_{rt} + \theta_t)\Delta t < -L_{max}$

$\delta_t + \delta_{rt} + \theta_t > \arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right)$

$\delta_{rt} < \arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t = -34.589$

Normalizzando: $\frac{-34.589}{v_t} = -1.38356$

Probabilità di uscire di strada:

$S \rightarrow S_X(v_t, \delta_t) = 1 - \phi\left(\frac{\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t}{v_t}\right) + \phi\left(\frac{\arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t}{v_t}\right)$

$d_{t+1} = d_t + [v_f - v_t\cos(\delta_t + \delta_{rt} + \theta_t)]\Delta t < g(v_t)$

$d_t + [v_f - v_t\cos(\delta_t + \delta_{rt} + \theta_t)]\Delta t < \left(\frac{v_t}{10}\right)^2$

$-v_t\cos(\delta_t + \delta_{rt} + \theta_t)\Delta t < \left(\frac{v_t}{10}\right)^2 - d_t - v_f\Delta t$

$v_t\cos(\delta_t + \delta_{rt} + \theta_t) > \frac{d_t - v_t^2}{100\Delta t} + v_f$

$\cos(\delta_t + \delta_{rt} + \theta_t) > \frac{1}{v_t}\left[\frac{d_t - v_t^2}{100\Delta t} + v_f\right]$

$\delta_t + \delta_{rt} + \theta_t > \arccos\left(\frac{1}{v_t}\left[\frac{d_t - v_t^2}{100\Delta t} + v_f\right]\right)$

$\delta_{rt} > \arccos\left(\frac{1}{v_t}\left[\frac{d_t - v_t^2}{100\Delta t} + v_f\right]\right) - \delta_t - \theta_t$

$\delta_{rt} > \arccos\left(\frac{1}{v_t}\left[\frac{d_t - v_t^2}{100\Delta t} + v_f\right]\right) - \delta_t - \theta_t = 10.003$

Normalizzando: $\frac{10.003}{v_t} = 0.40012$

Probabilità di entrare nella safety distance:

$S \rightarrow S_I(v_t, \delta_t) = 1 - \phi\left(\frac{1}{v_t}\left\{\arccos\left(\frac{1}{v_t}\left[\frac{d_t - v_t^2}{100\Delta t} + v_f\right]\right) - \delta_t - \theta_t\right\}\right)$

S_G = stato "buono"
S_X = stato fuori strada
S_I = stato rischio incidente
S_{XI} = stato fuori strada e rischio incidente

$\delta_{t_r} \sim \mathcal{N}(0, v_t)$ rumore sulla sterzata

$\theta_{t_c} \sim \mathcal{N}(0, \sigma)$ angolo della curva

p_1: probabilità di curvare

$g(v_t) = \frac{v_t}{2}$

p_2: probabilità di avere un veicolo davanti

- Probabilità di uscire di strada senza curva:

$y_{t+1} = y_t + v_t\sin(\delta_t + \delta_{t_r} + \theta_t)\Delta t > L_{max}$

$\sin(\delta_t + \delta_{t_r} + \theta_t) > \frac{L_{max} - y_t}{v_t\Delta t}$

$\delta_{t_r} > \arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t$

$P\left(\delta_{t_r} > \frac{1}{v_t}\left[\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) = 1 - \phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)$

$y_t + v_t\sin(\delta_t + \delta_{t_r} + \theta_t)\Delta t < -L_{max}$

$\sin(\delta_t + \delta_{t_r} + \theta_t) < \frac{-L_{max} - y_t}{v_t\Delta t}$

$\delta_{t_r} < \arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t$

$P\left(\delta_{t_r} < \frac{1}{v_t}\left[\arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) = \phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)$

$P_1 = (1-p_1)\left[1 - \phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) + \phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)\right]$

- Probabilità di uscire di strada con curva:

$\delta_{t_r} + \theta_{t_c} > \arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t$

$P\left(\delta_{t_r} + \theta_{t_c} > \frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) = 1 - \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)$

$\delta_{t_r} + \theta_{t_c} < \arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t$

$P\left(\delta_{t_r} + \theta_{t_c} < \frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) = \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)$

$P_2 = p_1\left[1 - \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) + \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{-L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)\right]$

- Probabilità di uscire di strada:

$P(s' = S_X \mid s = S_G, a = [v_t, \delta_t]) = P_1 + P_2 = P(s' = S_X \mid s = S_X, a = [v_t, \delta_t])$

- Probabilità di rischiare un incidente NON in curva:

$d_{t+1} = d_t + [v_f - v_t\cos(\delta_t + \delta_{t_r} + \theta_t)]\Delta t < g(v_t)$

$d_t + [v_f - v_t\cos(\delta_t + \delta_{t_r} + \theta_t)]\Delta t < \frac{v_t}{2}$

$v_f - v_t\cos(\delta_t + \delta_{t_r} + \theta_t) < \frac{v_t}{2\Delta t} - \frac{d_t}{\Delta t}$

$v_t\cos(\delta_t + \delta_{t_r} + \theta_t) > v_f - \frac{v_t}{2\Delta t} + \frac{d_t}{\Delta t}$

$\cos(\delta_t + \delta_{t_r} + \theta_t) > \frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)$

$\delta_{t_r} < \arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t$

$P\left(\delta_{t_r} < \frac{1}{v_t}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right) = \phi\left(\frac{1}{v_t}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right)$

$P_3 = p_2(1-p_1)\,\phi\left(\frac{1}{v_t}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right)$

- Probabilità di rischiare un incidente in curva:

$\delta_{t_r} + \theta_{t_c} < \arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t$

$P\left(\delta_{rt} + \theta_{t_c} < \frac{1}{\sqrt{v_t + \sigma}}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right) = \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right)$

$P_4 = p_2 p_1\,\phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{7.2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right)]$

- Probabilità di rischiare un incidente:

$P(s' = S_I \mid s = S_G, a = [v_t, \delta_t]) = P_3 + P_4$

- Probabilità di uscire di strada e rischiare un incidente:

$P(s' = S_{XI} \mid s = S_G, a = [v_t, \delta_t]) = (P_1 + P_2)(P_3 + P_4)$

$P(s' = S_G \mid s = S_i, a = [v_t, \delta_t]) = (1 - P_1 + P_2)(1 - P_3 + P_4)$ $\quad i \in \{G, X, I, XI\}$

$P(s' = S_X \mid s = S_i, a = [v_t, \delta_t]) = (P_1 + P_2)(1 - P_3 + P_4)$

$P(s' = S_I \mid s = S_i, a = [v_t, \delta_t]) = (1 - P_1 + P_2)(P_3 + P_4)$

$P(s' = S_{XI} \mid s = S_i, a = [v_t, \delta_t]) = (P_1 + P_2)(P_3 + P_4)$

$\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2}\right) > 1$

$v_f + \frac{2d_t - v_t}{2} > v_t$

$v_f + d_t > v_t + \frac{v_t}{2}$

$v_t\left(1 + \frac{1}{2}\right) < v_f + d_t$

$\frac{3}{2}v_t < v_f + d_t$

$v_t < \frac{2}{3}(v_f + d_t) \leftarrow$ verificare x domani

$\frac{1}{v_t}\left(v_f + \frac{2d_t - 3.6v_t}{2}\right) > 1$

$v_f + d_t - 1.8v_t > v_t$

$2.8v_t < v_f + d_t$

$v_t < \frac{v_f + d_t}{2.8}$

$\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2}\right) < -1$

$v_f + d_t - \frac{v_t}{2} < -v_t$

$\frac{v_t}{2} - v_t > v_f + d_t$

$v_t\left(\frac{1}{2} - 1\right) > v_f + d_t$

$-\frac{1}{2}v_t > v_f + d_t$

$v_t < -2(v_f + d_t)$ IMPOSSIBILE con $v_t > 0$

- Reward function:

$r_{center} = \begin{cases} r_1 & \Rightarrow |y_{t+1}| < \frac{1}{2}L_{max} \\ r_2 & \Rightarrow |y_{t+1}| < \frac{1}{4}L_{max} \end{cases}$

$|y_{t+1}| < \frac{1}{2}L_{max} \iff \frac{1}{2}L_{max} > y_{t+1} > -\frac{1}{2}L_{max}$

$r_1 \begin{cases} (1-p_1)\left[\phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{\frac{1}{2}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) - \phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{-\frac{1}{2}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)\right] \\ p_1\left[\phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{\frac{1}{2}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) - \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{-\frac{1}{2}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)\right] \end{cases}$

$|y_{t+1}| < \frac{1}{4}L_{max} \iff \frac{1}{4}L_{max} > y_{t+1} > -\frac{1}{4}L_{max}$

$r_2 \begin{cases} (1-p_1)\left[\phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{\frac{1}{4}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) - \phi\left(\frac{1}{v_t}\left[\arcsin\left(\frac{\frac{1}{4}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)\right] \\ p_1\left[\phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{\frac{1}{4}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right) - \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left[\arcsin\left(\frac{-\frac{1}{4}L_{max} - y_t}{v_t\Delta t}\right) - \delta_t - \theta_t\right]\right)\right] \end{cases}$

$r_{speed} = -|l - v_t|$

$r_{distance} = \begin{cases} r_3 & \Rightarrow d_{t+1} < g(v_t) \\ r_4 & \Rightarrow d_{t+1} > g(v_t) \end{cases}$

$r_3 \begin{cases} p_2(1-p_1)\,\phi\left(\frac{1}{v_t}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right) \\ p_2 p_1\,\phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{7.2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right)] \end{cases}$

$r_4 \begin{cases} p_2(1-p_1)\left[1 - \phi\left(\frac{1}{v_t}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right)\right] \\ p_2 p_1\left[1 - \phi\left(\frac{1}{\sqrt{v_t + \sigma}}\left\{\arccos\left[\frac{1}{v_t}\left(v_f + \frac{2d_t - v_t}{7.2\Delta t}\right)\right] - \delta_t - \theta_t\right\}\right)\right] \end{cases}$