International Trade and Globalization

Solutions To Problem Set Two

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Exercise 1 - Completing Riccardo's Example

Consider the Ricardian two country, two goods model presented in the lecture notes.

Autarky First, assume that countries cannot trade with each other. Focus on England first.

1. **What is the relative price of wine in England ($\frac{p_{W,E}}{p_{C,E}}$) ?**

We can compute the relative pricing by obtaining the first order conditions for the producers in England, hence $\forall\ i \in \{W, C\}$:

$$\max_{l_i^E \geq 0} z_W^E l_i^E p_i^E - \omega^E l_i^E$$

And hence we have that:

$$\forall\ i \in \{W, C\} \quad p_i^E \leq \frac{\omega^E}{z_i^E} \tag{1}$$

Recall from the consumer demand that:

$$c_{E,i} := \arg\max_{c_{E,j}} c_{C,E}^{0.5} c_{W,Z}^{0.5}$$

$$\text{subject to :}$$

$$\sum_{i \in \{W,C\}} p_i^E c_{E,i} \leq W$$

Claim. *In any equilibrium (under autarky and our cobb-douglas specification) it must be that* $c_{E,i} > 0\ \forall\ i \in \{W, C\}$ *and hence* (1) *can be read with equality.*

Proof. By contraposition. Suppose not then F.O.C. are not satisfied and hence the proposed allocation cannot be optimal. □

Given that we can eliminate the *complementary slacknees* from (1) we obtain by taking the ratio of the two production optimality conditions that:

$$\frac{p_W^E}{p_C^E} = \frac{\omega^E}{z_W^E} \frac{z_C^E}{\omega^E} = \frac{z_C^E}{z_W^E} = \frac{120}{100}$$

Recall in fact that in Riccardo's example $a_{j,i}$ is the number of workers needed in country j to produce one unit of good i, hence in our setting $z_i^j = \frac{1}{a_{j,i}}$.

2. **Deduce from this the relative consumption of wine in England ($\frac{c_{W,E}}{c_{C,E}}$).**

From the consumer FOCS it's immediate to get that:

$$\frac{c_{E,W}}{c_{E,C}} = \frac{p_C^E}{p_W^E} = \frac{5}{6}$$

3. **Using your response to Question 2 and the labour market clearing condition, determine how much wine and cloth England produces and consumes in autarky.**

Note that market clearing in the goods market requires that:

$$c_{E,i} = y_{E,i} = z_{E,i} l_{E,i}$$

From which is easy to see that:

$$\frac{c_{E,W}}{c_{E,C}} = \frac{5}{6} = \frac{l_{E,W} a_{c,E}}{l_{E,C} a_{W,E}}$$

Which implies that the share of workers in between sector is equal to 1 namely:

$$\frac{l_{E,W}}{l_{E,C}} = 1$$

Given that the labor supply is inelastic we have that:

$$l_{E,i} = \frac{1}{2} L_E \ \forall \ i \in \{W, C\}$$

From which we can get the consumption using the market clearing condition to be:

$$c_{E,W} = \frac{L_E}{240} \tag{2}$$

$$c_{E,C} = \frac{L_E}{200} \tag{3}$$

4. **Finally, what is the real wage of England in autarky? Does it depend on L_E ?**

Recall that given the structure of our economy real wages are equal to the marginal productivity of labor. Now one may ask, which of the following representation is correct?

$$\frac{w_E}{p_C} = z_{E,C}$$
$$\frac{w_E}{p_W} = z_{E,W}$$

Now they are both right just expressed in terms of wine price or clothes price.

5. **Repeat the same questions for Portugal. Is the real wage in Portugal higher or lower than in England? Why?**

Following the previous analyisis we can state that:

$$\frac{p_W^P}{p_C^P} = \frac{\omega^P}{z_W^P}\frac{z_C^P}{\omega^P} = \frac{z_C^P}{z_W^P} = \frac{9}{8}$$

Similarly:

$$\frac{c_{P,W}}{c_{P,C}} = \frac{p_C^P}{p_W^P} = \frac{8}{9}$$

Again we get that:

$$\frac{l_{P,W}}{l_{P,C}} = 1$$

$$c_{E,W} = \frac{L_E}{180}$$
$$c_{E,C} = \frac{L_E}{160}$$

The real wage in portugal is **higher** than in ingland given that portugal is much more productive in both sectors.

Complete specialization Let us guess that trade between Portugal and England leads to complete specialization, that is, that equilibrium wages hold $\frac{80}{120} < \frac{w_E}{w_P} < \frac{90}{100}$.

6. **How much wine and cloth does each country produce?**

Let's take a step back and analyze the general problem we are facing of two countries two good model. We can immediately have the following claim:

Claim. *In any equilibrium each country produce at least one good*

Proof. By contraposition. Suppose not, then it exist at least one country that does not produce anything. But given the fact that work is supplied inelastically we would not have market clearing on the labor market. ☐

Now let's go back to the problem of the producers and recalling the complementary slackness we have that:

$$p_{i,j} \leq \frac{w_j}{z_{i,j}} \text{ with inequality if } l_{ij} = 0$$

Now given the previous claim we have three possibilities (but we analyze only the case of perfect specialization):

1. **perfect specialization.** We see immediately that

$$p_i = \min\{\frac{w_E}{z_i^E}; \frac{w_P}{z_i^P}\} \; \forall \; i \in \{C, W\}.$$

   But recall from before the previous discussion that each country must produce at least one good hence we have that, w.l.o.g. assume that $\frac{w_E}{z_i^E} > \frac{w_P}{z_i^P} \rightarrow \frac{w_E}{w^P} > \frac{z_i^E}{z_i^P}$ by C.S. this implies that England does not produce good i but it must be (following the previous claim) that it produces good $\bar{i}$. Hence we know that we must have $\frac{w_E}{w^P} \leq \frac{z_{\bar{i}}^E}{z_{\bar{i}}^P}$ (with inequality if we want perfect specialization). This implies that we must have that $\frac{z_i^E}{z_i^P} < \frac{z_{\bar{i}}^E}{z_{\bar{i}}^P} \rightarrow \frac{z_i^E}{z_{\bar{i}}^E} < \frac{z_{\bar{i}}^P}{z_{\bar{i}}^P}$ hence **countries specialize on their comparative advantage**. In this case we would have that (given the assumptions on $a_{i,j} = \frac{1}{z_{i,j}}$) that **England** produces **clothes** and **portugal** produces **wine**. Moreover from our discussion it must be that $\frac{w_E}{w_P} \in \left(\frac{80}{120}, \frac{90}{100}\right)$.

Given the assumption that in equilibrium it holds perfect specialization is easy to get that (given the fact that the supply of labor is inelastic):

$$y_{E,C} = \frac{L_E}{100}, \quad y_{E,W} = 0$$
$$y_{P,C} = 0, \quad y_{E,W} = \frac{L_P}{80}$$

7. **Using the assumption of perfect competition, find an expression for the relative wage $\frac{w_E}{w_P}$ as a function of the relative price of wine, $\frac{p_C}{p_W}$.**

Using the FOC of producers its immediate to get that:

$$p_c = \frac{w_E}{z_{E,C}}$$
$$p_W = \frac{w_P}{z_{P,W}}$$

From which we get that:

$$\frac{p_c}{p_w} = \frac{100}{80}\frac{w_E}{w_P} \rightarrow \frac{w_E}{w_P} = 0.8\frac{p_c}{p_w}$$

8. **Using the demand equations for both countries, the goods market clearing conditions and the result of Question 6 to derive an expression for the relative price of wine as a function of the ratio $\frac{L_E}{L_P}$.**

Take the two resource constraints read:

$$\frac{L_E}{100} = \frac{1}{2}\frac{1}{p_c}(w_E L_E + w_P L_P)$$
$$\frac{L_P}{80} = \frac{1}{2}\frac{1}{p_w}(w_E L_E + w_P L_P)$$

Take the ratio to get that:

$$0.8\frac{L_E}{L_P} = \frac{p_w}{p_c}$$

9. **Conclude: which condition on the ratio $\frac{L_E}{L_P}$ must hold in order for the complete specialization equilibrium to apply? Interpret your results.**

It is immediate to have that:

$$\frac{p_w}{p_c} = 0.8\frac{w_P}{w_E} \rightarrow \frac{w_E}{w_P} = 0.8\frac{p_c}{p_w} = \frac{L_P}{L_W}$$

From the previous discussion on the bounds for specialization (see question 6) we have that:

$$\frac{w_E}{w_P} \in \left(\frac{80}{120}, \frac{90}{100}\right)$$

Hence we can derive the bounds on the labor composition combining (8) and (9) to get that:

$$\frac{80}{120} < \frac{L_p}{L_E} < \frac{90}{100}$$

10. **What are the real wages of both countries under complete specialization?**

$$\frac{w_E}{P} = \frac{U_E}{L_E} = \left(\frac{1}{200 \cdot 160}\right)^{\frac{1}{2}} \left(\frac{L_P}{L_E}\right)^{\frac{1}{2}}$$
$$\frac{w_P}{P} = \left(\frac{1}{200 \cdot 160}\right)^{\frac{1}{2}} \left(\frac{L_E}{L_P}\right)^{\frac{1}{2}}$$

11. **If the English population increased, would the English real wage increase or decrease? Why?**

Recall that we imposed perfect competition as market structure, this implies that the real wage stays fixed in terms of local good, but since now (recall that labor is supplied exogenously) production increased the relative price decreased and hence the real wage, while fixed in terms of local good has decreased in terms of foreign good.

Exercise 2 - Productivity increases in the DFS model

Consider the Dornbusch-Fischer-Samuelson model described in the notes. Assume that English and Portuguese productivities are given by

$$\forall i \in [0,1], \begin{aligned} z_E(i) &= 1 - i \\ z_P(i) &= i \end{aligned}$$

and that English and Portuguese labour endowments hold $L_E = L_P = 1$.

Autarky

1. **Determine English and Portuguese consumption levels for every good in autarky, and represent them graphically (that is, draw a graph with the goods' index $\boldsymbol{i}$ on the x -axis and with the consumption levels $\boldsymbol{c_E(i)}$ and $\boldsymbol{c_P(i)}$ on the y -axis).**

Given the fact that we have cobb-douglas preferences we can state that $c_j(i) \neq 0 \ \forall i, \ \forall \ j \in \{E, P\}$ hence the complementary slackness does not implies inequalities and hence we can state that:

$$p_j(i) = \frac{w_j}{z_j(i)} \ \forall \ j \in \{E, P\}$$

Using the autarky demand we have that:

$$c_j(i) = z_j(i)$$

From which we get that $\forall \ i \in [0,1]$:

$$c_E = 1 - i \tag{4}$$

$$c_P = i \tag{5}$$

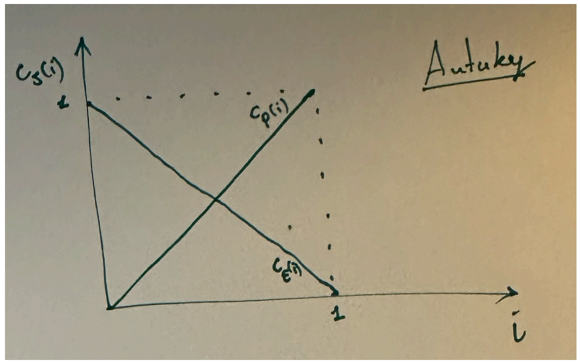

Figure 1: Autarky equilibrium

Free Trade

2. **Solve for the equilibrium values of the cut-off industry i^* and the equilibrium relative wage $\frac{w_E}{w_P}$.**

It's immediate that the cut off industry hereafter $i*$ is defined as (recall that it's of zero mesure):

$$i^* := p_E(i^*) = \frac{w_E}{1-i^*} = p_P(i^*) = \frac{w_P}{i^*} \rightarrow \frac{w_E}{w_P} = \frac{1-i^*}{i^*}$$

Now its even true that (recall that working population is normalized to one for both countries). Note that we can use Riemann Stieltes Integration to make it more clear but we hope it is enough to note that we integrate over di where $i \in [0,1]$ hence the two interval of integration should be "reversed" (consider i as uniform distributed where i increases):

$$w_E = \int_0^{i^*} p_E(i)c(i)\mathrm{d}i = (i^*)(w_E + w_P)$$
$$w_P = \int_{i^*}^1 p_P(i)c_P(i)\mathrm{d}i = (1-i^*)(w_E + w_P)$$

Taking the ratio we get that:

$$\frac{w_E}{w_P} = \frac{i^*}{1-i^*} \tag{6}$$

From which is immediate to state that:

$$\frac{w_E}{w_P} = 1 \tag{7}$$

$$i^* = \frac{1}{2} \tag{8}$$

3. **Determine English and Portuguese consumption levels for every good, and represent them graphically (that is, draw a graph with the goods' index i on the x-axis and with the consumption levels $c_E(i)$ and $c_P(i)$ on the y -axis). Comment on the changes with respect to autarky.**

Recall that consumer FOC gives us the following:

$$c_j(i) = \frac{w_j}{p(i)}$$

But we also know from the price setting of the firm that the real wage is equal to productivity hence:

$$c_E(i) = 1 - i \ \forall \ i \in \left[0, \frac{1}{2}\right]$$

$$c_E(i) = i \ \forall \ i \in \left[0, \frac{1}{2}\right]$$

And by symmetry of the problem we have that $c_E(i) = c_P(i) \ \forall \ i \in [0, 1]$.

Claim. *The gain from trade are very visible on both trhe countries, now England consume just as much from* $[0, 1/2]$ *but more in the other set of indeces. Note that now we have only one line representing world consumption not two separate representing the national ones and hence both countries can gain by exploiting the other country more productive firms.*

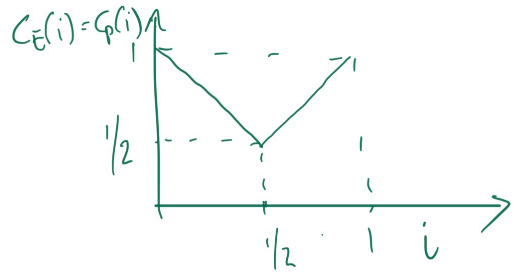

Figure 2: Free Trade

In the next two questions, you are asked to study what happens when England's productivity increases. However, unlike in the notes, productivity increases are not uniform across all goods, but biased towards a subset of them.

Productivity gains in England's competitive advantage industries

4. **Assume now that after a series of inventions, English productivities change to**

$$\forall i \in [0,1], z_E(i) = \begin{cases} 2-3i & \text{if } i \le \frac{1}{2} \\ 1-i & \text{if } i > \frac{1}{2} \end{cases}.$$

Note that this is a productivity shift which affects most the industries in which England was already strongest before. Portuguese productivities remain unchanged. Answer again questions 2 and 3 (i.e., solve for the equilibrium values of i^* and $\frac{w_E}{w_P}$, and for the consumption levels for every good in both country). Comment on your results: how does this asymmetric productivity shift affect both countries?

Claim. ***Gains are shared*** *It's not a novelty of ricardian models that increases in technology imply more welfare for everybody. This is possible since the price of commodities in England decreases* *(recall that the relative wage does not change but productivity for english firms increases hence there are more English firms) and hence part of the gain is transmitted to Portugal.*

Since the gains are before the previous cut off of i^* relative wage does not change as well as the cut off point (as discussed during lecture extreme change on very productive firms of one or the other country does not change anything in terms of relative wages and cut off point). What is interesting to analyze and it is new are the welfare consideration. It's immediate to see that countries are both better off since for goods from 0.5 to 1 nothing changes while the consumption before increases, de facto we have $c_E(i) = c_P(i) = 2 - 3i$ which is higher than before.

Productivity gains in England's competitive disadvantage industries

5. **Assume now that after a series of inventions, English productivities change to**

$$\boldsymbol{\forall i \in [0,1], z_E(i) = 1}$$

Note that this is a productivity shift which affects most the industries in which England was weakest before. Portuguese productivities remain unchanged. Answer again questions 2 and 3 (i.e., solve for the equilibrium values of $\boldsymbol{i^*}$ and $\boldsymbol{\frac{w_E}{w_P}}$, and for the consumption levels for every good in both country). Comment on your results: how does this asymmetric productivity shift affect both countries? In particular, draw Portugal's consumption profile in the basic case (Question 3) and now.

Again apply the same strategy as in exercise (3) to get that:

$$\frac{1}{i^*} = \frac{i^*}{1 - i^*}$$

and bounding the solutions to be in the interval $[0, 1]$ we get that $i^* = \frac{\sqrt{5}-1}{2} > 0.5$ Relative wage are now $\frac{w_E}{w_P} = \frac{2}{\sqrt{5}-1} > 1$. We get that for i in between 0 and $\frac{\sqrt{5}-1}{2}$ $C_E(i) = 1$ and $C_P(i) = \frac{\sqrt{5}-1}{2}$. For goods indexed between $\frac{\sqrt{5}-1}{2}$ and 1 $C_E(i) = \frac{2i}{\sqrt{5}-1}$ and $C_P(i) = i$ wheere the latter are produced in portugal.

6. **Has Portugal benefitted from the English productivity change? Interpret this. Hint: To answer this question, you need to calculate the Portuguese real wage (or equivalently, and perhaps easier, Portuguese utility per capita).**

The easiest way is to compute the two utilities before and after the change.

$$e^{\left(\int_0^{\frac{1}{2}} \ln(1-i)di+\int_{\frac{1}{2}}^1 \ln(i)di\right)} > e^{\left(\int_0^{\frac{\sqrt{5}-1}{2}} \ln\left(\frac{\sqrt{5}-1}{2}\right)di+\int_{\frac{\sqrt{5}-1}{2}}^1 \ln(i)di\right)}$$

Which could be also clear by noting that the real wage of portugal has decreased given an increase in the price index.

Claim. *Similar to the third exercise were a reduction in costs of trade reduces welfare for some countries, here we see somewhat the same thing: england changes in productivity have hurted portugal who has lost form the productivity increase in england. Note that however both countries are better off in the free trade equilibrium, just the change in similarity of close industries reduces the gains ffrom trade.*

Exercise 3 - Solving the Eaton-Kortum model

An analytical solution in a special case

Consider the Eaton-Kortum model described in the notes, but assume that there are no trade costs (that is, $d_{nj} = 1$ for every n and j). In this case, there is an explicit solution for the Equation system described by (13) in the notes.

1. **Derive this explicit solution (that is, derive an expression for $\frac{w_j}{w_{j'}}$ for any pair of countries j and j'). (Hint: write down (13) for countries j and j', and then divide both lines by each other).**

Note that there is an easy solution to this problem: Note that if trade costs are absent we have that

$$\pi_{nj} = \frac{T_j w_j^{-\theta}}{\phi_n}$$

And hence we have that:

$$w_j L_j = T_j w_j^{-\theta} \sum_{n=1}^{N} \frac{w_n L_n}{\phi_n}$$

Now consider the same expression for country $j\prime$:

$$w_{j\prime} L_{j\prime} = T_{j\prime} w_{j\prime}^{-\theta} \sum_{n=1}^{N} \frac{w_n L_n}{\phi_n}$$

Take the ratio of the two expression to get:

$$\frac{w_j}{w_{j\prime}} = \left(\frac{T_j L_{j\prime}}{T_{j\prime L_J}}\right)^{\frac{1}{1+\theta}}$$

Which is the expression we were required to find.

3. **Use your results from Question 1 to derive an expression for π_{nj}, the share of national income of country n spent on goods from country j.**

(we combined question 1 and question 2, see before)

3. **Comment on your results: how do the relative wage of country j and the spending shares π_{nj} depend on the model parameters ("average productivity" T_j, country size and productivity dispersion θ) ?**

Recall that the original model does not pose limit in setting the trading costs to 1 hence it remains the same thing: the wage of a country is increasing in technology level and decreasing in size. An increase in θ reduce the variance in countries and hence reduce gains from trade making the relative wage close to 1.

5. **a. Report your results for the basic parameter values, i.e., report wages, expenditure shares and real wages for each country.**

	Country 1	Country 2	Country 3	Country 4
Expenditure Shares				
Country 1	0.4554	0.2309	0.0784	0.2353
Country 2	0.2576	0.6555	0.0639	0.0230
Country 3	0.2384	0.2369	0.4469	0.0778
Country 4	0.1163	0.0514	0.4263	0.4061
Real Wages				
Real Wage	3.6320	3.7077	3.1753	1.9230
Real Trade Flows				
Imports 1 from 2	1.6773			
Imports 1 from 3	0.5693			
Imports 1 from 4	1.7094			
Imports 2 from 1		1.9105		
Imports 2 from 3		0.4735		
Imports 2 from 4		0.1708		
Imports 3 from 1			1.5141	
Imports 3 from 2			1.5048	
Imports 3 from 4			0.4938	
Imports 4 from 1				0.4471
Imports 4 from 2				0.1975
Imports 4 from 3				1.6393

Table 1: Basic Parameters

5. **b. In the "EKModel" file, change the value of d_{13} from 3 to 1 , and report your results again. Comment on the changes and interpret them: how does this fall in trade costs between countries 1 and 3 affect trade flows and the real wages of all countries? Interpret your results.**

	Country 1	Country 2	Country 3	Country 4
Expenditure Shares				
Country 1	0.3340	0.1613	0.3340	0.1707
Country 2	0.2724	0.6600	0.0436	0.0241
Country 3	0.2874	0.2720	0.3478	0.0927
Country 4	0.1381	0.0581	0.3268	0.4770
Real Wages				
Real Wage	4.2412	3.6951	3.5990	1.7743
Real Trade Flows				
Imports 1 from 2	1.3680			
Imports 1 from 3	2.8334			
Imports 1 from 4	1.4480			
Imports 2 from 1		2.0129		
Imports 2 from 3		0.3221		
Imports 2 from 4		0.1781		
Imports 3 from 1			2.0688	
Imports 3 from 2			1.9581	
Imports 3 from 4			0.6674	
Imports 4 from 1				0.4899
Imports 4 from 2				0.2061
Imports 4 from 3				1.1598

Table 2: Reduction of trade costs for 1 too acquire goods in 3

Claim. *Nothing surprising is happening. Countries 2 and 4 looses from this change in term of real wage, while countries 1 and 3 increases their wage (if one confronts the two table is easy to see that this reduction makes easy for country one to import goods of country 3, namely first table, first row, third column we see that the expenditure share of country 1 on goods purchased from country 3 increases from a 7.8% to a staggering 33.8%. Why is it bad for countries 2 and 4? For the same* *reason but reversed, they were exporting quite a lot in country 1 (23.1% and 23.5%. NB since we are talking of exporting we refer to the first row that represent the expenditure share of country 1) and now country 3 offeres much lower price in the countries shopping just because of less trade costs (that are dead lost costs) but usually is not without cost (whether technological or legal) and usually is not so much confined.*

Are pro free trade at wrong? We argue no. But while for the passage from autarky to free trade there are no doubts that countries are better off (simply their productivity frontier cannot be lower) sudden reduction in trade costs create winners and looser. But be awere that in this case we considered an exogenus and strong variation of trade costs between goods both from country 3 by country 1. It's not impossible (regional agreements may be the case why this happens)

5. **c. Put d_{13} back to 3 , and now change the value of θ from 2 to 4 . Report your results. How can you interpret this increase in θ, and what effects does it have on countries?**

We report the table in the next page (see Table 3 in the neext page)

Claim. *Recall that we can think of T_i as reflecting absolute advantage while θ generates comparative advantage. A reduction in generalized comparative advantage reduce the welfare of the countries because now dispersion is lower and the gains from trade are reduced.*

	Country 1	Country 2	Country 3	Country 4
Expenditure Shares				
Country 1	0.6161	0.1139	0.0165	0.2535
Country 2	0.1747	0.8134	0.0097	0.0022
Country 3	0.1982	0.1408	0.6285	0.0325
Country 4	0.0312	0.0044	0.3782	0.5862
Real Wages				
Real Wage	2.3263	2.4017	2.1541	1.6655
Real Trade Flows				
Imports 1 from 2	0.5300			
Imports 1 from 3	0.0766			
Imports 1 from 4	1.1794			
Imports 2 from 1		0.8393		
Imports 2 from 3		0.0465		
Imports 2 from 4		0.0103		
Imports 3 from 1			0.8540	
Imports 3 from 2			0.6066	
Imports 3 from 4			0.1400	
Imports 4 from 1				0.1039
Imports 4 from 2				0.0146
Imports 4 from 3				1.2599

Table 3: Change in θ