International Trade and Globalization

Solutions To Problem Set Four

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1 EXERCISE 1 - Population growth in the Krugman model

Consider the Krugman (1979) model as described in the lecture notes. Assume that in the autarky equilibrium, the population of a country increases from L to 2L

1. What are the effects of this increase in population on the PP and on the ZZ curve?

The PP curve is obtained by solving the problem of profit maximization in the supply side, that is:

$$\max_{p(i)} \quad \pi = p(i)c(i) - \left(\frac{c(i)L}{z} + f\right)w \tag{1}$$

The resulting F.O.C. is as follows:

$$c(i) + p(i)c(i)p(i) - \frac{w}{z}c(i)p(i) = 0 \tag{2}$$

Multiplying both sides by $\frac{p(i)}{c(i)}$ and considering that the elasticity of demand is given by $\epsilon(c(i)) = c(i)p(i)\frac{p(i)}{c(i)}$, after rearranging we get:

$$\frac{p(i)}{w(i)} = \frac{\epsilon(c(i))}{\epsilon(c(i)) - 1}\frac{1}{z} \tag{3}$$

In the model all varieties are symmetric and we get: $p(i) = p$ and $c(i) = c$. Hence

$$\frac{p}{w} = \frac{\epsilon(c)}{\epsilon(c) - 1}\frac{1}{z} \tag{4}$$

is the PP curve. As we can observe it is not affected by the doubling of the population. Multiplying the demand function by 2 does not affect elasticity, while population is not present in the first order condition of the producers.

Regarding the ZZ cruve, since the model assumes free entry profits must be equal to zero, otherwise new firms would enter to reap the benefits of the market. In the model the zero profit condition is possible if and only if:

$$\frac{p}{w} = \frac{f}{Lc} + \frac{1}{z} \tag{5}$$

And we call this equation the ZZ curve. When population moves from L to 2L the curve shifts leftward. In words, this mean that for the same level of c $\frac{p}{w}$ will be lower in equilibrium. The two curve in equilibrium solves for c (consumption of each variety) and $\frac{p}{w}$, the level of the markup that firms can impose.

Intuitively, the basic assumption of the model is increased return to scale. A larger market means increased return for the firms, that have lower average cost, and positive profits lure in new firms until π goes to zero. The outcome is icreased competition, which reduced the markup that firms can impose and $\frac{w}{p}$ goes up.

2. What are the effects of this increase in population on the mass of varieties produced?

To solve for the mass of varieties produced we need to introduce the labor market clearing condition $Ml = L$ rewritten as (since each variety uses $(f + \frac{Lc}{z})$ units of labor):

$$M\left(f + \frac{Lc}{z}\right) = L \tag{6}$$

which implies:

$$M = \frac{L}{f + \frac{Lc}{z}} \tag{7}$$

Whose first derivative is positive in L: $\frac{fz^2}{(cL+fz)^2}$

This means that when the size of the population doubles, the variety of products present on the market increases less than proportionately. There is more variety overall, but less "per-capita" variety.

This is a consequence of higher competition, as seen in the previous point. Indeed, proportionately less firms are able to stay in the market with at least zero profits. However, the overall number of firms is higher thanks to increased productivity due to increasing return to scale.

3. Is there any difference between population growth and international trade in this model (i.e., would anything be different if instead of multiplying its population by 2, the country would have opened up to another identical country with population L?) Why or why not

There is no difference. This happens because in the Krugman model of 1979 there are no trade costs, and production costs are homogeneous in the two trading countries. Hence, the movement from autarky to trading actually corresponds to a doubling of the population. The situation would be different if assumprions were relaxed. For example, if there were trading costs or countries were assymmetric.

2 EXERCISE 2 - Trade costs in the Krugman Model

Equilibrium masses of varieties.

1. What is the marginal cost of a home firm when delivering a good to the foreign market? Therefore, what is the price at which a home firm sells on the foreign market? Denote this price by p_H^*.

$MC_H^* = \tau w_H$ therefore, $p_H^* = \frac{\epsilon}{\epsilon-1}\tau w_H$

2. Write down the profits of a home firm as a function of p_H (the price charged on the home market), p_H^*, c_H, c_H^* and w_H **Ho dei dubbi sulla funzione dei profitti che ho usato.**

Total Revenue:

- *Home sales*:

$$R_H = p_H \cdot L_H \cdot c_H$$

- *Foreign sales*:

$$R_F = p_H^* \cdot L_F \cdot c_H^*$$

Total Cost:

- *Domestic cost (marginal cost)*:

$$\text{Cost}_H = \frac{w_H}{z} \cdot L_H \cdot c_H$$

- *Export costs:*

$$Cost_F = \tau \cdot \frac{w_H}{z} \cdot L_F \cdot c_H^*$$

- *Fixed cost (in labor units)*:

$$\text{Cost}_{\text{fixed}} = w_H \cdot f$$

- *Total cost*:

$$C = w_H \cdot \left[f + \frac{L_H \cdot c_H}{z} + \tau \cdot \frac{L_F \cdot c_H^*}{z} \right]$$

Total Profit:

$$\pi = R_H + R_F - C = p_H L_H c_H + p_H^* L_F c_H^* - w_H \left[f + \frac{L_H c_H}{z} + \tau \cdot \frac{L_F c_H^*}{z} \right]$$

Pricing Rules and Substitutions:

- Domestic price:

$$p_H = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{w_H}{z} \Rightarrow \frac{w_H}{z} = \frac{\varepsilon - 1}{\varepsilon} \cdot p_H$$

- Foreign price:

$$p_H^* = \frac{\varepsilon}{\varepsilon - 1} \cdot \tau \cdot \frac{w_H}{z} \Rightarrow \tau \cdot \frac{w_H}{z} = \frac{\varepsilon - 1}{\varepsilon} \cdot p_H^*$$

Plug back into cost:

$$C = w_H \cdot f + \frac{w_H}{z} \cdot L_H c_H + \tau \cdot \frac{w_H}{z} \cdot L_F c_H^* = w_H \cdot f + \frac{\varepsilon - 1}{\varepsilon} \left(p_H L_H c_H + p_H^* L_F c_H^* \right)$$

Profit expression:

$$\pi = p_H L_H c_H + p_H^* L_F c_H^* - \left[w_H f + \frac{\varepsilon - 1}{\varepsilon} \left(p_H L_H c_H + p_H^* L_F c_H^* \right) \right]$$

$$\Rightarrow \pi = \frac{1}{\varepsilon} \left(p_H L_H c_H + p_H^* L_F c_H^* \right) - w_H f$$

Replace w_H using pricing rule:

$$w_H = \frac{\varepsilon - 1}{\varepsilon} \cdot p_H \cdot z \Rightarrow w_H f = \frac{\varepsilon - 1}{\varepsilon} \cdot p_H \cdot z \cdot f$$

Final Profit Formula:

$$\pi = \frac{1}{\varepsilon} \left(p_H L_H c_H + p_H^* L_F c_H^* \right) - \frac{\varepsilon - 1}{\varepsilon} \cdot p_H \cdot z \cdot f$$

3. Imposing free entry and using your response to question 1, deduce an expression for the total production of a home firm, $c_h + \tau c_h^*$, as a function of f and ϵ.

Zero profit (free entry condition):

$$\pi = 0 \Rightarrow \frac{1}{\varepsilon} \left(p_H L_H c_H + p_H^* L_F c_H^* \right) = \frac{\varepsilon - 1}{\varepsilon} \cdot p_H \cdot z \cdot f$$

Multiply both sides by ε:

$$p_H L_H c_H + p_H^* L_F c_H^* = (\varepsilon - 1) \cdot p_H \cdot z \cdot f$$

Divide both sides by $p_H \cdot z$:

$$L_H c_H + \frac{p_H^*}{p_H} \cdot \frac{L_F c_H^*}{z} = (\varepsilon - 1) f$$

Use the pricing rule: $\frac{p_H^*}{p_H} = \tau$

$$L_H c_H + \tau \cdot \frac{L_F c_H^*}{z} = (\varepsilon - 1) f$$

Express total production per firm:

Recall total production per firm is:

$$y = c_H + \tau c_H^*$$

We do not multiply both terms by population, but we observe that:

$$L_H c_H + \tau \cdot \frac{L_F c_H^*}{z} = (\varepsilon - 1) f \Rightarrow \frac{L_H}{z}(c_H + \tau c_H^*) = (\varepsilon - 1) f$$

Final expression:

$$c_H + \tau c_H^* = \frac{(\varepsilon - 1) \cdot f \cdot z}{L_H}$$

This is the total quantity produced per firm serving L_H home and L_F foreign consumers.

4. Finally, use the labour market clearing condition to deduce the equilibrium masses of varieties in Home and in Foreign, M_H and M_f. Do they depend on trade costs? Which country produces more varieties?

Labour required per firm:

$$\ell = f + \frac{1}{z}\left(c_H + \tau c_H^*\right)$$

$$c_H + \tau c_H^* = \frac{(\varepsilon - 1) \cdot f \cdot z}{L_H}$$

$$\ell = f + \frac{1}{z} \cdot \frac{(\varepsilon - 1)fz}{L_H} = f + \frac{(\varepsilon - 1)f}{L_H} = f\left(1 + \frac{\varepsilon - 1}{L_H}\right)$$

Labour market clearing:

$$M_H \cdot \ell = L_H \Rightarrow M_H = \frac{L_H}{f\left(1 + \frac{\varepsilon - 1}{L_H}\right)} = \frac{L_H^2}{f(L_H + \varepsilon - 1)}$$

$$M_F = \frac{L_F^2}{f(L_F + \varepsilon - 1)}$$

Relative wages.

5. Write down an expression for the ideal price index in home, as a function of M_H, M_F, p_H and p_F^*

Ideal price index in Home. Maintaining the same structure of the ideal price index as in the CES general equilibrium model (Problem Set 1)

$$P_H = \left(\sum_{i \in \text{Home}} p_H^{1-\varepsilon} + \sum_{i \in \text{Foreign}} (p_F^*)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \quad \text{(CES price index over all varieties)}$$

$$P_H = \left(M_H \cdot p_H^{1-\varepsilon} + M_F \cdot (p_F^*)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \quad \text{(using symmetry: home imports } M_f \text{ from foreign at the price } p_F^*)$$

6. Use the result from Question 5, the results from the first part of the exercise, and the CES demand function to derive an expression for c_H and c_H^* as a function of only wages w_H and w_F (and parameters).

CES demand per variety

$$c(i) = \left(\frac{p(i)}{P}\right)^{-\varepsilon} \cdot \frac{w}{P} \quad \text{(CES demand with symmetric varieties)}$$

Home demand for a domestic variety

$$c_H = \left(\frac{p_H}{P_H}\right)^{-\varepsilon} \cdot \frac{w_H}{P_H}$$

Foreign demand for a Home variety (exports)

$$c_H^* = \left(\frac{p_H^*}{P_F}\right)^{-\varepsilon} \cdot \frac{w_F}{P_F} \quad \text{with } p_H^* = \tau \cdot p_H$$

Foreign price index

$$P_F = \left(M_F \cdot p_F^{1-\varepsilon} + M_H \cdot (p_H^*)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Prices with markup rule

$$p_H = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{w_H}{z} \quad , \quad p_F = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{w_F}{z} \quad , \quad p_H^* = \tau \cdot p_H = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{\tau w_H}{z}$$

Price index in Home with substitution

$$P_H = \left(M_H \cdot \left(\frac{\varepsilon}{\varepsilon - 1} \cdot \frac{w_H}{z}\right)^{1-\varepsilon} + M_F \cdot \left(\frac{\varepsilon}{\varepsilon - 1} \cdot \frac{\tau w_F}{z}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Factor out the common constant:

$$P_H = \left(M_H \cdot w_H^{1-\varepsilon} + M_F \cdot (\tau w_F)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \cdot \text{const}$$

Final expression for c_H:

$$c_H = \frac{w_H}{\left(M_H \cdot w_H^{1-\varepsilon} + M_F \cdot (\tau w_F)^{1-\varepsilon}\right)^{\frac{\varepsilon}{1-\varepsilon}}}$$

Similarly, for exports c_H^*:

$$c_H^* = \frac{w_F}{\left(M_F \cdot w_F^{1-\varepsilon} + M_H \cdot (\tau w_H)^{1-\varepsilon}\right)^{\frac{\varepsilon}{1-\varepsilon}}}$$

7. Combining these expressions with the result from Question 3, show that you get

$$1 = \frac{L_H}{L_H + \tau^{1-\varepsilon}\left(\frac{w_F}{w_H}\right)^{\varepsilon-1}} + \frac{\tau^{1-\varepsilon} \cdot L_F \cdot \left(\frac{w_F}{w_H}\right)^{\varepsilon}}{L_F + \tau^{1-\varepsilon}\left(\frac{w_F}{w_H}\right)}$$

3 EXERCISE 3 - Empirics of the Melitz model

Download the dataset melitz.xlsx or melitz.dta, depending on whether you prefer to work with Excel or Stata. I recommend Stata, but if you are not familiar with it, Excel is fine. Note that this is no real-world data, but data which I have artificially created. It gives you information for 168 firms of an imaginary country which has done a radical trade liberalization on December 31st, 2014, passing from autarky to free trade. You are given, for every firm, an identifier (firm_id), and the firms' sales, physical output and employment for 2014 and 2015 (missing information in a given cell means that the firm is not active in that year). Furthermore, the dataset contains a dummy variable called X, equal to 1 if a firm exports in a given year, and equal to 0 if it does not. Assume that this data has been generated by a Melitz model.

1. Assume you know from independent evidence that the fixed production cost in this imaginary country is f = 1 unit of labour. Use this information to calculate the productivity of every firm in the dataset (you don't need to report your complete results for this question, but you will need them to answer the next ones)

In 2014 average productivity is 2.114324, standard deviation is 2.432035. The highest level of productivity is 17.762, while the lowest is 0.40023.

In 2015 average productivity is 2.608473, standard deviation is 2.612875. The highest level of productivity is still 17.762, while the lowest is now 0.8013279.

2. How would you identify the productivity cut-off $z*$? What is your estimate for that cut-off in autarky and after the trade liberalization?

The production cut-off is given by the intersection of two functions which cross only once. The first is the free entry condition (FE), a positive relationship between the production cut-off and the "aggregate demand term" $\frac{B}{w}$, which states that expected profits before entry must be equal to the entry fee to be paid. The second is the Production cut-off, a positive relation between the same two variabls, that states that profit must be equal to zero for a firm with the cut-off level of productivity.

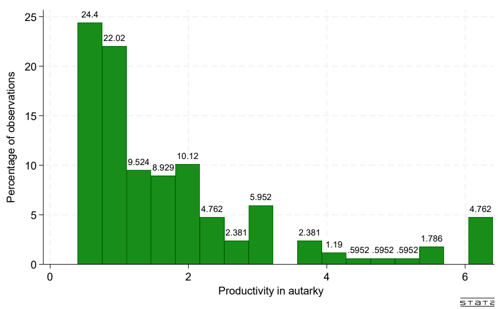
The data is generated according to a Melitz model. Thus, the fact that a firm is present in the dataset shows that it took the decision to pay the entry fee and enter the market, meaning that the FE condition must be respected for all levels of productivity observed. As for production, in autarky it is positive for all firms too.
(We are assuming that production lower than one is still positive, as could be the case of the number of tons of coal produced by a mine. If instead production could take place only in discrete numbers, for example number of cars produced, 70 firms would be choosing not to produce in the dataset).
When trade happens it is zero for some firms and not all firms that remain in the market choose to export.
In 2014, the firm with the lowest level of productivity has z = 0.40023, which represents the cutoff $z*$ in autarky given what was said above.
In 2015 trade takes place and we would expect the Free Entry curve to shift up, raising the cut-off. Indeed, the lowest productivity of a firm with positive production becomes 0.8013279. This suggests that the cutoff is between this number and 0.78078 , the firm with lower productivity closer to 0.80123279 in 2014. $z*_T \in (0.78078, 0.80123279]$ All firms with productivity below the cutoff exit the market (41 firms in the sample).

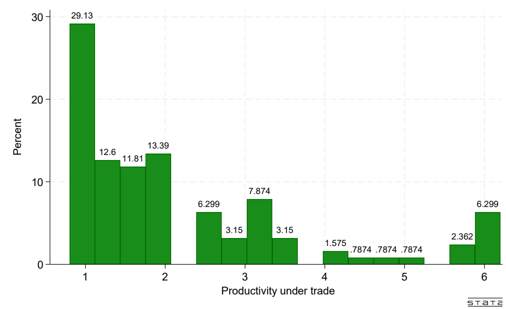
3. What is your estimate for the export cut-off $z*_X$?
The reasoning is the same for exporters. The firm taking part in trade with lowest productivity

has z = 2.042619, the closest among non-exporters is 1.984692. Hence, the cut-off must be included between this two numbers with the upper bound included.

4. Histograms:

i)

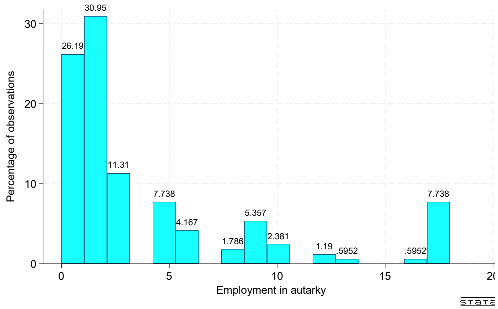


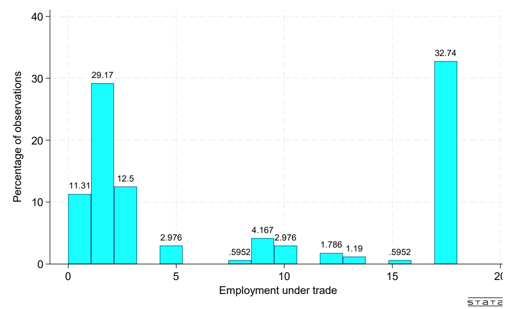


In line with predictions of the model, the percentage of high productivity firms increased with trade, as those below the cutoff exit the market.

ii)

Employment remains stable overall. However, workers from low productivity firms that exit the market concentrate more in high productivity size.


Percentage of observations
Employment in autarky
26.19
30.95
11.31
7.738
4.167
1.786
5.357
2.381
1.19
.5952
.5952
7.738
0
10
20
30
0
5
10
15
20


Percentage of observations
Employment under trade
11.31
29.17
12.5
2.976
.5952
4.167
2.976
1.786
1.19
.5952
32.74
0
10
20
30
40
0
5
10
15
20

5. Estimate exporter premia for sales and for employment in 2015

The estimate for the coefficient of the regression of sales on the dummy of being an exporter in 2015 is 62.92648.
The estimate for the coefficient of the regression where unemplotment is the dependent variable and the dummy of being an exporter the undependent variable in 2015 is 20.97549.

6. In this exercise, everything went smoothly, as all variables of the artificially created dataset were completely in line with the model. What do you think would be potential issues (for measurement, for econometric biases, etc.) when trying to test the predictions of the Melitz model with real-world data?

In terms of measurment, in the real world it could be more difficult to assess productivity of different firms. In the dataset we just used the only factor of production is labor, something that is not true for the vast majority of real firms. In general, it might be difficult to collect definite information on entry cost (here assumed equal to 1), fixed costs and variable costs in large and complex firms.
Biases might be an issue too: Firstly, the model assumes that exporters are more productive due to a selection process. The opposite might be true, with firms that are already exporters becoming more efficient thanks to the exposure to innovation and higher competition. Secondly, international markets are not completely free, as trade between countries is higly regulated. Thus, exporting firms might be favored by government policies such as subsidies. In the same vein, firms producing for the local market might be less productive due to tariffs that protect them from competion. Lastly, the least productive firms might survive in local markets by not paying taxes when the institutional framework allows it. Then, it would be very difficult to collect data on them and the resulting distribution of productivities would be biased.