International Trade and Globalization

Solutions To Problem Set Five

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1 Exercise 1 -Analytical solution of the Melitz-Chaney model

Consider the Melitz-Chaney model described in the lecture notes on "heterogenous firms". Recall that the cdf of the Pareto distribution is $G(z) = 1 - z^{-\theta}$, and its density is $g(z) = \theta z^{-\theta-1}$.

1. **Consider the system of equations defining the autarky equilibrium (Equation (9) on Page 10). Use the density of the Pareto distribution to solve for the autarky values of z^*, $\frac{B}{w}$ and the real wage $\frac{w}{P}$.**

Consider the following system that allows us to pin down z^* and $\frac{B}{w}$:

$$\begin{cases} z^{*\varepsilon-1}\frac{B}{w} = f \\ \int_{z^*}^{+\infty}\left(z^{\varepsilon-1}\frac{B}{w} - f\right)g(z)dz = f_E \end{cases}$$

Solving the integral we get that (we used the fact that $z^{*\ -(\varepsilon-1)}f = \frac{B}{w}$):

$$\frac{\varepsilon-1}{\theta-\varepsilon-1} f z^{*-\theta} = f_E$$

That with some computations yield: $z^* = \left(\frac{f}{f_E}\frac{\varepsilon-1}{\theta-\varepsilon+1}\right)^{\frac{1}{\theta}}$ hence:

$$z^* = \left(\frac{f}{f_E}\frac{\varepsilon-1}{\theta-\varepsilon+1}\right)^{\frac{1}{\theta}}$$
$$\frac{B}{w} = \frac{f}{z^{*\varepsilon-1}}$$

Please note that we have solved the integral under the fundamental assumption $\theta > \varepsilon - 1$. For sake of completeness we re derive equation (12) of the Lecture Notes for which:

$$\frac{w}{P} = \frac{\varepsilon-1}{\varepsilon}M^{\frac{1}{\varepsilon-1}}\left(\int_{z^*}^{+\infty} z^{\varepsilon-1}\frac{g(z)}{1-G(z^*)}dz\right)^{\frac{1}{\varepsilon-1}}$$

Since we still need many of the results. By the law of large numbers in equilibrium $1 - G(z^*)$ are producing hence the total number of varieties is:

$$M = 1 - G(z^*)M_E = z^{*\theta} M_E \text{with our assumption}$$

Where M_E is the mass of firms that payed the entrance cost. Labor supply is inelastic and is L and labor demand is the sum of the entrance fee $f_E M_E$ (in aggregate terms) and the labor used to produce. Each firm requires:

$$f + \frac{c(i)}{z(i)}$$

Now recall that:

$$c(i) = z(i)^{\varepsilon} \frac{B}{w}(\varepsilon - 1)$$

Combining I get that:

$$l(i) = f + z(i)^{\varepsilon-1} \frac{B}{w}(\varepsilon - 1) = (\varepsilon - 1)\left(z(i)^{\varepsilon-1} \frac{B}{w} - f\right) + \varepsilon f$$

Now we can compute:

$$\int_{z^*}^{+\infty} \left[(\varepsilon - 1)\left(z^{\varepsilon-1} \frac{B}{w} - f\right) + \varepsilon f\right] \frac{g(z)}{1 - G\left(z^*\right)} dz = \frac{(\varepsilon - 1) f_E}{z^{*-\theta}} + \varepsilon f$$

Market clearing condition now reads:

$$L = \frac{f_E M}{z^{*-\theta}} + M\left(\frac{(\varepsilon - 1) f_E}{z^{*-\theta}} + \varepsilon f\right)$$

From which is immediate to get:

$$M = \frac{L z^{*-\theta}}{\varepsilon (f_E + f z^{*-\theta})}$$

Now using the expression for z^* we get that:

$$M = \frac{L}{\varepsilon f} \cdot \frac{\theta - \varepsilon + 1}{\theta} \tag{M AUT.}$$

And so:

$$M_E = \frac{L}{\varepsilon(f_E + f z^{*-\theta})} = \frac{\varepsilon - 1}{\varepsilon\theta}\frac{L}{f_E} \tag{M_E AUT.}$$

Now the CES index is:

$$P = \left(\int_0^M \left(\frac{\varepsilon}{\varepsilon - 1}\frac{w}{z(i)}\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

And hence:

$$\frac{w}{P} = \frac{\varepsilon - 1}{\varepsilon} M^{\frac{1}{\varepsilon - 1}} \left(\int_{z^*}^{+\infty} z^{\varepsilon - 1}\frac{g(z)}{1 - G\left(z^*\right)} dz\right)^{\frac{1}{\varepsilon - 1}}$$

Now using everything together, the previous equation, (M_E AUT), (M AUT.) and the expression for the cutoff we get:

$$\frac{w}{P} = \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon - 1}} (\theta - \varepsilon + 1)^{-\frac{1}{\theta}} (\varepsilon - 1)^{\frac{1}{\theta} - \frac{1}{\varepsilon - 1}} L^{\frac{1}{\varepsilon - 1}} f_E^{-\frac{1}{\theta}} f^{\frac{1}{\theta} - \frac{1}{\varepsilon - 1}} \tag{1}$$

2. **How does the autarky level of z^* depend on f, f_E and L ? Interpret these results.**

1. Higher fixed costs increase the productivity trash-old since only the more productive firms can offset this costs ans till have non negative profits. Mathematically, indeed: $\frac{\partial z^*}{\partial f} > 0$
2. Higher entry costs implies higher barriers to entry: old firms are protected even tough they are not super productive, now firms enter only if their production is sufficiently large. Mathematically, indeed: $\frac{\partial z^*}{\partial f_E} < 0$
3. There is no heterogeneity in wages hence no role for labor supply.

3. **How does the autarky level of the real wage $\frac{w}{P}$ depend on f, f_E and L ? Interpret these results.**

1. Fixed costs only affect the cut off. Hence consumer loose utility by wasting resources, in a sense, so negative ffect on real wages. Mathematically, indeed: $\frac{\partial \frac{w}{p}}{\partial f} < 0$

2. Similar to before on the conclusions, BUT now not only the cut off is affected but also trough the mass of firms.

3. Labor supply impact M_E hence increasing real wage. Mathematically, indeed: $\frac{\partial \frac{w}{D}}{\partial L} > 0$

Now, we switch to the trade equilibrium (and keep making the assumption $\tau^{\varepsilon-1} f_X > f$, which ensures that not all firms export in equilibrium).

4. **Show that the percentage of active firms that export in equilibrium is equal to $s_X = \left(\frac{z_X^*}{z^*}\right)^{-\theta}$.**

$$s_X = \frac{\text{Prob}\left(z > z_X^*\right)}{\text{Prob}\left(z > z^*\right)} = \frac{1 - G\left(z_X^*\right)}{1 - G\left(z^*\right)} = \left(\frac{z_X^*}{z^*}\right)^{-\theta}$$

5. **Using the system of equations defining the trade equilibrium, solve for z^*, $\frac{B}{w}$ and the real wage $\frac{w}{P}$ in the trade equilibrium.**

The system reads:

$$\begin{cases} z^{*\varepsilon-1} \frac{B}{w} = f \\ z_X^{*\varepsilon-1} \tau^{1-\varepsilon} \frac{B}{w} = f_X \\ \int_{z^*}^{+\infty} \left(z^{\varepsilon-1} \frac{B}{w} - f\right) g(z) dz + \int_{z_X^*}^{+\infty} \left(z^{\varepsilon-1} \tau^{1-\varepsilon} \frac{B}{w} - f_X\right) g(z) dz = f_E \end{cases}$$

And using the fact that the cut off for trade is:

$$z_X{}^* = \tau \left(\frac{f_X}{f}\right)^{\frac{1}{\varepsilon-1}} z^*$$

I Can reduce the system to:

$$\begin{cases} z^{*\varepsilon-1} \frac{B}{w} = f \\ \int_{z^*}^{+\infty} \left(z^{\varepsilon-1} \frac{B}{w} - f\right) g(z) dz + \int_{z_X^*(z^*)}^{+\infty} \left(z^{\varepsilon-1} \tau^{1-\varepsilon} \frac{B}{w} - f_X\right) g(z) dz = f_E \end{cases}$$

Now start from the first integral, we can write it as

$$\frac{B}{w} z^{*\varepsilon-1-\theta} \left[\frac{\varepsilon - 1}{\theta - \varepsilon + 1}\right]$$

The second integral can be written as (using the first equation in the 3x3 system):

$$\frac{B}{w}\tau^{1-\varepsilon} z_x^{*\varepsilon-1-\theta}\left[\frac{\varepsilon-1}{\theta-\varepsilon+1}\right]$$

That we can re write as (using the cut off for trade):

$$\frac{B}{w}\tau^{-\theta} z^{*\varepsilon-1-\theta}\left[\frac{\varepsilon-1}{\theta-\varepsilon+1}\right]\left(\frac{f_x}{f}\right)^{\frac{\varepsilon-1-\theta}{\varepsilon-1}}$$

Sum the two integrtals to get that:

$$f_E = \frac{B}{w} z^{*\varepsilon-1-\theta}\left[\frac{\varepsilon-1}{\theta-\varepsilon+1}\right]\left[1+\tau^{-\theta}\left(\frac{f_x}{f}\right)^{\frac{\varepsilon-1-\theta}{\varepsilon-1}}\right]$$

Which delivers:

$$z^*_{\text{Trade}} = \left(\frac{\varepsilon-1}{\theta-\varepsilon+1}\left[1+\tau^{-\theta}\left(\frac{f_x}{f}\right)^{\frac{\varepsilon-\theta-1}{\varepsilon-1}}\right]\frac{f}{f_E}\right)^{\frac{1}{\theta}} \tag{2}$$

The remaining follow trivially from previous formulas (In any case are later reported to solve for the ACR formula)

6. **Using your previous results, solve for the mass of entrants M_E and the mass of active firms M, both in autarky and with free trade. In particular, show that the mass of entrants is not affected by trade, as claimed in the lecture notes.**

To compute the expression for the real wage in Autarky Equilibrium we already computed the mass of entry firms and the mass of firms. We called those equations (M AUT) and (M_E AUT).

Following similar passages to before (with the addition of the trade cut off) we start from the labor market condition:

$$L = M_E f_E + M_E\int_{z^*}^{\infty}\left(f+(\varepsilon-1)z^{\varepsilon-1}\frac{B}{w}\right)g(z)dz + M_E\int_{z_X^*}^{\infty}\left(f_X+(\varepsilon-1)\tau^{1-\varepsilon}z^{\varepsilon-1}\frac{B}{w}\right)g(z)dz$$

Wich can be rewritten as:

$$\frac{L}{M_E} = \varepsilon \left(f_E + f \left(1 - G\left(z^* \right) \right) + f_X \left(1 - G\left(z_X^* \right) \right) \right)$$

And substituting we find exactly (M AUT) and (M_E AUT).

7. **Using your results from questions 1 and 5, calculate the ratio between real wages with trade and in autarky. Then, using equation 26 on page 22, compute the value of π_{nn} in the trade equilibrium, and use these results to show that the ACR formula holds.**

The equation we need is:

$$\pi_{nn} = \frac{\int_{z^{\text{Trade}\,*}}^{+\infty} z^{\varepsilon-1} g(z) dz}{\int_{z^{\text{Trade}\,*}}^{+\infty} z^{\varepsilon-1} g(z) dz + \tau^{1-\varepsilon} \int_{z_X^*}^{+\infty} z^{\varepsilon-1} g(z) dz} \tag{26 LN}$$

Which under our parametric assumption becomes:

$$\pi_{nn} = \frac{z_{\text{Trade}}^{*\varepsilon-\theta-1}}{z_{\text{Trade}}^{*\varepsilon-\theta-1} + \tau^{1-\varepsilon} z_X^{*\varepsilon-\theta-1}}$$

Similarly to before we find The basket price to be:

$$P = \frac{\varepsilon}{\varepsilon - 1} w \left[M_E \left(\frac{\theta}{\theta - \varepsilon + 1} \right) \left(z_{\text{Trade}}^{*\varepsilon-1-\theta} + \tau z_X^{*\varepsilon-1-\theta} \right) \right]^{\frac{1}{1-\varepsilon}}$$

Hence the ACR formula becomes:

$$\frac{\left(\frac{w}{P}\right)_{\text{Trade}}}{\left(\frac{w}{P}\right)_{\text{Autarky}}} = \pi_{nn}^{\frac{1}{1-\varepsilon}} \cdot \frac{z_{\text{Trade}}^{*\frac{\varepsilon-1-\theta}{\varepsilon-1}}}{z^{*\frac{\varepsilon-1-\theta}{\varepsilon-1}}} = \pi_{nn}^{-\frac{1}{\theta}}$$

8. **Discuss the effect of trade on:**

   - The number of domestic varieties.

   - The price/wage ratio of domestic varieties.

   - The number of foreign varieties. Which of these channels contributes to gains

from trade, and which of it limits them?

1. The number of domestic varieties $\downarrow$
2. The price/wage ratio of domestic varieties $\uparrow$
3. The number of foreign varieties $\uparrow$

Exercise 2 - Some more results on the Bustos model

Consider the Bustos model, as described in section 6.2.2 of the lecture notes. Assume that, as in the Melitz-Chaney model, the distribution of productivity is Pareto, such that $G(z) = 1 - z^{-\theta}$

Autarky

Consider first a country in autarky:

1. **Show that if $\eta > \gamma^{\epsilon-1}$, there are two cut.offs, z^* and z_A^*, such that all firms with productivity draws higher than z^* produce, and all firms with productivity draws higher than z_A^* use the advanced technology. Derive an expression for these cut-offs as a function of $\frac{B}{w}$. Why is$\eta > \gamma^{\epsilon-1}$ a necessary and sufficient condition for this costellation to occur in equilibrium?**

   In line with Bustos(2011) we consider η to be an increase in fixed costs for firms and γ as the related growth in productivity.

   The cut-offs z^* and z_A^* are found respectively at the minimum level of productivity that allows firm to produce, that is when profits are zero, and at the minimum level of productivity that makes a firm indifferent between producing with the old techology or adopting the advancement. that is:

$$\pi^B(i) = z^*(i)^{\epsilon-1}B - wf = 0$$

And

$$\pi^A(i) = (\gamma z_A^*)^{\epsilon-1}B - w(\eta f) = \pi^B(i)(z_A^*)$$

Where B is an auxiliary variable which depends on wages, the aggregate price level, and aggregate income.

The first condition is respected when:

$$z^* = \frac{1}{\left(\frac{B}{w}\right)^{\frac{1}{\epsilon-1}}}(fw)^{\frac{1}{\epsilon-1}}$$

And the second when:

$$z_A^* = \left[\frac{(\eta-1)f}{\frac{B}{w}(\gamma^{\epsilon-1}-1)}\right]^{\frac{1}{\epsilon-1}}$$

Note that both functions are negative in $\frac{B}{w}$ and that $z_A^* > z^*$ if and only if $\frac{\eta}{\gamma^{\epsilon-1}} > 1$. We can see this by writing z_A^* as a function of z^*:

$$z_A^* = z^*\left(\frac{\eta-1}{\gamma^{\epsilon-1}-1}\right)^{\frac{1}{\epsilon-1}}$$

Otherwise, all producing firms would adopt the advanced technology, contraddicting Bustos' assumptions and data.

2. **Write down the free entry condition, and use this to solve for the equilibrium levels of z^* and z_A^*. What is the percentage of producing firms that use the advanced technology in equilibrium? How does this percentage depend on the parameters γ and η?**

   The free entry condition requires that expected profits before entry are equal to zero. To compute this we need. To form expectations, note that $g(z) = \theta z^{-(\theta+1)}$, and that no firm will produce with productivity $z' < z^*$, or $G(z') = 0$. Hence, FE is written like:

$$\mathbb{E}[\pi(z)] = \int_{z^*}^{z_A^*} \left(\frac{B}{w} z^{\epsilon-1} - f\right) \theta z^{-(\theta+1)}\, dz + \int_{z_A^*}^{+\infty} \left(\frac{B}{w}(\gamma^{\epsilon-1} - 1) z^{\epsilon-1} - (\eta - 1) f\right) \theta z^{-(\theta+1)}\, dz = 0$$

It is a positive relation between the productivity cut-off z^* (we can do without z_A^* since it can be expressed as a function of the other). The equilibrium level is given by the intersection of the curve above with the cut-off curve. The intersection always exist in the model as both curves are linear in $z(i)^{\epsilon-1}$ and they go in opposite directions.

The share of producing firms that use advanced technology can be computed as: $\mathbb{P}(z \geq z_A^* | z \geq z^*) = \frac{\Pr(z \geq z_A^*)}{\Pr(z \geq z^*)} = \left(\frac{z^*}{z_A^*}\right)^{\theta}$.

Using the expressions for the cut-offs found above :

$$\left(\frac{z^*}{z_A^*}\right)^{\theta} = \left(\frac{\frac{f}{(B/w)}}{\frac{\eta f}{(B/w)(\gamma^{\epsilon-1}-1)}}\right)^{\frac{\theta}{\epsilon-1}} = \left(\frac{\gamma^{\epsilon-1} - 1}{\eta}\right)^{\frac{\theta}{\epsilon-1}}$$

When γ increases the percentage grows and more producing firms adopt the advanced technology as the benefits are higher. When the cost of adoption η increases only very productive firms will choose the technology and the percentage decreases. The parameters ϵ and θ are elasticities that regulated how sharply responses to changes are. (See point 4 for the explicit computation of z^*)

Trade Now assume that the country opens up to trade with another identical country

3. **Still assuming that, as in autarky, $\eta > \gamma^{\epsilon-1}$, find a sufficient condition on parameters for the equilibrium to be defined by three cut-offs, as in the LN. The cutoffs are $z^* < z_X^* < z_A^*$, such that all firms with productivity draws higher than z^* produce, all firms with productivity draws higher than z_X^* export, and all firms with productivity draws higher than z_A^* use the advanced technology**

In line with the model we assume both variable (iceberg) and fixed trade costs for exporting, as reported in the profits.

The method to be used to find the cut-offs is the same as in question 1, with the difference that now we hav three crossing points:

Firms are expected to become producers when their level of productivity is enough to grant them non-negative profits:

$$\pi^B(i) = z^*(i)^{\epsilon-1}B - wf = 0$$

Firms can decide to export when, considering the costs, the profits from selling on both markets are at least the same of selling only on the domestic market:

$$\pi(i)^B_X = z^*_X(i)^{\epsilon-1}(1+\tau^{1-\epsilon})B - w(f+f_X) = \pi(i)(z^*_X)^B$$

Finally, they decide to adopt the advanced technology to serve both markets when, given productivity, their profits are at least as high as the level they reach while exporting with the basic technology, the cut-off is given by:

$$\pi(i)^A_X = (\gamma z^*_A(i))^{\epsilon-1}(1+\tau^{1-\epsilon})B - w(\eta f + f_X) = \pi(i)(z^*_A)^B_X$$

To write the cut-offs in this form we are assuming that in a graph with z on the y-axis and $\frac{B}{w}$ on the x-axis (a) the crossing between π^B and zero comes earlier (for a lower level of z) than the one between π^B_X and π^B, and (b) that the latter cross earlier than π^A_X and π^B_X.

(a) is achieved in the model when

$$f < f_X\tau^{\epsilon-1} < \frac{(\eta-1)f}{\gamma^{\epsilon-1}-1}$$

and for (b) we have:

$$f_X \tau^{\epsilon-1} < \frac{(\eta-1)f}{(\gamma^{\epsilon-1}-1)(1+\tau^{1-\epsilon})}$$

Merging the two inequalities we get a sufficient condition for our model to reflect assumptions:

$$f < f_X \tau^{\epsilon-1} < \frac{(\eta-1)f}{(\gamma^{\epsilon-1}-1)(1+\tau^{1-\epsilon})}$$

4. **Write down the free entry condition, and use it to solve for the cutoffs z^*, z_X^* and z_A^* :**

As in exercise 2, the free entry condition is given by putting expected profits equal to zero given the distribution of productivities.

$$\begin{aligned} f_E = \int_{z^*}^{z_X^*} \left(z(i)^{\epsilon-1}\frac{B}{w} - f \right) \theta z^{-(1+\theta)}\, dz + \int_{z_X^*}^{z_A^*} \left(z(i)^{\epsilon-1}(1+\tau^{1-\epsilon})\frac{B}{w} - (f+f_X) \right) \theta z^{-(1+\theta)}\, dz \\ + \int_{z_A^*}^{\infty} \left((\gamma z(i))^{\epsilon-1}(1+\tau^{1-\epsilon})\frac{B}{w} - (\eta f + f_X) \right) \theta z^{-(1+\theta)})\, dz = \mathbb{E}\left[\pi(z)\right] \end{aligned}$$

The cut-off levels of productivity are given by:

$$z^* = \frac{1}{\left(\frac{B}{w}\right)^{\frac{1}{\epsilon-1}}} (fw)^{\frac{1}{\epsilon-1}}$$

Exactly like exercise 1

$$z_X^* = \left(\frac{f_X}{(B/w)\tau^{1-\epsilon}} \right)^{\frac{1}{\epsilon-1}}$$

And

$$z_A^* = \left[\frac{f(\eta - 1) + f_X}{(B/w)(1 + \tau^{1-\epsilon})(\gamma^{\epsilon-1} - 1)}\right]$$

Again, the higher cut-offs can be expressed as functions of z^*

$$z_X^* = z^* \left(\frac{f_X}{f}\right)^{\frac{1}{\epsilon-1}}$$

$$z_A^* = z^* \left(\frac{\eta f + f_X}{f(1 + \tau^{1-\epsilon})\gamma^{\epsilon-1}}\right)^{\frac{1}{\epsilon-1}}$$

N.B. with CES preferences ϵ is assumed to be larger than one and $1 - \epsilon$ is always a negative number.

By substituting the values with the functions of z^* in the free entry equation we get a system of two equations that can be solved to find the equilibrium given data on the initial parameters. In particular,

$$z^* = \left(\frac{\kappa f \xi}{f_E}\right)^{\frac{1}{\theta}}$$

where

$$\kappa = \frac{\theta}{\theta - (\epsilon - 1)}$$

and

$$\xi = 1 + \left(\frac{f_X}{f}\right)^{1-\frac{\theta}{\varepsilon-1}} \tau^{-\theta} + \left(\frac{\eta f + f_X}{f(1 + \tau^{1-\varepsilon})\gamma^{\varepsilon-1}}\right)^{\frac{-\theta}{\varepsilon-1}} \left(\eta - 1 - \frac{f_X}{f}\right)$$

The remaining cut-offs can be computed using their form as functions of z^*

5. **Calculate again the percentage of producing firms which use the advanced technology. How does this compare to autarky?**

   The method used to compute this percentage doesn't change with respect to point 2. Now the result is:

$$\left(\frac{z_A^*}{z^*}\right)^\theta = \left[\frac{(\gamma^{\epsilon-1}-1)(1+\tau^{1-\epsilon})}{\eta-1}\right]^{\frac{\theta}{\epsilon-1}}$$

   It is immediate to see that this percentage is higher than the one we found in point 2. Hence, trade increases the percentage of firms adopting the new technology, as it was observed in the data for the MERCUSOR by Bustos (2011).

   a) The percentage is inversely proportional to τ (again, because we assume $\epsilon > 1$), meaning that when trade variable costs are reduced more firms consider adoption of the new technology.

   b) The percentage is not affected by fixed trade costs. This is due to the fact that all firms that adopt the new technology are already exporters, thanks to the condition outlined in point 3.

   c) This happens because there is no complementarity between technology adoption and exporting in terms of fixed costs. If we disentangle the profit function and isolate the part of export from innovating, we get:

$$(\gamma - 1)z^{\epsilon-1}(\tau^{1-\epsilon})P^{\epsilon-1}W$$

   And we can see that fixed costs for exporting do not enter into it. Instead, fixed costs are going to affect the percentage of firms that decide to export given basic technology.

Exercise 3 - Multinational activity and firm heterogeneity

Consider the baseline Melitz model described in section 3.3 of the lecture notes. Maintain all assumptions of that model, but assume now additionally that firms are given the opportunity to serve the market of the foreign country through FDI. Opening a foreign subsidiary entails a fixed cost f_{FDI} (holding $f_{FDI} > f$), but allows the firm not to pay iceberg transport costs.

1. Write down the profit function of a firm which does FDI, a firm which exports, and a firm which serves only the domestic market.

   - The profit function of a firm which serves only the **domestic market** is:

   $$\pi = z(i)^{\varepsilon-1}B - wf$$

   - The profit function of a firm which **exports** is:

   $$\pi_X = z(i)^{\varepsilon-1}B(1+\tau^{1-\varepsilon}) - w(f+f_X)$$

   - A firm which does **FDI** has profits:

   $$\pi_{\text{FDI}} = 2z(i)^{\varepsilon-1}B - w(f+f_{\text{FDI}})$$

2. Focus on a firm which has decided to produce domestically, but needs to decide whether to export or do FDI on top of that

   (a) Assume first $f_{FDI} \leq f_X$. Do firms' decisions on how to serve the foreign market depend on productivity? **Under the assumption that $f_{FDI} < f$, firms always prefer to do FDI. The profit expressions:**

   $$\pi_{FDI} = 2z(i)^{\varepsilon-1}B - w(f+f_{\text{FDI}}) \quad \text{and} \quad \pi_X = z(i)^{\varepsilon-1}B(1+\tau^{1-\varepsilon}) - w(f+f_X)$$

   are such that $\pi_{FDI} \geq \pi_X$ regardless of productivity.

   (b) Now, assume $f_{FDI} > f_X$. Do firms' decisions on how to serve the foreign market depend on productivity? Derive a cut-off productivity level z^*_{FDI}, above which firms decide to engage in FDI. **Firms engage in FDI only if their productivity exceedes a certain threshold, which is given by the inequality above:**

   $$\pi_{FDI} = 2z(i)^{\varepsilon-1}B - w(f+f_{\text{FDI}}) \geq \pi_X = z(i)^{\varepsilon-1}B(1+\tau^{1-\varepsilon}) - w(f+f_X)$$

   $$z(i)^{\epsilon-1}B(1-\tau^{1-\epsilon}) \geq -w(f_X - f_{FDI})$$

   $$z^*_{FDI} = \left(\tfrac{-w(f_X - f_{FDI})}{B(1-\tau^{1-\epsilon})}\right)^{\frac{1}{\epsilon-1}}$$

3. Under the assumption $f_{FDI} > f_X$, calculate the ratio $\frac{z^*_{FDI}}{z^*_X}$ (which, for most productivity distributions, is inversely related to the percentage of exporting firms who do FDI). Discuss how it depends on the different parameters of the model. **Firms decide to export if:**

$$\pi_X \geq \pi, \text{ i.e., } z(i)^{\epsilon-1}B(1+\tau^{1-\epsilon}) - w(f+f_X) \geq z(i)^{\epsilon-1}B - wf$$

$$z(i)^{\epsilon-1}B\tau^{1-\epsilon} \geq wf_X$$

$$z^*_X = \tfrac{w}{B}\tfrac{f_X}{\tau^{1-\epsilon}}$$

The final ratio is then:

$$\tfrac{z^*_{FDI}}{z^*_X} = (\tfrac{f_{FDI}-f_X}{f_X}\tfrac{1}{\tau^{\epsilon-1}-1})^{\frac{1}{\epsilon-1}}$$

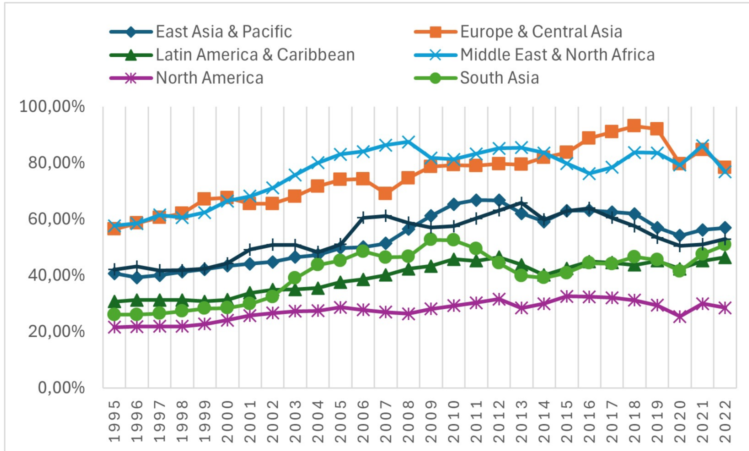
This ratio increases with f_{FDI}, decreases with τ, and decreases with ε.

Exercise 4 - Trade openness and tariffs in different parts of the world

1. Using WITS data for trade flows (https://wits.worldbank.org/) and any data source of your choice for GDP, compute a time series for the trade-to-GDP ratios of the seven large world regions defined by the World Bank (see https://data.worldbank.org/country). Plot these time series in one or in a series of graphs, using the software of your choice.

Again using WITS we can find imports for the 7 regions in USD$Thousands. Similarly using WITS we find exports. From the World Bank we can find the GDP at current dollars for the 7 regions.

2. Discuss your findings. What are the big trends? Are there significant differences across regions?



The figure display (again excluding more recent years with turmoil in trade) that trade is accounting for a much higher percentage of gdp in recent years. Variation among the 7 regions are not that substantial, they are all increasing, worth of mention the middle east that thanks to his products (oil prevalently) is one of the more open countries along the time span considered.

3. Using again WITS data, generate and plot time series for average tariffs in the seven regions of Question 1.

We can retrieve the data from WITS. Unfortunately despite some observation are starting from 1988, they become consistent in 1992 but unfortunately 1994 is aa NA for North America, hence we select our time series by 1995 till the last date which (oddly) is only 2022.

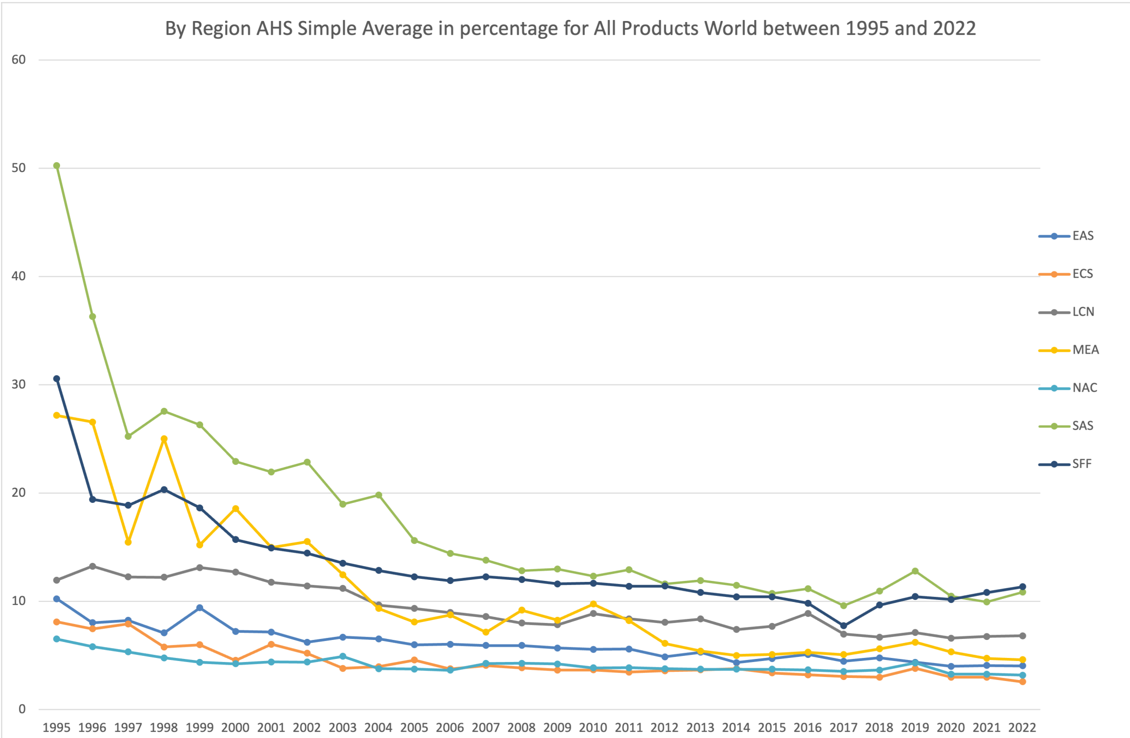


Figure 1: AHS Simple Average (%) across regions,*i.e* Effectively Applied Simple Average tariff (%)

4. Discuss your findings for tariffs. What are the big trends? Are there significant differences across regions?

The trend is very clear: worldwide (Again they stop at 2022 so what about 2025 and Trump policies and counter tariffs?) there has been a steadily decrease in tariffs across al the regions, especially before 2008 for developing countries which started from higher tariffs values.