Politecnico di Milano
School of Industrial and Information Engineering
Master of Science in Mathematical Engineering

CRYPTOASSETS IN ASSET ALLOCATION:

a new Asset Class

Supervisor: Prof. Daniele Marazzina
Prof. Ferdinando M. Ametrano

Master thesis by: Matteo Avigni
ID: 883093

Academic year 2018-2019

Abstract

Cryptoassets are becoming increasingly popular in the financial markets as a new investment instrument. This thesis aims to study their use in this sense from several points of view. After deepening on the state of art, this work analyzes cryptoassets properties such as returns and volatility in relation to the properties of common financial instruments such as equity, bonds, currencies and commodities represented by financial indices. Then the correlations are studied both within the world of cryptoassets and with all the other standard assets. What catches the eye is the fact that cryptoassets are uncorrelated to the market but strongly correlated to each other. This suggests to consider them as a new asset class. Finally an analysis of the optimal allocation of a portfolio composed with and without cryptoassets is done with the Markowitz model. The results show that it is useful in terms of diversification to include cryptoassets in a portfolio but having multiple cryptoassets or just bitcoins doesn't make much difference.

Ringraziamenti

Ci tengo a ringraziare tutte le persone che mi hanno accompagnato in questi anni di studio e che mi hanno permesso di raggiungere questo importante traguardo.

Ringrazio in particolare il professor Ferdinando Ametrano, che per primo mi ha introdotto all'appassionante tema di Bitcoin; Paolo Mazzocchi per la pazienza con la quale mi ha seguito nel lavoro di tesi; i professori che ho avuto lungo il mio percorso formativo per aver contribuito alla mia crescita.

Ringrazio la mia famiglia per il sostegno e i sacrifici fatti, e per aver sempre creduto in me.

Grazie a Eleonora per l'energia con cui mi è stata accanto, anche nei momenti di difficoltà.

Un grazie a tutti gli amici con cui ho condiviso lo studio e non solo, per essermi stati di ispirazione.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

Cryptoassets are a type of private asset that depend primarily on cryptography and distributed ledger technology as part of their perceived or inherent value. Since the launch of Bitcoin on the 3^{rd} of January 2009 a wide range of cryptoassets have been established, each one with slightly different characteristics.

Although the number of existing cryptoassets is vast, the biggest share of market in terms of capitalization and exchanged volumes is detained by only few of them. In the *Crypto Index* [Digital Gold Institute, 2019] daily exchanged volumes have been collected for 12 major cryptoassets from several exchanges. The collection of data has considered only the crytoassets which satisfy some consistencty criteria. Using this dataset one can plot the 1-month rolling average of the percentage of exchanged volumes for each of them, where 100% is the sum of the volume exchanged for all of them. This configuration is shown in figure 1.1: the black line represents the sum of the percentage of Bitcoin, Ethereum, Ripple and Litecoin. This line lies always above 90%. This implies that, in the period between January 2016 and May 2019, considering those cryptoassets allows the analysis produced in this thesis to cover at least the 90% of the market.

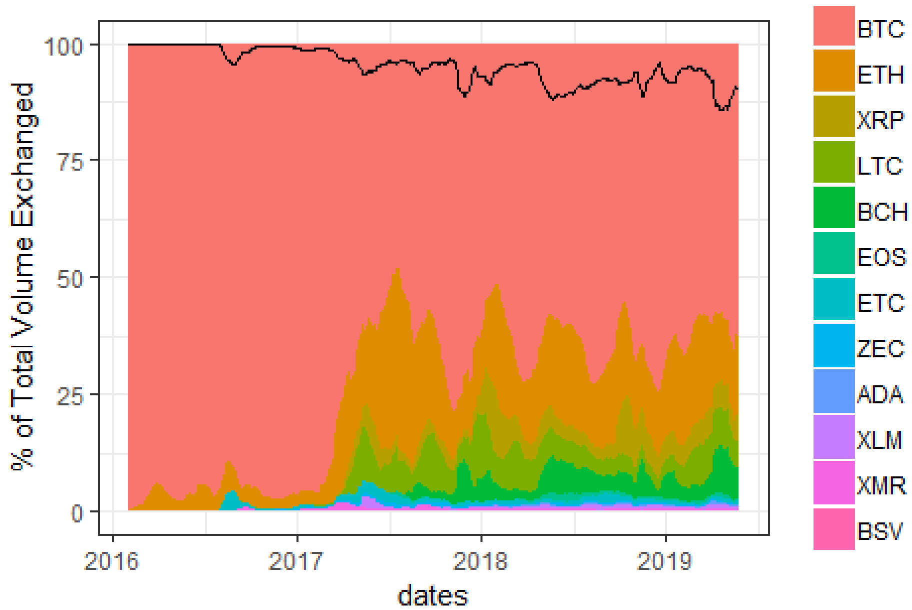


Figure 1.1: 1-month rolling average percentage of exchanged volumes by cryptoasset

Bitcoin, Ethereum, Ripple and Litecoin are, therefore, the cryptoassets this thesis is focusing on and below are rapresented their characteristics, similarities and differences.

Bitcoin (BTC) was presented in [Nakamoto, 2009] as a peer-to-peer protocol for electronic cash that would allow online payments to be sent directly from one party to another without going through a financial institution. Initially it was only used by a niche of people in online cryptology forums to experiment with the transaction protocol. It took a few years for Bitcoin to gain public notoriety, while the price kept rising and having quick crashes. In particular, Bitcoin generated a lot of buzz in 2017, a year that registered the price increase from 998$ to 13,412$ in January 2018 with an all-time high of 19,666$ on the 17^{th} of December. As usually seen during any of the history's gold rushes, many people joined the trend and tried to jump on the train of quick and easy money and were let down when the price collapsed to a level of six thousand dollars in 2018 and has lately stabilized around three thousands. Recently there has been again more and more talks about Bitcoin and cryptoassets in general due to a group of important payment firms that, together with Facebook, have announced a project of a new global cryptocurrency: Libra. Setting aside the mere numerical value of the price, Bitcoin was the first protocol to solve the problem of double-spending without the need for a centralized party: bitcoins can be transferred but not duplicated, as they only exist as validated transactions in the distributed blockchain. In the same way that e-mail substituted post mail, Wikipedia and other knowledge-based website outdated paper encyclopedias, music and film streaming services are becoming the new user friendly experience for the two industries, Bitcoin presents itself as a system to exchange wealth between users without the need for banks or other trusted third parties. Furthermore, Bitcoin is the first digital currency to achieve scarcity in the digital realm: its monetary policy based on deterministic supply, mimics the progressive scarcity of gold. It is this deterministic supply that make Bitcoin a suitable long-term investment, since it prevents arbitrary increase in its total available amount, unlike what happens with fiat money. For these reasons I believe that Bitcoin can be digital gold with an embedded secure network and its characteristics make it resemble more closely to a cryptoasset rather than a cryptocurrency. Even though there are a number of supporters of this idea, for instance in [Dyhrberg, 2016] it is shown that Bitcoin possesses the same hedging abilities of gold, there is yet no general consensus on the matter and different studies get to the opposite result: see for example [Klein et al., 2018]. Bitcoin has been called many names: a bubble, a Ponzi scheme, but also defined as sound money and store of value. I agree with the latter and believe that if Bitcoin is true digital gold, then its value will express the huge potential that has been so far limited by scepticism and misunderstandings.

Ethereum (ETH) is the name of a blockchain company that has created the digital token ether. But Ethereum and ether are now used interchangeably to refer to the cryptocurrency. Ether is backed by a blockchain, much like bitcoin, but the technology is slightly different and aimed at a specific use case: smart contracts. The cryptocurrency ether is required by developers who want to build *apps* on the Ethereum blockchain and by users who want access to interact with the smart contracts on the platform.

Ripple (XRP) markets itself as a cross-border payments solution for large financial institutions based on blockchain technology. At the moment, an international payment may take a few days to be made with a very high cost. A challenge for banks is high-volume, but low-value, transactions. These can often be expensive and unprofitable for banks because it takes a lot of effort to move the money and the percentage cut won't be as high as for a larger transaction. Ripple is trying to solve this problem via its technology. The Ripple digital currency, known as XRP, can be used by enterprise to get instant liquidity needed in a high-value transaction, without having to pay fees. XRP acts as a bridge between fiat currencies during a transaction.

Ripple said transactions in XRP can be settled in four seconds, faster than any major cryptocurrency right now.

Litecoin (LTC) is probably bitcoin's closest rival in terms of the use case. Founder Charlie Lee has, on numerous occasions, said that this cryptocurrency can be used for payments because it's faster than bitcoin. Litecoin transactions take just over two minutes to go through, compared to an average of around nearly 300 minutes for bitcoin. There is a limited supply of 84 million litecoins, compared to 21 million bitcoins.

This work intends to study the correlation between the crypto-world, represented by the four major cryptoassets in terms of market capitalization, and other types of standard assets. In particular, I tried to understand if there is significant difference among these four cryptoassets in terms of correlation with the market. Then, diversification property of cryptoassets added to a portfolio is explored by computing the optimal allocation for different levels of risk and expected return in a Markowitz mean-variance framework.

In literature there are several papers about the inclusion of Bitcoin in an investment portfolio, for example [Bouri et al., 2017] where the authors arrive to the conclusion that Bitcoin acts well in terms of diversification, less as a hedge or safe haven. In [Liu, 2018] the authors analysed a portfolio composed only by ten major cryptoassets and found out that diversifying between crypto yield higher returns with lower risk with respect to single cryptoassets. Then in [Corbet et al., 2018] the authors analysed the role of btc, ltc and xrp in an investment portfolio with other indexes. It turned out that cryptoassets can be considered as a new investment asset class since they are interconnected with each other and have similar patterns of connectedness with other asset classes.

The present thesis has been written during the author's collaboration with the Digital Gold Institute (https://www.dgi.io/), a research and development center focused on teaching, consulting and advising about scarcity in the digital domain (Bitcoin and cryptoassets) and the underlying blockchain technology.

1.1 Thesis structure

This work is structured into 6 chapters.

Chapter 2 presents the state of the art in cryptoassets usage for diversification in asset allocation.
In chapter 3 a study on the empirical correlation between the assets taken into considerations is done. Here is also checked the significance of each correlation value by performing two statistical tests, Pearson's t-test and a permutation test. Finally, there's a deeper analysis of correlations between cryptoassets, which is done by investigating the rolling correlations and their significance.
In chapter 4 some consideration about cryptoassets returns are done.
Chapter 5 analyses the optimal allocation of a portfolio composed of cryptoassets and "classic" instruments
Chapter 6 concludes this work and sums up the main results. It also includes some final remarks to this study.

1.2 Dataset Presentation

The dataset on which we focus our study contains 886 observations of the prices of 20 assets valued daily (excluding holidays and weekends) from the 1^{st} of January 2016 till the 24^{th} of May 2019 (some data provided by Bloomberg and others, the ones related to the cryptoassets, by Coinmarketcap). The assets we included in our analysis are grouped into five different classes, as explained in the following list (in brackets we indicate the shortened name that is used in the tables and graphs):

1. Cryptoassets:
    - Bitcoin (btc): Value of a single bitcoin, quoted in dollars.
    - Ethereum (eth): Value of a single ether, quoted in dollars.
    - Litecoin (ltc): Value of a single litecoin, quoted in dollars.
    - Ripple (xrp): Value of a single ripple, quoted in dollars.
2. Stock Indexes:
    - S&P500 (sp500): American stock market index based on 500 large company with stock listed either on the NYSE or NASDAQ.
    - EURO STOXX 50 (eurostoxx): equity index of eurozone stocks, covering 50 stocks from 11 eurozone countries.
    - MSCI BRIC (bric): market cap weighted index designed to measure the equity market performance across the emerging country indexes of Brazil, Russia, India and China.

- NASDAQ (nasdaq): market cap weighted index including all NASDAQ tiers (Global Select, Global Market and Capital Market).

3. Bond Indexes:

   - BBG Pan European (bond europe): Bloomberg Barclays Pan European Aggregate Index that tracks fixed-rate, investment-grade securities issued in different European currencies.
   - BBG Pan US (bond us): Bloomberg Barclays US Aggregate Bond Index, a benchmark that measures investment grade, US dollar-denominated, fixed-rate taxable bond market.
   - BBG Pan EurAgg (bond eur): similar to the Pan European but it only considers securities issued in Euros.

4. Currenncies

   - EUR/USD (eur): spot value of one Euro in US dollars.
   - GBP/USD (gbp): spot value of one British Pound in US dollars.
   - CHF/USD (chf): spot value of one Swiss Franc in US dollars.
   - JPY/USD (jpy): spot value of one Japanese Yen in US dollars.

5. Commodities:

   - Gold (gold): price of gold measured in USD/Oz.
   - WTI (wti): price of crude oil used as benchmark in oil pricing and as the underlying commodity in the NYMEX oil future contracts.
   - Grain (grain): S&P GPSCI index that measures the performance of the grain commodity market.
   - Metals (metal): S&P GSCI Industrial Metals index that measures the movements of industrial metal prices including aluminium, copper, zinc, nickel and lead.

Chapter 2

Present literature

In literature we still can find few studies about the correlations inside the world of cryptoassets.

Cryptoassets market has grown continuously through the years: since the born of Bitcoin a lot of new coins have been launched and all of them have been increasingly exchanged during time. Figure 2.1 shows the market capitalization of all the cryptoasset taken from coinmarketcap.com.

Figure 2.1: Market Cap of the cryptocurrencys (source: coinmarketcap.com)

The value reached between the last years suggests that they may be seen as a new category of investment assets.

For this reason one may wonder if there are properties which characterize this new asset-class. As the bond market is considered a hedging against bad times for stock markets, or the gold is considered a safe haven where to keep money safe from volatility it's interesting to investigate if this kind of properties hold also for cryptoassets.

2.1 Asset Class

In [Corbet et al., 2018] there is one of the first attempt to analyse these properties. Authors considered the returns of Bitcoin, Litecoin and Ripple to represent cryptoassets and other indexes to represent all the other asset classes: MSC GSCI Total Returns Index, the US$ Broad Exchange Rate, the SP500 Index and the COMEX closing gold price, VIX and the Markit ITTR110 index. All the data where collected in a period from January 2013 to July 2017, that is just before the well-known rally of bitcoin that pushed its price near to 20'000$. Authors focused on studying spillover indexes both in time and frequency domain between all the instruments in the dataset. They found that cryptoassets are highly connected to each other and disconnected from mainstream assets. Their results also support the position that cryptocurrency market is a new investment asset class, since they are interconnected with each other and have similar patterns of connectedness with other asset classes.

It's still necessary to go deeper in the analysis: cryptoassets can be considered a new asset-class but it's necessary to keep in mind that this is a "young" market and correlations may vary over time.

For example, in [Liu, 2018] a further study on the diversification between cryptoassets is performed. The dataset considered includes Bitcoin, Ethereum, Ripple, Litecoin, Stellar, Monero, Dash, Tether, NEM and Verge, and it covers the period from 07-Aug-15 to 09-Apr-18, that contains the rally previously stated and also the come-back (to 7'000$ for bitcoin).

By using this time window the authors observe that the highest correlation is between Bitcoin and Litecoin and it's just 0.52. Globally they get very low correlations between cryptocurrencies as shown in figure 2.2:

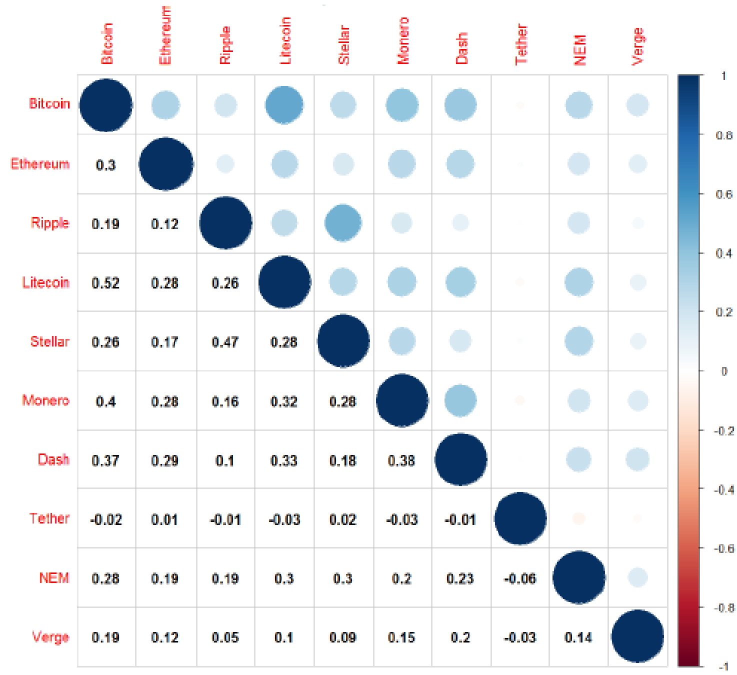


Figure 2.2: Correlation Matrix for cryptocurrencies, source: [Liu, 2018]

Then, authors proceed by performing the efficient frontier of a portfolio composed with all of these cryptoassets and then by looking for the optimal allocation. To get it, they use different methods that are summarized in table 2.1:

Model	Objective Function	Type	Constraints
$1/N - Rule$	-	-	-
Minimum Variance	$w'\Sigma w$	Minimize	$w'\mathbb{1}w = 1, w \geq 0$
Risk Parity	$\sum_{i=1}^{N} [w_i (\Sigma w)_i - w'\Sigma w/N]$	Minimize	$w'\mathbb{1}w = 1, w \geq 0$
Markowitz	$w'\Sigma w$	Minimize	$w'\mu \geq \mu_0, w'\mathbb{1}w = 1, w \geq 0$
Maximum Sharpe	$w'\mu/\sqrt{w'\Sigma w}$	Maximize	$w'\mathbb{1}w = 1, w \geq 0$
Maximum Utility	$w'\mu - \frac{\gamma}{2}w'\Sigma w$	Maximize	$w'\mathbb{1}w = 1, w \geq 0$

Table 2.1: Models used in the paper

μ_0 in the Markowitz model is set to be the corresponding mean under 1/N-Rule, and the risk aversion parameter γ is 1 in the maximum utility model. The authors show that almost any of that models can beat the 1/N-Rule in terms of Sharp ratio. This result is, of course, due to the low correlations between cryptoassets but, as we said, they depend on the time period one considers. Further considerations on these results are discussed in chapter 3.

2.2 Optimal Allocation

More research has been done about the usage of bitcoin in an investment portfolio to diversify and reach higher returns. A contribution has been given in [Vianello, 2018].

In that work the author starts by studying the correlation of Bitcoin with some index representative of the market. Then, given that Bitcoin turns out to be uncorrelated with the market, he performs optimal portfolio allocation analyses to investigate its diversification properties, both with Markowitz mean-variance optimization and with the CVaR as portfolio risk measure. Evidence shows that allocating a small percentage of wealth in the digital assets proves to be extremely beneficial in terms of lowering the risk and increasing the expected returns.

The dataset considered is composed by the same instruments we consider in our thesis in order to explore the relation of Bitcoin with all the possible asset classes. Differently from us, the period he considers is from the 19^{th} of July 2010 till the 2^{nd} of November 2018.

First, the author assess the correlation of the cryptoasset with the indexes by observing the historical correlation of the daily returns.

	btc	bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eur	eur	gbp	chf	jpy	gold	wti	grain	metal
btc	1.0	1.4	4.4	4.1	3.6	1.4	-1.8	1.9	2.3	0.7	2.5	-1.1	-0.2	0.8	3.5	2.7

Table 2.2: Historical Correlations between bitcoin and other instruments, source: [Vianello, 2018]

After that he calibrate several more sophisticated models and look for the correlations in these models. The first is the Jump Diffusion model presented in 1976 by R.C. Merton where log-normal jumps are added to the simple Black&Scholes dynamics of the asset price.

$$\frac{dS_t}{S_t} = (\mu - \lambda\mu_J)\, dt + \sigma dW_t + Y_t dN, \tag{2.1}$$

The results are showed in the following table:

	btc	bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eur	eur	gbp	chf	jpy	gold	wti	grain	metal
btc	1.0	2.7	7.7	6.3	6.4	1.9	-2.4	2.7	2.7	1.2	4.4	-1.8	-0.6	1.1	4.2	4.2

Table 2.3: Correlations under Jump Diffusion model, source: [Vianello, 2018]

Then the stochastic volatility (SV) model of Heston (1993) is presented and calibrated. This model introduced a new stochastic process that accounts for the variance of the underlying price which evolves as in B&S but with a stochastic volatility term.

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_T} dW_t^S, \tag{2.2}$$

$$dV_t = k\,(\Theta - V_t)\, dt + \sigma_V \sqrt{Vt} dW_t^V, \tag{2.3}$$

	btc	bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eur	eur	gbp	chf	jpy	gold	wti	grain	metal
btc	1.0	1.8	6.3	5.9	5.4	1.9	-2.2	2.6	2.8	0.9	3.2	-1.8	-0.5	0.9	4.5	3.3

Table 2.4: Correlations under Heston model, source: [Vianello, 2018]

The last model that is presented is the Bates one, which was introduced in 1996 and it is the combination of the former two: an asset dynamics which includes jumps and is driven by a stochastic volatility.

$$\frac{dS_t}{S_t} = (\mu - \lambda\mu_J)\, dt + \sqrt{V_T} dW_t^S + Y_t dN, \tag{2.4}$$

$$dV_t = k\,(\Theta - V_t)\, dt + \sigma_V \sqrt{Vt} dW_t^V, \tag{2.5}$$

	btc	bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eur	eur	gbp	chf	jpy	gold	wti	grain	metal
btc	1.0	1.5	5.2	4.7	4.2	1.4	-2.3	2.2	3.1	0.5	2.5	-1.2	-0.1	1	3.3	2.8

Table 2.5: Correlations under Bates model, source: [Vianello, 2018]

With all these models the values for the correlations of Bitcoin with the indexes is a single digit result, almost always lower than 5%. This result is exactly in line with the empirical correlation. For simplicity we do not report the correlation matrix but all the values are very similar even calculated with different methods.

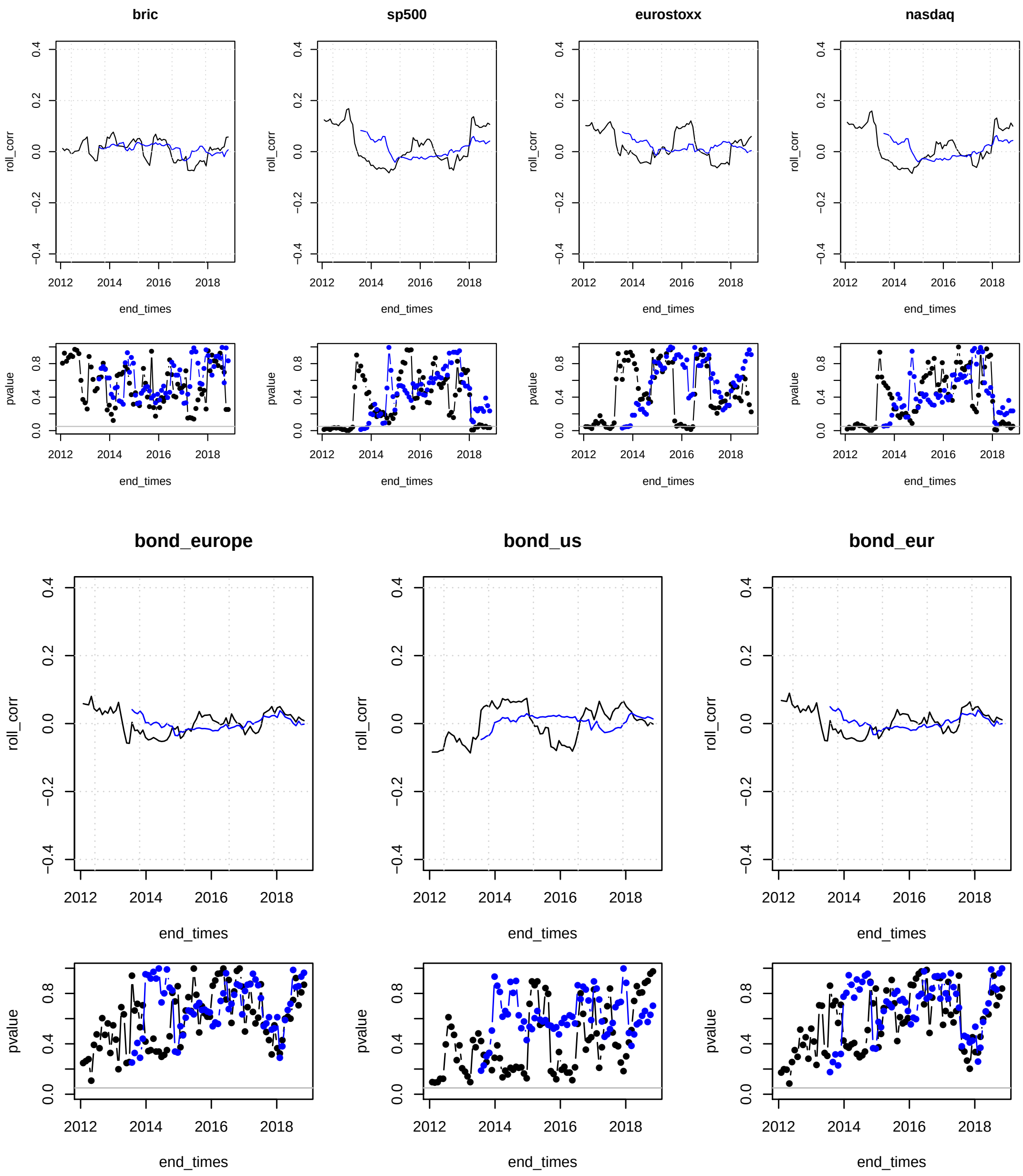
A further analysis done by the author is to compute the 18-months and 36-months rolling correlation between Bitcoin and the indexes. In figure 2.3 there are two graphs for each asset in the dataset: in the top plots levels of the rolling correlations are represented using two different colours, blue for the 3-years and black for the 18-months windows; in the bottom plots the significance of each rolling correlation through its p-value. The grey horizontal line represents the 5% level of significance.

From these pictures one can immediately see that the correlation of any asset with Bitcoin is hardly ever significantly different from zero, and when it is, its absolute level is never greater than 15% unless for a small period of time. Moreover, there is no line that is always above zero, nor below. This indicates that there is no underlying trend, whether positive or negative, and the correlation one might happen to find is only temporary.

Given these results the author try to understand the impact of including Bitcoin in an investment portfolio by drawing the efficient frontier using 2 different measures of risks: volatility and CVaR or expected shortfall.

2.2.1 Mean-Variance method

The first method used to draw the efficient frontier is the one introduced by Harry Markowitz in 1952.


bric
sp500
eurostoxx
nasdaq
bond_europe
bond_us
bond_eur
roll_corr
pvalue
end_times

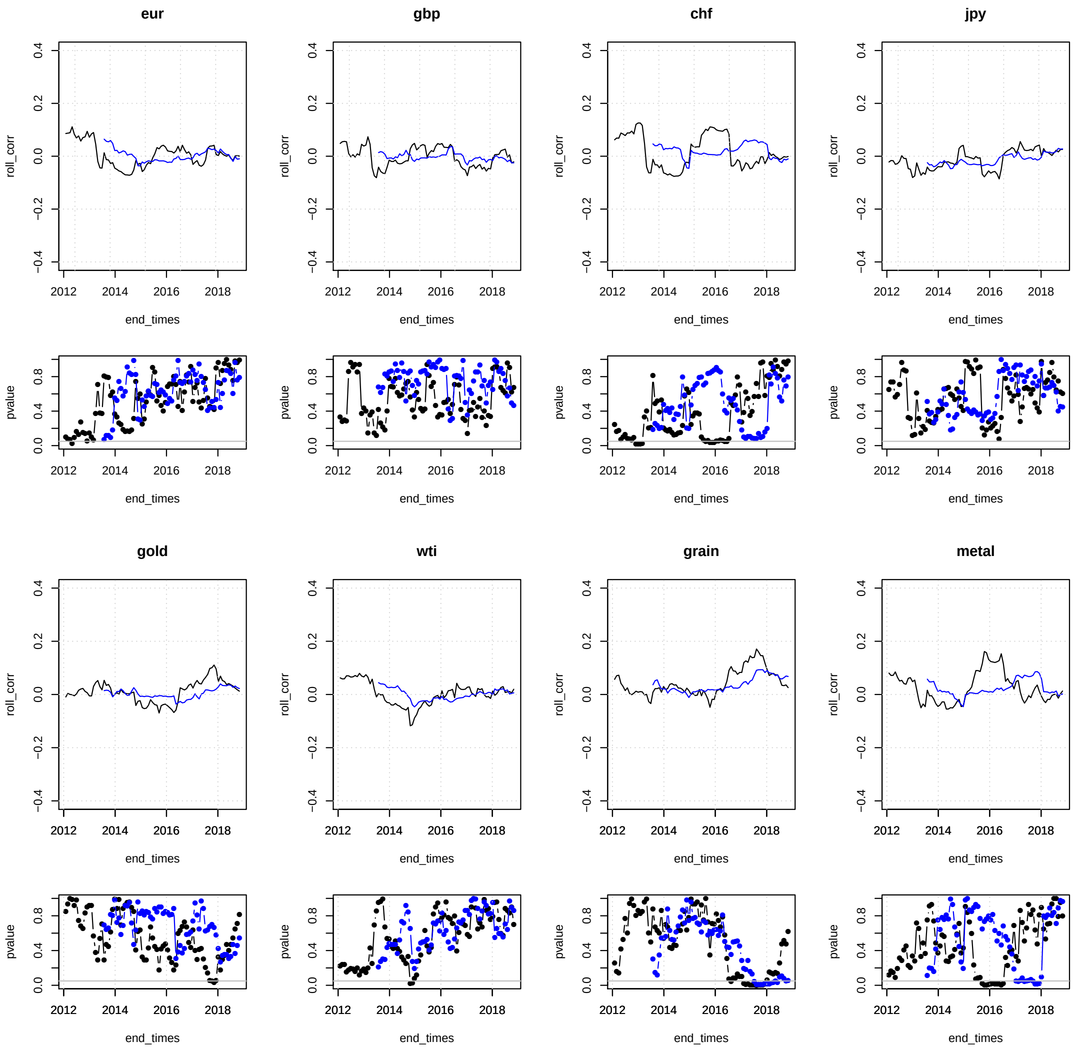


Figure 2.3: Rolling correlation between BTC and other instruments of the portfolio considered in [Vianello, 2018]

It is the first mathematical framework which take into account the diversification principle in the field of asset allocation. Its key point is that an asset's return and risk should not be assessed by itself, but rather by how it affects the overall portfolio risk and return. To do so, the variance is used as a proxy for risk. Hence it is also called mean-variance analysis.

The main assumptions that are the building blocks of this theory are the following:

1. investors are risk averse: they will always choose the less risky asset, when two assets offer the same return. At the same time, an investor wanting a higher return has to be willing to accept a higher risk. This equally holds for portfolios as a whole: given two portfolio with different risk profiles, he will choose the less risky in case of same return and the most remunerating in case of same risk;

2. portfolio return is the weighted sum of the single assets' returns: in general, $E\left[R_p t f\right] = \sum_{i=1}^{N} w_i E\left[R_i\right]$;

3. portfolio variance is a function of both the assets variances and their correlations: $V_p t f = \sum_{i=1}^{N} w_i \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i, j=1}^{N} w_i w_j \rho_{i,j} \sigma_i \sigma_j$

Now the optimization problem to be solved is:

$$\begin{aligned} \underset{\mathrm{w} \in R^N}{\text{minimize}} \quad & \sigma_{ptf}^2\left(\mathrm{w}\right) \\ \text{subject to} \quad & \mathrm{e}^T \mathrm{w} = 1, \\ & \mathrm{r}^T \mathrm{w} = r_t arget, \\ & w_i \geq 0, fori = 1...N. \end{aligned}$$

where e indicates a vector of ones and the first constraint makes sure that the sum of the weights always equals to one. This is to represent a portfolio in which all the money available is allocated in the assets we are taking into consideration. The second constraint ensures that the portfolio allocation w produces the target expected return $r_t arget$. Finally, the last constraint is in fact optional and is only used to exclude the possibility to go short on any asset.

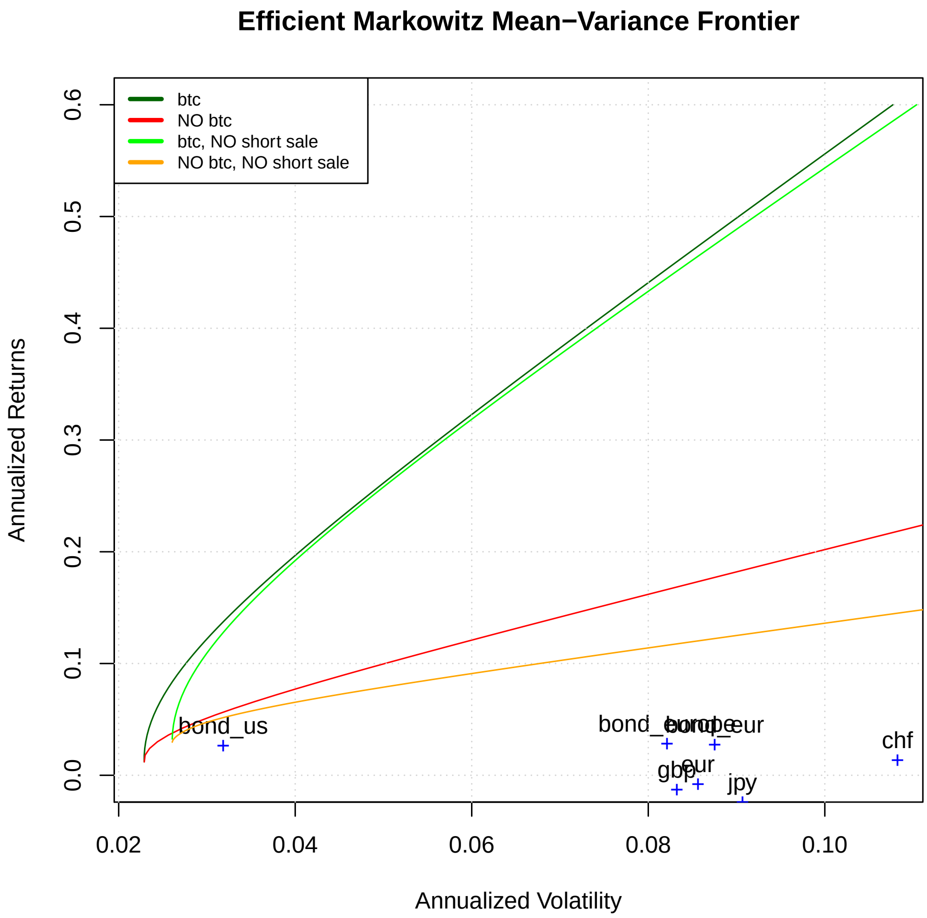


Figure 2.4: Efficient Frontier under Mean-Variance method, source: [Vianello, 2018]

In this framework, as shown n figure 2.4, the efficient frontier with Bitcoin in the portfolio is much better than the one without Bitcoin, both in the case of no-short sale constraints and in the other case.

In table 2.6 the expected annualized returns of the portfolio reported from the author for different levels of volatility:

Volatility Level	Return without Bitcoin	Return including Bitcoin
2.61%	3.00%	3.00%
2.75%	3.89%	7.70%
3.00%	4.59%	10.94%
3.25%	5.05%	13.37%
3.50%	5.43%	15.48%
3.75%	5.76%	17.41%
4.00%	6.06%	19.21%
4.25%	6.34%	20.94%
4.50%	6.61%	22.60%
4.75%	6.87%	24.21%
5.00%	7.12%	25.79%
5.25%	7.37%	27.34%
5.50%	7.61%	28.86%
5.75%	7.85%	30.37%
6.00%	8.08%	31.85%

Table 2.6: Results under Mean-Variance optimization, source: [Vianello, 2018]

2.2.2 CVaR method

Then the author studies the allocation in a slightly different framework: by using as risk measure of the portfolio the CVaR (conditional VaR).

To use this approach it is required only to change the objective function with no alterations on the constraints. The formulation of the problem become the following:

$$
\begin{aligned}
\underset{\mathrm{w}\in R^N}{\text{minimize}} \quad & PtfRisk\,(\mathrm{w}) \\
\text{subject to} \quad & \mathrm{e}^T\mathrm{w} = 1, \\
& \mathrm{r}^T\mathrm{w} = r_t arget, \\
& w_i \geq 0, for i = 1...N.
\end{aligned}
$$

where the $PtfRisk\,(\mathrm{w})$ is defined as:

$$CVaR = \frac{1}{1-\alpha}\int_{\alpha}^{1} VaR_{\gamma} d\gamma, \tag{2.6}$$

The introduction of this method is justified by the author by saying that the classic mean-variance framework, which consider the volatility as a proxy for the portfolio risk, does not distinguish between the volatility part caused by positive and negative returns. High returns yield higher volatility and so are penalized. The CVaR method solve this problem since it considers the loss distribution of the portfolio, and so just negative returns yield penalization to an asset.

The previous results are confirmed: the efficient frontier of the portfolio which includes Bitcoin is better than the one without Bitcoin also in this analytical framework.

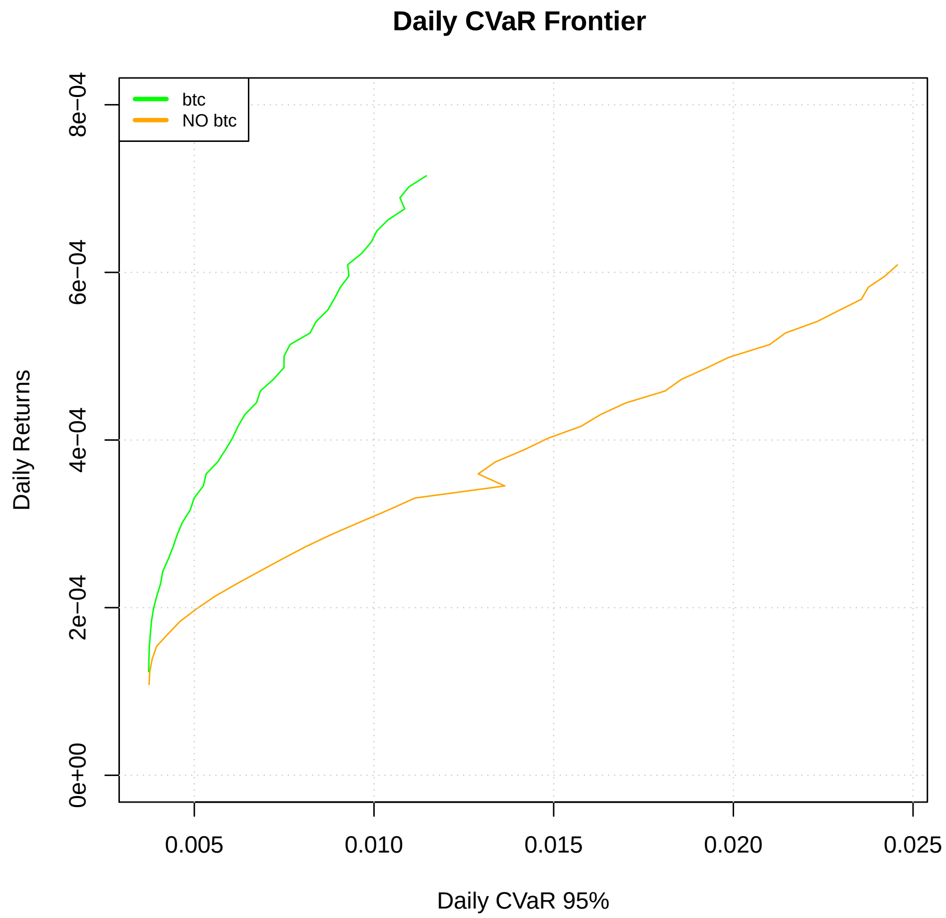


Figure 2.5: Efficient Frontier under CVaR method, source: [Vianello, 2018]

The results of this work lead to think of Bitcoin as a useful diversification instrument: it is uncorrelated to all the other instruments and it produces high returns.

Chapter 3

Correlations Analysis

A correlation analysis is needed to assess the properties of cryptoassets in an investment portfolio. From here on we refer to logarithmic returns simply as returns.

3.1 Empirical Correlations

First of all, we calculate the empirical correlation between all the instruments in the portfolio based on the empirical time series of our dataset. For this part, we will consider our data as successive samples of a N-dimensional vector in RN, where N is the number of assets:

$$\mathrm{x}_j = \begin{pmatrix} x_{1,j} \\ x_{2,j} \\ . \\ . \\ . \\ x_{N,j} \end{pmatrix}, j = 1...N_{sample} \tag{3.1}$$

Each element i of the vector x_j represents the j^{th} realization of the returns for asset i. Following basic statistics, we can now compute the sample mean of our vectors of returns as:

$$\bar{\mathrm{x}} = \frac{1}{N_{sample}} \sum_{j=1}^{N_{sample}} \mathrm{x}_j = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ . \\ . \\ . \\ \bar{x}_N \end{pmatrix} \tag{3.2}$$

where $\bar{x}_i = \frac{1}{N_{sample}} \sum_{j=1}^{N_{sample}} x_{i,j}$ is the sample mean of component i.
We then compute the sample covariance matrix through the following formula:

$$\bar{\Sigma} = \frac{1}{N_{sample} - 1} \sum_{j=1}^{N_{sample}} (\mathrm{x}_j - \bar{\mathrm{x}}) (\mathrm{x}_j - \bar{\mathrm{x}})^T \tag{3.3}$$

where $\bar{x}$ represent the sample mean of the returns just introduced. All the information needed to obtain the correlation matrix C are already included in $\bar{\Sigma}$, we only need to perform some further calculations:

$$C_{i,j} = \frac{\bar{\Sigma}_{i,j}}{\sqrt{\bar{\Sigma}_{i,i}\bar{\Sigma}_{j,j}}} \tag{3.4}$$

We have thus obtained an empirical estimate of the correlation between our assets returns. The formula of $C_{i,j}$ is often referred to as Pearson correlation coefficient, from the name of the English mathematician Karl Pearson who first formulated it.

See below the table with expected returns and volatility of returns:

	Annualized Mean Return	Annualized Mean Volatility
btc	131.09%	72.73%
eth	395.64%	120.92%
ltc	161.96%	109.69%
xrp	231.37%	129.85%
bric	9.10%	15.61%
sp500	10.27%	12.58%
eurostoxx	1.66%	15.51%
nasdaq	13.62%	15.75%
bond_europe	1.52%	2.90%
bond_us	2.85%	2.83%
bond_eur	2.08%	2.47%
eur	0.91%	7.32%
gbp	-4.18%	10.36%
chf	-0.01%	6.88%
jpy	2.81%	9.03%
gold	5.28%	11.66%
wti	14.39%	33.78%
grain	-8.70%	17.44%
metal	6.03%	15.47%

Table 3.1: Annualized Mean Returns and Variances

And then the correlation matrix:

	btc	eth	ltc	xrp	bric	sp500	eurostoxx	nasdaq
btc	100,00%							
eth	45,17%	100,00%						
ltc	58,62%	43,32%	100,00%					
xrp	34,90%	29,10%	38,51%	100,00%				
bric	0,77%	2,58%	6,52%	7,77%	100,00%			
sp500	3,16%	2,73%	5,37%	5,81%	45,87%	100,00%		
eurostox	0,71%	-2,74%	4,00%	3,98%	52,72%	54,18%	100,00%	
nasdaq	4,10%	4,02%	6,71%	5,70%	45,93%	94,63%	48,77%	100,00%
bond_eu	2,44%	2,29%	2,93%	2,49%	5,44%	2,82%	8,44%	5,71%
bond_us	-1,76%	0,39%	-3,85%	0,88%	-11,36%	-27,51%	-28,58%	-24,07%
bond_eu	3,29%	3,55%	3,09%	1,16%	2,44%	-0,26%	-0,51%	3,06%
eur	1,99%	5,37%	-3,18%	0,68%	10,76%	3,58%	-8,36%	1,67%
gbp	1,68%	-1,13%	-3,17%	0,41%	18,08%	18,99%	22,40%	16,46%
chf	0,19%	7,78%	-1,84%	0,68%	1,88%	-11,99%	-21,59%	-12,55%
jpy	2,01%	2,94%	-0,64%	-0,18%	-18,01%	-33,72%	-42,19%	-30,14%
gold	2,61%	5,19%	-0,04%	1,28%	-2,68%	-19,01%	-30,77%	-17,16%
wti	-1,15%	-6,77%	-0,63%	1,56%	29,40%	33,06%	33,24%	25,20%
grain	5,80%	1,52%	1,91%	1,06%	11,73%	7,90%	10,18%	7,27%
metal	0,82%	2,91%	5,13%	3,38%	35,38%	23,36%	24,36%	20,38%
vix	-6,99%	-6,94%	-6,35%	-8,03%	-38,26%	-78,81%	-47,75%	-75,76%

	bond_europ	bond_us	bond_eur	eur	gbp	chf	jpy	gold	wti	grain	metal	vix
btc												
eth												
ltc												
xrp												
bric												
sp500												
eurostox												
nasdaq												
bond_eu	100,00%											
bond_us	45,80%	100,00%										
bond_eu	87,19%	50,49%	100,00%									
eur	-13,73%	15,19%	-7,59%	100,00%								
gbp	15,00%	-5,42%	-12,04%	55,07%	100,00%							
chf	-2,87%	27,99%	-0,24%	78,14%	42,26%	100,00%						
jpy	15,12%	51,97%	21,30%	37,30%	5,15%	49,78%	100,00%					
gold	10,92%	42,55%	17,84%	43,84%	13,09%	46,58%	61,45%	100,00%				
wti	2,06%	-18,21%	-1,99%	2,99%	14,31%	-4,37%	-12,17%	0,78%	100,00%			
grain	0,32%	-0,14%	-0,94%	9,53%	6,13%	4,87%	3,07%	7,34%	10,81%	100,00%		
metal	-8,00%	-15,73%	-8,42%	15,74%	14,45%	7,29%	-8,83%	12,57%	28,52%	11,21%	100,00%	
vix	-6,86%	21,86%	-5,50%	-3,93%	-14,16%	11,32%	29,09%	13,88%	-25,82%	-10,52%	-16,83%	100,00%

Figure 3.1: Historical correlations matrix

There are mainly two things that we may notice. First of all, we can see very low correlation between the cryptoassets and all the other instruments: none of these correlations is double digit and almost all of them are below 5% in absolute value. On the contrary, inside the "world" of the cryptoassets, correlations are much higher.
To better analyse the first point, we proceed by performing a statistical test on the correlation values between each cryptoasset and other instruments.

3.2 Correlation Significance

We would investigate whether it is correct to assume that the cryptoasset world is uncorrelated with all the other instruments. If it will be the case this would mean that investing in cryptoassets could improve a portfolio in terms of diversification.
We will perform an hypothesis test. In particular we are interested in testing if the sample correlation coefficients are significantly different from zero or not. Both of the following tests are presented in the most general form for a sample of two variables, with their distribution correlation ρ and their sample correlation $\hat{\rho}$.

Following standard testing procedure, we specify the null hypothesis and the alternative hypothesis:

$$\mathrm{H}_0 : \rho = 0 \quad vs. \quad \mathrm{H}_1 : \rho \neq 0. \tag{3.5}$$

These will be common to both presented tests which are the *Pearson's t-test* and the *Permutation test*.

3.2.1 Pearson's t-test

Our first test is based on Student's t-distribution and the following t-statistic:

$$t = \hat{\rho}\sqrt{\frac{n-2}{1-\hat{\rho}^2}}, \tag{3.6}$$

which under the null hypothesis is distributed as a Student's t with n-2 degrees of freedom, where n stands for the cardinality of the sample. We can thus proceed by computing the relative p-value and compare it to a given level of confidence α (usually $\alpha = 95\%$). The result of the test will be deduced as follows:

- $p - value < 1 - \alpha$: we have statistical evidence to state that the correlation is significantly different from zero;
- $p - value \geq 1 - \alpha$:there is no statistical evidence to state that the correlation is significantly different from zero.

3.2.2 Permutation test

The permutation test is based on building an empirical distribution of values for the correlation by sampling different pairs of X and Y variable and then computing Pearson's correlation. Let (x_i, y_i) be the original pairs for $i = 1, \ldots, N_{sample}$. Create a new dataset (x_i, y_i^*) by replacing y_i with one of the possible N_{sample} permutations and then compute the new sample correlation for the new dataset. If this is done a large enough number of times, we obtain an empirical distribution of possible values for the correlation of x and y. From this distribution we can then obtain the $p - value$ of the test and thus get the final result in the same way as in the previous case.

3.2.3 Significance results

Here we report the first results of our analysis: in tables 3.2, 3.3, 3.4 and 3.5 there are the values and relative $p - values$, given from Pearson $t - test$ and permutation test, of the correlations between these 4 cryptoassets and the indexes.
One can immediately notices that very few correlations are above 0.05 or below -0.05 and no one is above 0.10 or below -0.10. Moreover the $p - Values$ are in general high enough to reject the alternative hypothesis and when this is not the case the correlation is still considerably low.

		bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eu
btc	Correlation	0.01	0.03	0.01	0.04	0.02	-0.02	0.03
	Pearson %	82.01	34.80	83.23	22.26	46.85	60.01	32.79
	Permutation %	81.20	35.40	81.60	25.20	42.80	63.40	33.00

		eur	gbp	chf	jpy	gold	wti	grain	metal
btc	Correlation	0.02	0.02	0.00	0.03	0.03	-0.01	0.06	0.01
	Pearson %	55.38	61.86	95.44	55.09	43.73	73.35	8.44	80.79
	Permutation %	52.60	61.40	96.80	59.40	44.80	73.80	9.00	82.40

Table 3.2: hypothesis test btc correlations

		bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eu
eth	Correlation	0.03	0.03	-0.03	0.04	0.02	0.00	0.04
	Pearson %	44.35	41.67	41.48	23.22	49.71	90.66	29.09
	Permutation %	47.40	41.60	41.20	22.20	51.00	88.80	30.80

		eur	gbp	chf	jpy	gold	wti	grain	metal
eth	Correlation	0.05	-0.01	0.08	0.03	0.05	-0.07	0.02	0.03
	Pearson %	11.04	73.73	2.06	38.20	12.31	4.41	65.20	38.74
	Permutation %	11.80	70.40	1.80	38.40	13.40	3.00	65.40	36.60

Table 3.3: hypothesis test eth correlations

		bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eu
ltc	Correlation	0.07	0.05	0.04	0.07	0.03	-0.04	0.03
	Pearson %	5.24	11.05	23.44	4.60	38.33	25.24	35.83
	Permutation %	4.40	12.20	19.60	5.40	34.00	26.40	35.00

		eur	gbp	chf	jpy	gold	wti	grain	metal
ltc	Correlation	-0.03	-0.03	-0.02	-0.01	0.00	-0.01	0.02	0.05
	Pearson %	34.41	34.69	58.46	84.82	99.16	85.25	57.12	12.70
	Permutation %	33.20	37.20	58.20	87.40	99.00	87.40	56.20	13.20

Table 3.4: hypothesis test ltc correlations

		bric	sp500	eurostoxx	nasdaq	bond_europe	bond_us	bond_eu
xrp	Correlation	0.08	0.06	0.04	0.06	0.02	0.01	0.01
	Pearson %	2.08	8.41	23.69	9.03	45.93	79.43	73.08
	Permutation %	2.80	7.40	24.40	9.00	46.40	81.40	71.40

		eur	gbp	chf	jpy	gold	wti	grain	metal
xrp	Correlation	0.01	0.00	0.01	0.00	0.01	0.02	0.01	0.03
	Pearson %	84.02	90.38	83.88	95.71	70.48	64.27	75.19	31.56
	Permutation %	83.40	88.80	83.40	92.40	73.00	60.00	72.40	30.40

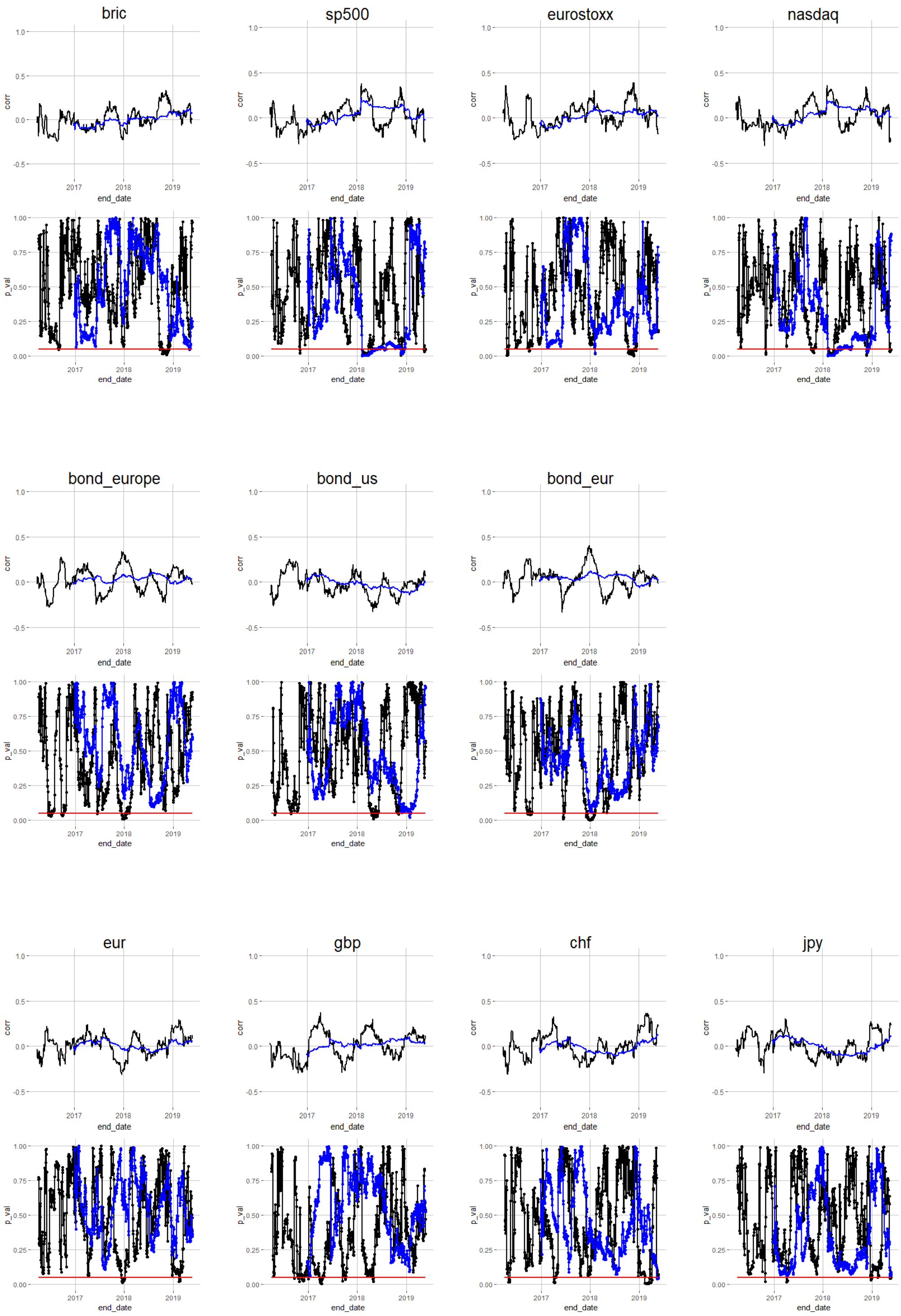
Table 3.5: hypothesis test xrp correlations

The meaning of these results is that all 4 the cryptoassets under observation are uncorrelated to the market, that roughly speaking means that they do not care of how other asset classes move. These results lead us to think of cryptoassets as diversification assets in an investment portfolio.

3.3 Rolling Correlations

In this section are considered smaller time windows on which compute the correlations. Until now the whole dataset has been considered but since the world of cryptoassets is in some way younger than all the other asset classes under analysis, it could be interesting to have a look at what is the path of correlations.

In the following graphs we can see the rolling correlations between Bitcoin and the indexes: the blue line is the 1 year correlation reported in the last day of the relative time window while the black one is the 3-months correlation. Under each graph there are the t-test p-values of each correlation value with a red line which stands for the 5% significance level.


bric
sp500
eurostoxx
nasdaq
bond_europe
bond_us
bond_eur
eur
gbp
chf
jpy
corr
p_val
end_date
2017
2018
2019

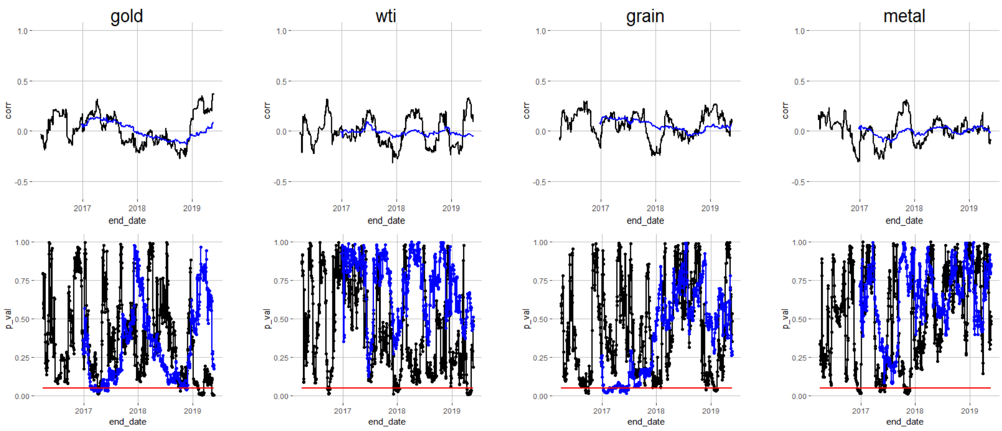
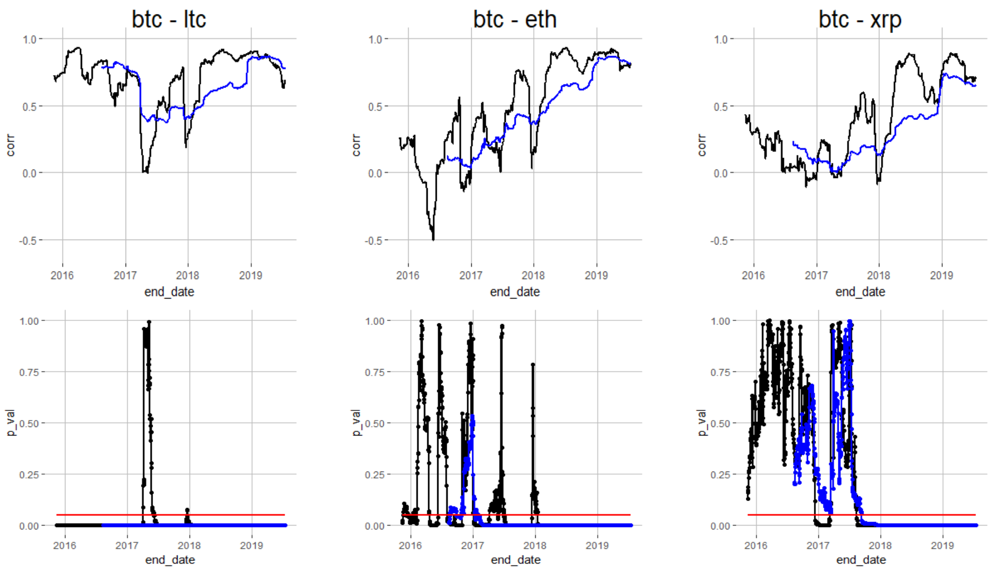


Figure 3.2: btc rolling correlations

While the P-values are almost always high, the correlations for both the 1-year and the 3-months period vary between negative and positive values. This confirms that there is no reason to think that there could be a relation between Bitcoin returns and any other index returns. Even for the other cryptoassets the same holds.

At this point it is clear that cryptoassets are uncorrelated to the market but what is the relationship between each other? Could it be significant?



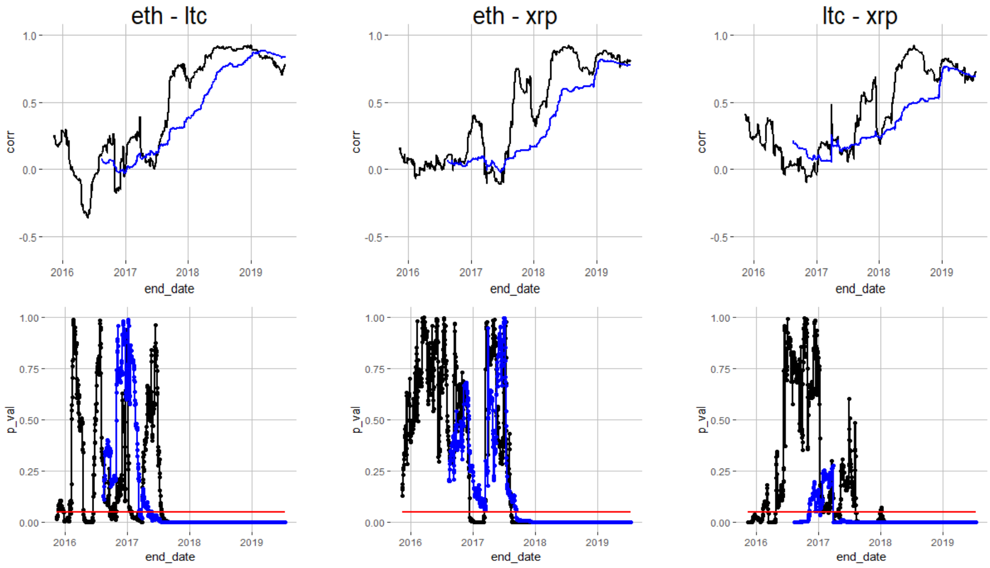


Figure 3.3: Rolling correlations between cryptoassets

From these graphs we can see a significant growing path for the correlations with p-values that, after a starting period of moves, finally it returns smoothly to a value closed to zero..

For all the paths the finale correlation is about 70% that means a very high positive correlation. This result suggests that, in order to reduce the portfolio's volatility, and assuming that the diversification properties hold, it is best to invest in one only cryptoasset inside each portfolio.

Chapter 4

Returns Analysis

when considering cryptoassets in asset allocation, one of the most critic factor is the high volatility that in much cases keep investors away from this asset class. With this in mind it's interesting to analyse cryptoassets returns and compare them to the returns of all the other asset classes.

In this chapter is exploited the role of returns in asset allocation and then a look at the stylized facts that characterize returns of financial instruments such as *fat tails* and *volatility clustering* is given.

4.1 Fundamentals

Starting from the 70's, thanks to the development of mathematical finance as a study subject, lots of mathematicians have tried to model financial markets in a stochastic framework with the objective to make the best possible asset allocation decision.

More clearly, the goal is to estimate the distribution of the price of financial instruments at a future time horizon $t + \tau$ (the investment horizon). A rational approach should link the market model, i.e.the distribution of the price at the investment horizon, with the observations, i.e. the past realizations of some market observables. The way to go from historical data to future prices consists of four building blocks:

1. Detecting the invariants
2. Determining the distribution of the invariants
3. Projecting the invariants into the future

4. Mapping the invariants into the market prices

The first step consists of finding an element which characterizes the market and that repeats itself identically throughout history in the market. These elements are the invariants.

In [Meucci, 2009] a definition of invariants is provided:

Definition 4.1.1 *Consider a starting point $\tilde{t}$ and a time interval $\tilde{\tau}$, which we call estimation interval. Consider the set of equally-spaced dates:*

$$\mathcal{D}_{\tilde{t},\tilde{\tau}} \equiv \left\{ \tilde{t}, \tilde{t}+\tilde{\tau}, \tilde{t}+2\tilde{\tau}, ... \right\} \tag{4.1}$$

consider a set of random varibales:

$$X_t, \quad t \in \mathcal{D}_{\tilde{t},\tilde{\tau}} \tag{4.2}$$

the random variables X_t are market invariants for the starting point $\tilde{t}$ and the estimation interval $\tilde{\tau}$ if they are independent and identically distributed and if the realization x_t of X_t becomes available at time t.

This is still not enough since we would like to find a model that describe these invariants independently by the starting point $\tilde{t}$. Therefor, we give the following definition:

Definition 4.1.2 *A time homogeneus invariant is an invariant whose distributione does not depend on the reference time $\tilde{t}$*

The second step of the process is to infer a distribution of the invariants thanks to their repetitive behaviour, the third step consists of projecting this estimated distribution to the generic time horizon τ that is relevant for the investment decision while the last step is to map the forecasted invariant in the future price of the instrument under analysis.

In an investment portfolio composed by different asset classes it is crucial to find for every instrument its own invariant.

In the case of equity market the returns are considered invariants. This can be simply verified by looking for uncorrelation between returns in different days. In the scatter plots in figure 4.1 there are on the x-axis the returns at time t while on the y-axis the ones at time $t+1$.

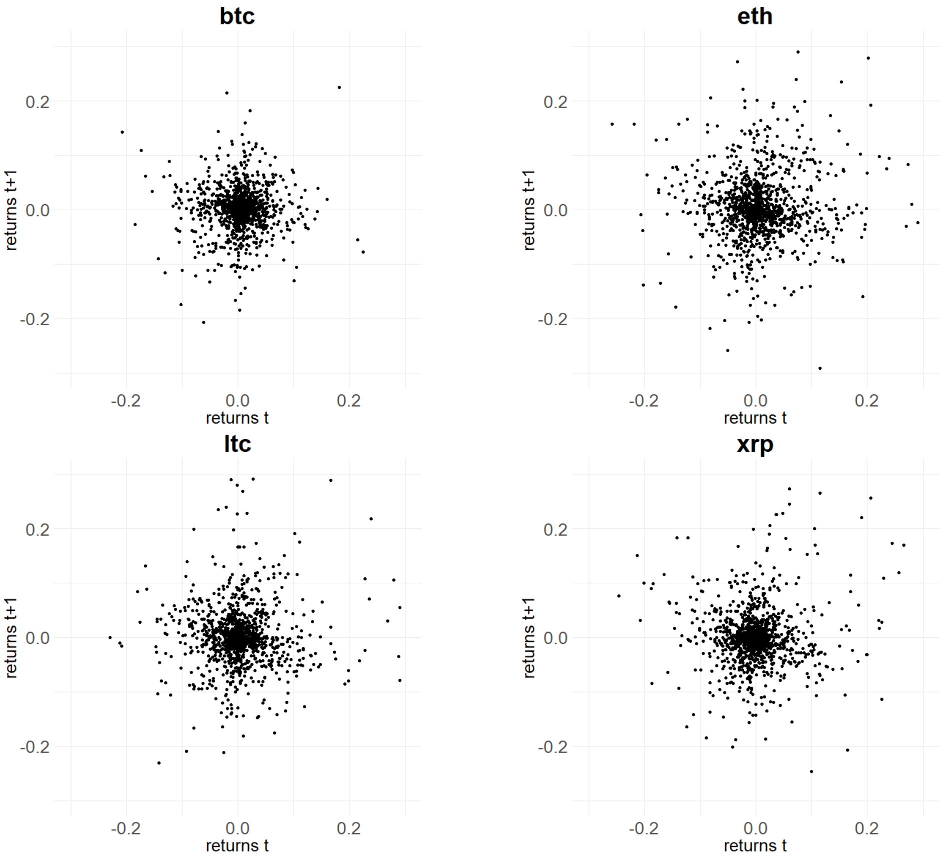


Figure 4.1: Lagged returns scatter plot

The absence of trends means that returns satisfies the independence requirement. To check the identically distributed requirement we look through the empirical distribution of returns, in particular we draw the rolling 2-years mean, standard deviation, kurtosis and skewness of daily returns. We expect to see almost flat lines for every of these parameters if we assume that returns are the invariants we are looking for.

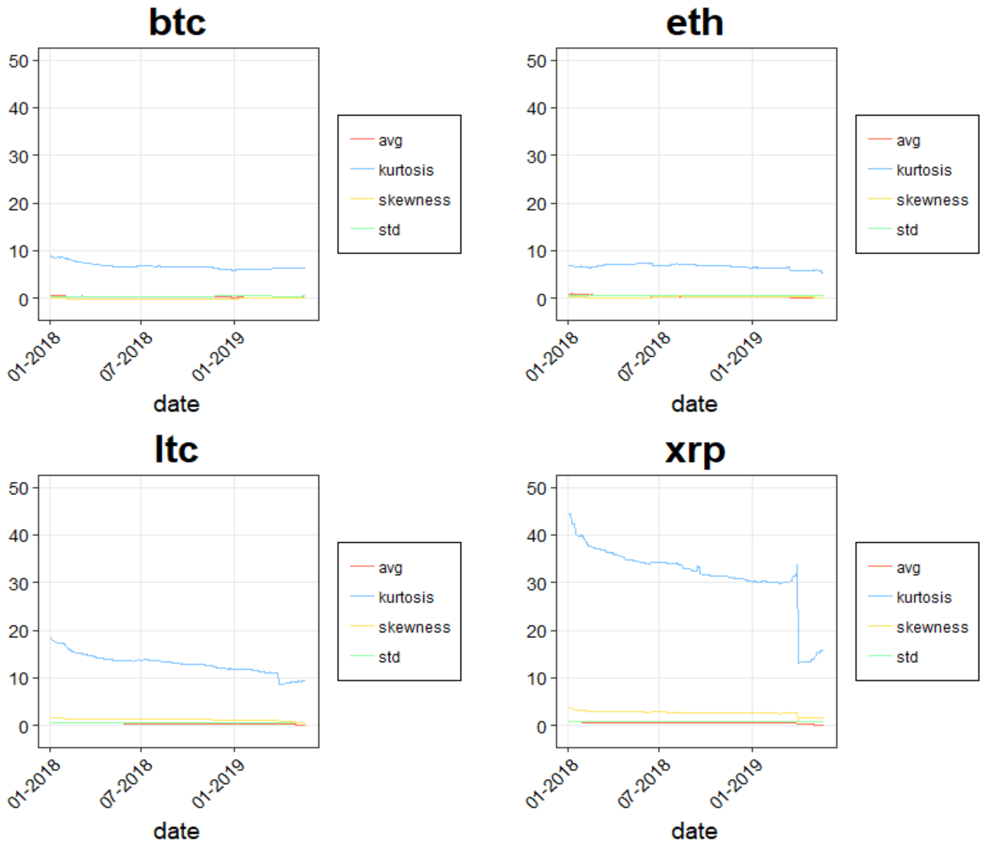


Figure 4.2: Rolling 2-years values of parameters for empirical distribution of cryptoassets

All the parameters seems to remain almost flat during time except for the kurtosis of Ripple. It's due to the sudden growth of its value at the end of 2017 which goes out of sample where we see the jump from about 32 to about 14. This flatness allow us to assume that retuns act as invariants for cryptoassets. Even for equity, indexes, currencies and commodities the returns are considered invariants for asset allocation purposes. Below the lagged returns and parameters of empirical distribution of daily returns for the S&p500 are reported. The differences with respect to the cryptoassets are the magnitudes of all these values but the absence of trend for lagged returns and the flatness of the parameters are still valid.

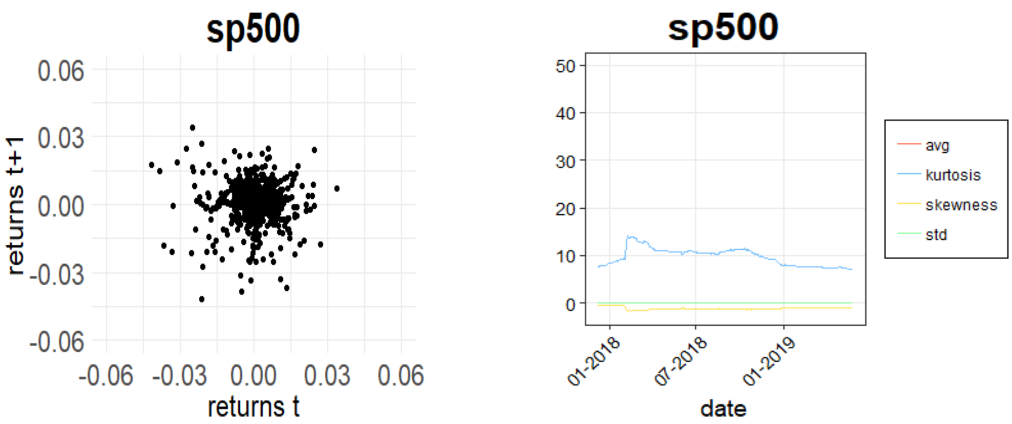


Figure 4.3: Lagged returns and rolling 2-years values of parameters for empirical distribution of S&P500

From this section's analysis we can conclude that the properties needed for invariants are satisfied by returns also for cryptoassets. This suggests that one can insert cryptoassets in the usual models for asset allocation such as the Markowitz approach.

4.2 Stylized Facts

At this point we go deeper in the analysis of returns. We want to assess their empirical characteristics, in particular we check if also for this asset class there are the "$problems$" of $fattails$ and $volatilityclustering$.

4.2.1 Fat Tails

Following the 2008 Financial Crisis, more and more doubts about usual risk managements metrics have risen. Usual asset pricing models assume Gaussian distribution for market invariants but what happens is that extremal events are more likely than estimated. Mathematically speaking this means that in reality the tails of model parameters distributions are fatter than the Gaussian ones. For a normal distribution, a majority of asset variation fall within 3 standard deviations of its mean which subsequently understates risk and volatility.

To verify this financial markets property for cryptoassets we look at the empirical distribution of them versus the Gaussian one with same mean and variance.

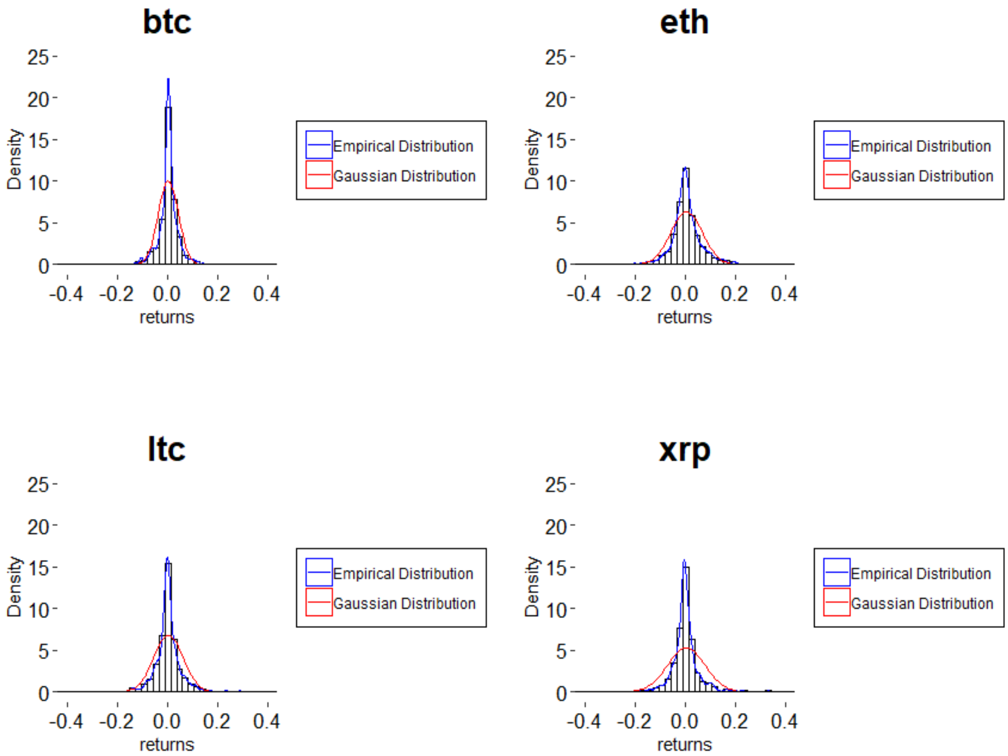


Figure 4.4: Empirical versus Gaussian distribution for cryptoassets

Figure 4.4 shows that also in this case the Gaussian distribution has some drawback fitting the real distribution of returns. Indeed, as we saw also in figure 4.2, all of these distributions have a leptokurtic shape. If we look at the tails of the distributions we see that the red ones goes to zero slightly faster than the blue ones.

4.2.2 Volatility Clustering

Observing empirical returns, we encounter another drawback: in financial markets *"large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes"* (Mandelbrot, 1963). This means that the volatility tends to have periods of high magnitudes followed by periods of low magnitude. The consequence of this clustering is that the hypothesis of *iid* returns is no longer acceptable since they are no more completely independent.

To check this property for cryptoassets, we look at the autocorrelation function of the absolute values of returns.

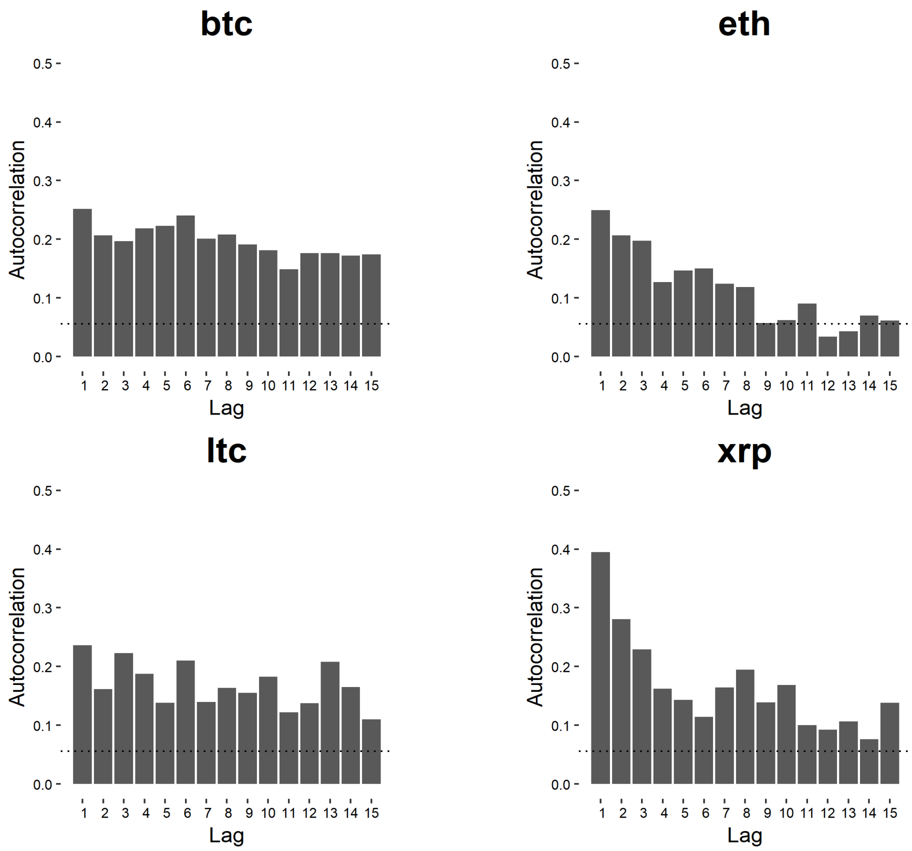


Figure 4.5: Autocorrelation function of absolute values of cryptoassets returns

The autocorrelation function is the correlation of a signal with a delayed copy of itself as a function of delay. Thanks to this function what we can say is that the magnitude of returns at the generic time t has a positive and significantly different from zero correlation with the magnitude of the returns in the previous periods ($t-1$, $t-2$,...) and volatility clustering property still holds.

From this chapter we understood that even with high volatility and returns, the cryptoassets can be modelled like all the other financial instruments. The main properties and drawbacks hold also in this case and so the literature about financial returns is still true for cryptoassets. The next step is to insert them in an investment portfolio and to see if one can take some profit from their returns even with such high volatilities.

Chapter 5

Asset Allocation

The goal of this chapter is to investigate wether including cryptoassets in an investment portfolio could generate profits and advantages in terms of diversification or not. This investigation has been done by working on the idea of efficient frontier and optimal allocation. In particular, the considered framework is the well known Markowitz's Portfolio Theory.

After that, the focus will move on the differences from an investment portfolio that includes multiple cryptoassets and an investment porfolio that includes bitcoin as the only cryptoasset.

5.1 Markowitz's Portfolio Theory

Harry Markowitz is highly regarded as a pioneer for his theoretical contributions to financial economics and corporate finance. In 1990, he won a Nobel Prize for his contributions to these fields, which he explained in his "Portfolio Selection" [Markowitz, 1952] essay first published in The Journal of Finance, and more extensively in his book, "Portfolio Selection: Efficient Diversification" [Markowitz, 1959]. His studies gave rise to the 'Modern Portfolio Theory' (MPT). The foundation for this theory was substantially later expanded upon by Markowitz' fellow Nobel Prize co-winner, William Sharpe, who is widely known for his 1964 Capital Asset Pricing Model work on the theory of financial asset price formation.

Essentially, MPT is an investment framework where the goal is to select financial instruments in order to maximize the portfolio's expected returns with the minimun risk, that is measured by the portfolio volatility. In order to get the minimum possible volatility what comes to our help is the principle of diversification: this idea was well summarized in [Fabozzi et al., 2002] with the sentence *"don't put all your eggs in one basket"*.

Even if cryptoassets have higher volatility with respect to other instrument, they have almost zero correlations with the market, and thus they could increase returns without affecting much the overall portfolio volatility.

Consider now a set of N financial instrument represented through a vector S by which we would like to construct our investment portfolio. Let's call w the vector of which element w_i represent the percentage of our wealth that we invest in the instrument S_i.
Consider $\mathbb{E}[R_p]$ the expected return of our portfolio: it is defined as

$$\mathbb{E}[R_p] = \sum_{i=0}^{N} \mathbb{E}[w_i R_i] \tag{5.1}$$

where R_i is the return of the instrument S_i.
We now define the portfolio volatility σ_p as

$$\sigma_p = \sqrt{\sigma_p^2} \tag{5.2}$$

with

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{5.3}$$

where σ_i is the (sample) standard deviation of the periodic returns of the instrument S_i, and ρ_{ij} is the correlation coefficient between the returns on instruments S_i and S_j. Finally let's call Σ the covariance matrix of the N instruments returns R_i.
Now we can define the Portfolio Frontier $\mathcal{W}^*$:

consider $\mathcal{W}$ the set of all possible portfolios, then $\mathcal{W}^*$ is the subset of $\mathcal{W}$ which contains all the portfolios w that solve the following quadratic problems

$$\begin{aligned} \min_{w \in \mathcal{W}} \quad & \frac{1}{2} w^t \Sigma w \\ \text{s.t.} \quad & \mathbb{E}[R_p] = \mu \\ & e^T w = 1 \end{aligned} \tag{5.4}$$

for $\mu \in (-\inf, \inf)$.

The condition $e^T w = 1$ means that the portfolio is full-invested. For simplicity in this thesis we only focus on the sub-problems with the additional condition

$$w_i > 0 \qquad for \quad i = 1, ..., N \tag{5.5}$$

which does not allow investor to sell-short the instruments.

We solved this quadratic problem by using the function *solve.QP* of the R [R Core Team, 2013] package *quadprog* [original by Berwin A. Turlach

R port by Andreas Weingessel, 2019] which solves quadratic programming problems with the dual method of Goldfarb and Idnani (1982, 1983) (this method is based on the fact that the unconstrained minimum of the objective function can be used as a starting point for positive definite quadratic programming problems).

The outcome is the set of portfolios with the minimum volatility for every reachable returns. An horizontal parabola is obtained by plotting the return versus volatility of those portfolios. Let us call the points on this parabola $w_i = (\sigma_i, r_i)$. Note that due to the properties of an horizontal parabola, for every σ_i there are two points lying on the parabola, let's say (σ_i, r_j) and (σ_i, r_k). This means that for every level of volatility one can chose between two portfolios with different returns. Of course an investor will always choose the one with highest return, which is $(\sigma_i, max(r_j, r_k))$. By taking these points for each possible σ_i we got the part of the parabola which is above the horizontal axis: this is called *Efficient Frontier*.

5.2 Numerical results

We constructed the efficient frontier using our dataset in order to check which instruments yield a positive contribute to the portfolio. In figure 5.1 there are the efficient frontiers of three portfolios:

1. in black the portfolio composed of the standard instruments only (no cryptoassets)
2. in blue the portfolio composed of the standard instruments plus Bitcoin
3. in red the portfolio composed of all the instruments of the dataset

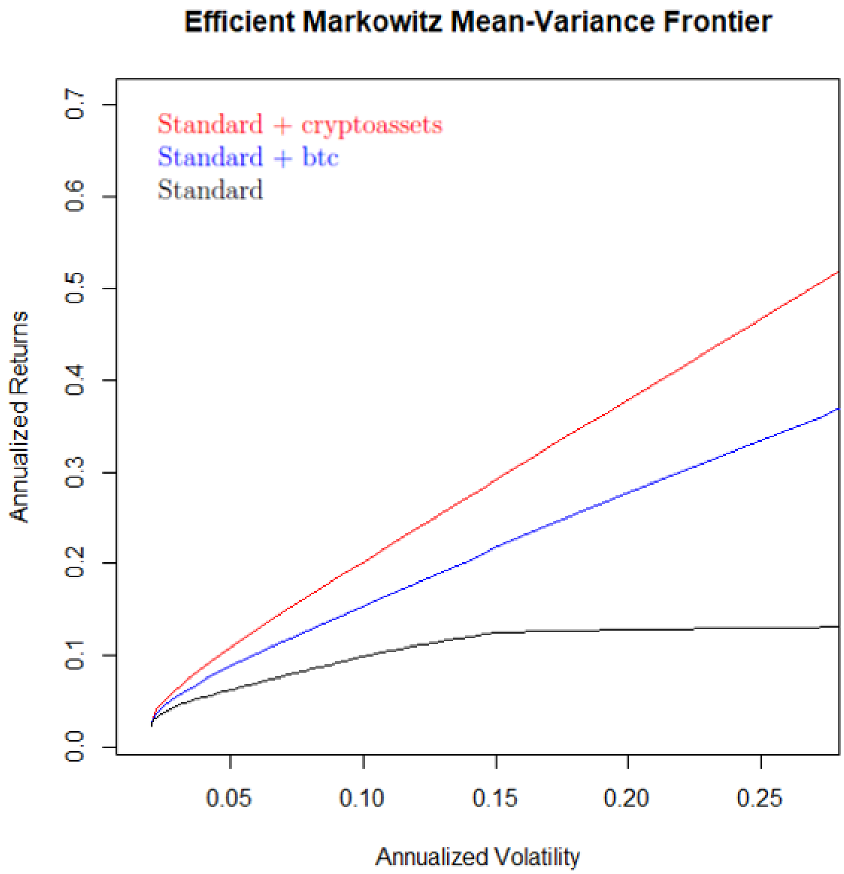


Figure 5.1: Efficient frontier over the whole dataset time period

Here we see that each of these frontiers are different from each other. In particular the more instruments we include, the higher return we get (by fixing a target volatility). From this first analysis one could conclude just that including more instruments is in general positive. This consideration is just another way to define the diversification principle.

Something strange is that after our considerations about the correlations inside the world of cryptoassets one could expect to see almost no differences between the portfolio with just Bitcoin and the one with all the cryptoassets since it's hard to reduce the portfolio volatility by combining them. Another important thing to say is that this asset class was born not so far and this could affects results: in general, when there is a new instrument in the market it takes some time for it to be priced at the right level and to become efficient. Just thinking about an IPO for a stock, in the first days of trading it usually move faster than usual stocks.

For these reasons we then repeated the calculations to obtain the efficient frontier considering just the second half of the dataset time period, that means starting from the 14^{th} July of 2017.

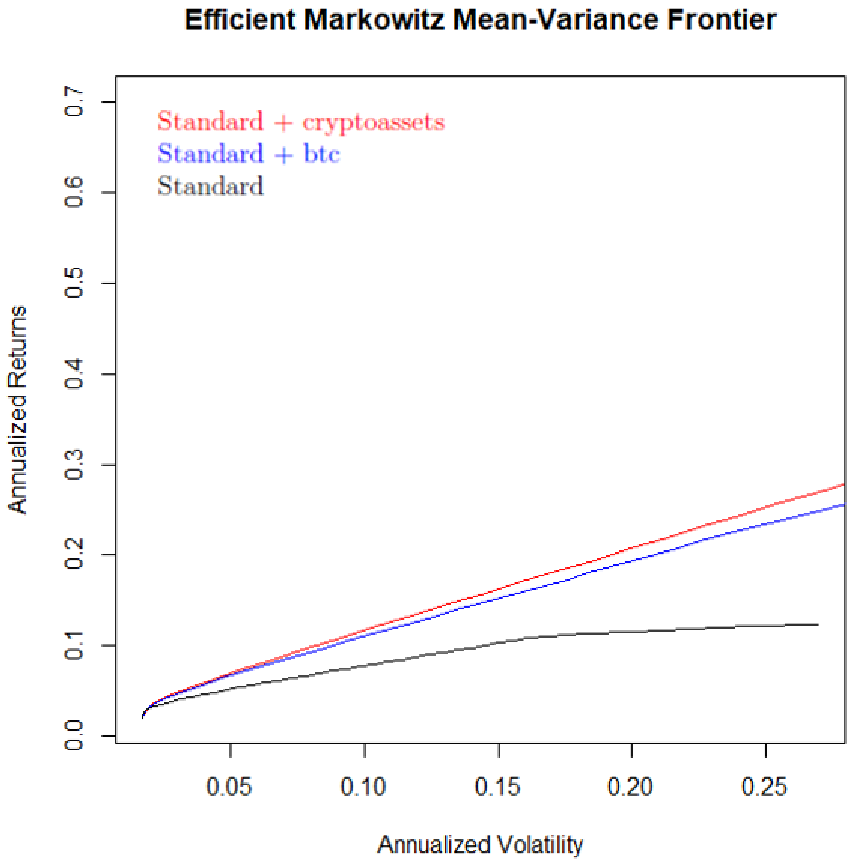


Figure 5.2: Efficient frontier over the periof from the 13^{th} July of 2017 to the 4^{th} of May of 2019

The black line remains almost unchanged while the red and blue ones have gone down. This happens even if the considered time window still contains the period of biggest increase of cryptoassets, which is the last part of 2017. That is because that period was characterized by a very high volatility and so it causes a lower wealth allocation in cryptoassets.

Even more important is the fact that the red and blue lines get closer with respect to the previous case. The distance between them is very small and we can consider them almost equals. This means that including Bitcoin or including all the top four cryptoassets by exchanged volumes makes no differences in terms of performance versus volatility.

In chapter 3 we studied the correlations between cryptoassets and we have seen that, after a period where the correlations were relatively small, they go up and the p-values go down. Of course this is one of the reasons that makes the frontiers closer after the first half of the data sample: the high correlations don't allow to reduce the volatility by including different cryptoassets.

In the following table we report the return of the best portfolios which are the one that have the best Sharpe ratio.

	Whole dataset	Second half
Standard + cryptoassets	8.87%	3.59%
Standard + btc	5.87%	3.62%
Standard	3.88%	3.05%

Table 5.1: Return of the best portfolio for each frontier in terms of Sharpe ratio

The Sharpe ratio SR is a measure of the goodness of a portfolio. It is defined as:

$$SR = \frac{R_p - R_f}{\sigma_p} \tag{5.6}$$

where R_p and σ_p are respectively the return and the volatility of the portfolio and R_f is the risk free rate. In our analysis we considered a risk free rate equal to 0 since we are doing a comparison analysis and not an absolute one: we're not interested in evaluating in this way the absolute goodness of these portfolios.

In the Table 5.1 there is no new information with respect to the graphs above. We report these values just to identify a precise portfolio over each of the three frontiers. We now analyse the allocation for each of them.

	Whole dataset		
	Standard + cryptoassets	Standard + btc	Standard
bric	0.34%	1.13%	0.83%
sp500	4.31%	4.57%	4.35%
eurostoxx	0.00%	0.00%	0.00%
nasdaq	6.21%	6.24%	6.83%
bond_europe	0.00%	0.00%	0.00%
bond_us	76.14%	75.16%	70.52
bond_eur	4.86%	5.69%	12.78%
eur	0.00%	0.00%	0.00%
gbp	0.00%	0.00%	0.00%
chf	0.00%	0.00%	0.00%
jpy	0.00%	0.00%	0.00%
gold	0.00%	0.00%	0.52%
wti	1.87%	1.44%	1.26%
grain	0.00%	0.00%	0.00%
metal	2.28%	2.84%	2.91%
btc	1.19%	2.93%	
eth	1.78%		
ltc	0.00%		
xrp	1.03%		

Table 5.2: Allocations for the best Sharpe ratio portfolios considering the whole dataset

In Table 5.2 the most significant differences among the portfolios are the increasing weight of bond_eur and the decreasing weight of bond_us in removing the cryptoassets. Another important factor is of course the weight assigned to the cryptoassets: it goes from 2.93% in the Standard + btc case to a total of 4.00% in the Standard + cryptoassets case while the weight of btc goes from 2.93% to 1.19%. From this table it seems there is a difference in terms of performance and allocation if we include in our investable universe all the cryptoassets or just Bitcoin.

	Second half		
	Standard + cryptoassets	Standard + btc	Standard
bric	0.00%	0.00%	0.00%
sp500	4.35%	3.37%	3.90%
eurostoxx	0.00%	0.00%	0.00%
nasdaq	0.69%	1.94%	1.58%
bond_europe	0.00%	0.00%	0.00%
bond_us	35.99%	39.03%	35.17%
bond_eur	55.32%	51.87%	56.77%
eur	0.00%	0.00%	0.00%
gbp	0.00%	0.00%	0.00%
chf	0.00%	0.00%	0.00%
jpy	0.85%	0.66%	0.76%
gold	0.00%	0.00%	0.00%
wti	1.86%	2.11%	1.82%
grain	0.00%	0.00%	0.00%
metal	0.00%	0.00%	0.00%
btc	0.63%	1.02%	
eth	0.00%		
ltc	0.00%		
xrp	0.30%		

Table 5.3: Allocations for the best Sharpe ratio portfolios considering the second half of the dataset

In Table 5.3 we get in general a smaller allocation in cryptoassets even if in the second half of the dataset the peak reached in 2017 by the cryptoassets in general is still included. One more interesting fact is that the total allocation in cryptoassets remains at about 1% in both the case of Standard + btc and Standard + cryptoassets. Moreover, the frontiers of these 2 cases are so closed that the difference may not justify the addition of one instrument in the portfolio.

5.3 Allocation in time

After these studies it is relevant to analyse how the allocation changes in time.

We considered a rolling time window of 2 years on which we calculated the optimal allocation in terms of Sharpe ratio.

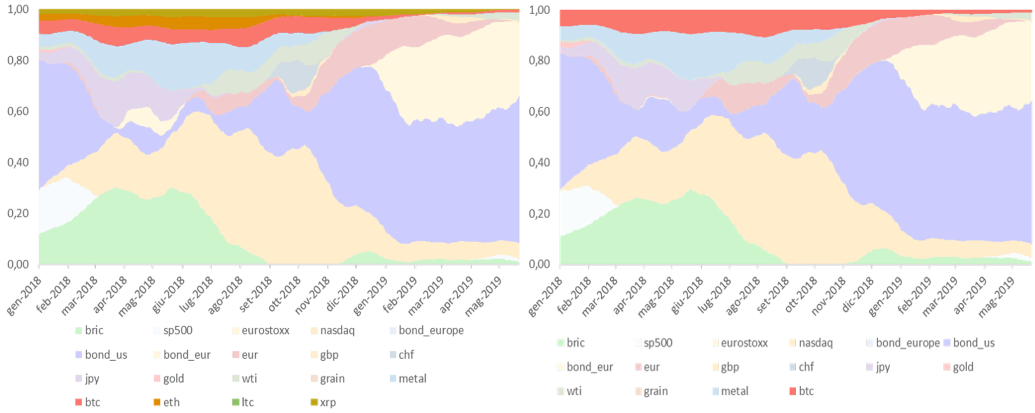


Figure 5.3: Optimal allocation over a rolling time window of 2 years

The figure on the left represents the portfolio that contains, along with the standard instruments, also btc, ltc, eth and xrp while the one on the right includes just standard instruments and bitcoin. Allocation of standard instruments remains almost unchanged in both cases.
It's interesting to observe that by removing xrp, ltc and eth from the portfolio we can say that their allocation is moved completely to btc. This fact confirms what we've seen in section 5.2: adding cryptoassets different from btc yield no extra return and it doesn't change the portfolio allocation in standard instruments.

It is also interesting to observe that the relative allocation of standard instruments doesn't change that much by including or not cryptoassets.

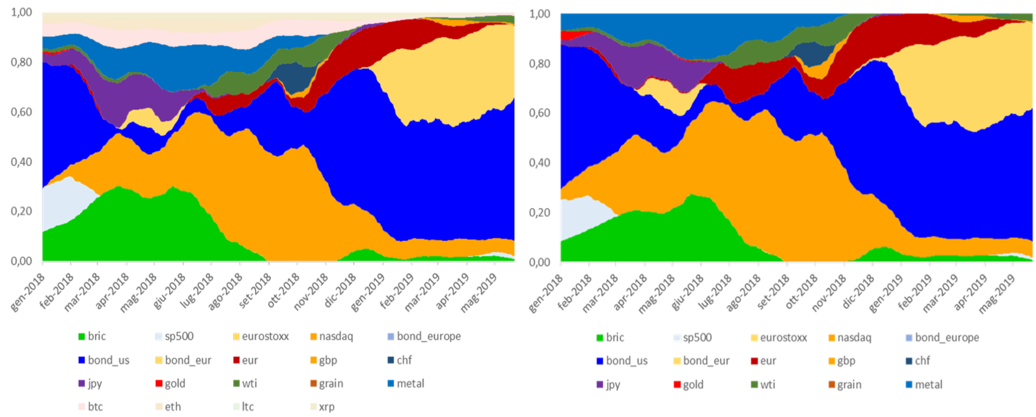


Figure 5.4: Optimal allocation over a rolling time window of 2 years with and without cryptoassets

This result suggests that cryptoassets don't affect correlations between other asset classes, meaning that one can consider them as a new uncorrelated asset class.

Chapter 6

Conclusions

Let's sum up what we've done in this work:

in chapter 2 we've gone through present literature about the inclusion of cryptoassets in an Optimal Portfolio Allocation strategy. Some works studied correlations between cryptoassets with different results due to different time window for the data sample. An introduction of different cryptoassets in a portfolio allocation framework is rarely founded in literature. Just Bitcoin has been analysed in these terms. The presented literature states that having a small percentage of wealth invested in Bitcoin can yield much better returns with respect to not having it at all.

In chapter 3 we performed a correlation analysis to check whether it is reasonable to consider the cryptoassets as a new asset class: first we've studied the correlation matrix over the whole dataset, after that we analysed rolling correlations. Results show that btc, ltc, eth and xrp are uncorrelated with the other asset classes but they have high correlations between each other (in particular they're increasing over time, since these market is going to be more efficient).

To consider this new asset class in the Markowitz's Portfolio Theory it was necessary to wonder if it can be modelled as all other asset classes. In chapter 4 we studied returns distributions and stylized facts of btc, ltc, eth and xrp by comparing them with the other standard instruments. We found that all the usual characteristics (fat tails, volatility clustering, ...) hold also in the case of cryptoassets even if they are very volatile.

In chapter 5 we finally moved to asset allocation. We've considered three cases: the *Standard*, where we considered an investable universe without

cryptoassets, the *Standard + btc*, where we included also Bitcoin in the set of assets one may invest in and the *Standard + cryptoassets*, where we included also ltc, eth and xrp. A first result shows that the more instruments one considers the better return he gets, but this results does not hold when the dataset considered has a shorter time window: the difference between the *Standard + btc* case and the *Standard + cryptoassets* becomes very small. By going into the allocation over rolling time windows we saw that the total allocation in cryptoassets remain unchanged in the these two cases. Then we've seen that by including cryptoassets in the portfolio the relative weights of other instruments also remain unchanged. Thanks to these observations, we can refer to cryptoassets as a new asset class.
Moreover, as one can see from the figure 1.1 in the introduction, Bitcoin rapresent more than 5o% of cryptoassets in terms of exchanged volume.

Since cryptoassets have a very short history with respect to other financial instruments, a possible further development of this work could be to update the dataset and to analyze consistency of high correlations between them and low correlation with other instruments. An other interesting development could be to include cryptoassets in portfolio theory by using other frameworks, such as the Black-Littermann one, which shows more stability in allocation results with respect to the classical Mean-Variance portfolio theory and allow to take into account investors views on market returns.

Thanks to the numerical results obtained in this work we can say that it makes no big differences to include just Bitcoin or Bitcoin plus other cryptoassets in an investment portfolio. Moreover we believe that the infrastructure and the community of Bitcoin make it much more durable and resilient with respect to all the other cyrptoassets.

Bibliography

Samuele Vianello. Bitcoin as a digital asset: Correlation and optimal portfolio allocation. Master's thesis, Politecnico di Milano, 2018.

Weiyi Liu. Portfolio diversification across cryptocurrencies. *Finance Research Letters*, 29, 07 2018. doi: 10.1016/j.frl.2018.07.010.

Digital Gold Institute. Crypto index, 2019. URL `https://github.com/dginst/crypto-index`.

Satoshi Nakamoto. Bitcoin: A peer-to-peer electronic cash system. 03 2009.

Anne Haubo Dyhrberg. Hedging capabilities of bitcoin. is it the virtual gold? *Finance Research Letters*, 16(C):139–144, 2016. URL `https://EconPapers.repec.org/RePEc:eee:finlet:v:16:y:2016:i:c:p:139-144`.

Tony Klein, Hien Pham Thu, and Thomas Walther. Bitcoin is not the new gold - a comparison of volatility, correlation, and portfolio performance. 03 2018.

Elie Bouri, Peter Molnár, Georges Azzi, David Roubaud, and Lars Ivar Hagfors. On the hedge and safe haven properties of bitcoin: Is it really more than a diversifier? *Finance Research Letters*, 20, 02 2017. doi: 10.1016/j.frl.2016.09.025.

Shaen Corbet, Andrew Meegan, Charles Larkin, Brian Lucey, and Larisa Yarovaya. Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Economics Letters*, 165(C):28–34, 2018. URL `https://EconPapers.repec.org/RePEc:eee:ecolet:v:165:y:2018:i:c:p:28-34`.

Attilio Meucci. *Risk and Asset Allocation*. Springer Finance, 01 2009. doi: 10.1007/978-3-540-27904-4.

Harry Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, March 1952. doi: 10.2307/2975974. URL https://www.jstor.org/stable/2975974.

Harry Markowitz. *Portfolio Selection: Efficient Diversification of Investments*. New York : Wiley, 1959.

Frank J. Fabozzi, Francis Gupta, and Harry M. Markowitz. The legacy of modern portfolio theory. *The Journal of Investing*, 11(3):7–22, 2002. ISSN 1068-0896. doi: 10.3905/joi.2002.319510. URL https://joi.pm-research.com/content/11/3/7.

R Core Team. *R (A Language and Environment for Statistical Computing)*. R Foundation for Statistical Computing, Vienna, Austria, 2013. URL http://www.R-project.org/.

S original by Berwin A. Turlach R port by Andreas Weingessel. *quadprog: Functions to Solve Quadratic Programming Problems*, 2019. URL https://CRAN.R-project.org/package=quadprog. R package version 1.5-7.

Darko Stosic, Dusan Stosic, Teresa B. Ludermir, and Tatijana Stosic. Collective behavior of cryptocurrency price changes. *Physica A: Statistical Mechanics and its Applications*, 507(C):499–509, 2018. doi: 10.1016/j.physa.2018.05.0. URL https://ideas.repec.org/a/eee/phsmap/v507y2018icp499-509.html.

Peter M. Krafft, Nicolas Della Penna, and Alex Sandy Pentland. An experimental study of cryptocurrency market dynamics. In *Proceedings of the 2018 CHI Conference on Human Factors in Computing Systems*, CHI '18, New York, NY, USA, 2018. Association for Computing Machinery. ISBN 9781450356206. doi: 10.1145/3173574.3174179. URL https://doi.org/10.1145/3173574.3174179.

Rosario N. Mantegna and H. Eugene Stanley. *An Introduction to Econophysics: Correlations and Complexity in Finance*. Cambridge University Press, USA, 1999. ISBN 0521620082.

E.J. Elton, M.J. Gruber, S.J. Brown, and W.N. Goetzmann. *Modern Portfolio Theory and Investment Analysis*. John Wiley & Sons, 2009. ISBN 9780470388327. URL https://books.google.it/books?id=a0tcTEQ3DAUC.

Jean-Philippe Bouchaud and Marc Potters. *Theory of financial risk and derivative pricing*. Cambridge Univ. Press, Cambridge [u.a.], 2. ed edition, 2003. ISBN 0521819164. URL http://gso.gbv.de/DB=2.1/CMD?ACT=SRCHA&SRT=YOP&IKT=1016&TRM=ppn+368093751&sourceid=fbw_bibsonomy.

J. N. *Journal of the Royal Statistical Society*, 102(2):295–298, 1939. ISSN 09528385. URL http://www.jstor.org/stable/2980009.

Vasiliki Plerou, Parameswaran Gopikrishnan, Bernd Rosenow, Luís A. Nunes Amaral, and H. Eugene Stanley. Universal and nonuniversal properties of cross correlations in financial time series. *Phys. Rev. Lett.*, 83: 1471–1474, Aug 1999. doi: 10.1103/PhysRevLett.83.1471. URL https://link.aps.org/doi/10.1103/PhysRevLett.83.1471.

B. Podobnik, D. Wang, D. Horvatic, I. Grosse, and H. E. Stanley. Time-lag cross-correlations in collective phenomena. *EPL (Europhysics Letters)*, 90(6):68001, jun 2010. doi: 10.1209/0295-5075/90/68001. URL https://doi.org/10.1209%2F0295-5075%2F90%2F68001.

Gang-Jin Wang, Chi Xie, Kaijian He, and H. Eugene Stanley. Extreme risk spillover network: application to financial institutions. *Quantitative Finance*, 17(9):1417–1433, September 2017. doi: 10.1080/14697688.2016.127. URL https://ideas.repec.org/a/taf/quantf/v17y2017i9p1417-1433.html.

Gang-Jin Wang, Chi Xie, and H. Stanley. Correlation structure and evolution of world stock markets: Evidence from pearson and partial correlation-based networks. *Computational Economics*, 51:607–635, 03 2018. doi: 10.1007/s10614-016-9627-7.