

Model-based testing under uncertainty

L1: Theoretical aspects & practical applications

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Outline

- Markov Decision Process
 - Structure
 - Rewards
 - Policy, best policy, value iteration
- Model-based testing (MBT)
 - Offline vs online approaches
 - Conformance relation
 - Probabilistic alternating simulation and refinement
- Online MBT under uncertainty
 - Problem statement
 - Uncertain model paramenters
 - Bayesian inference
 - Framework and test case generation strategies

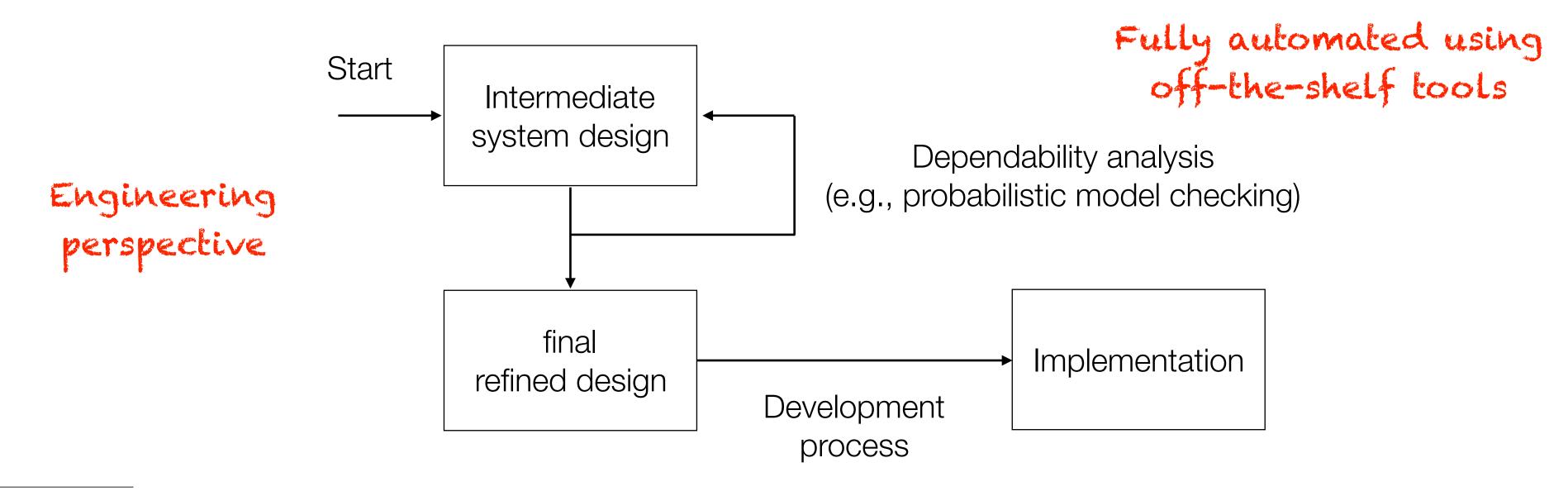
Markov decision processes

- Structure
- Examples
- Rewards
- Policy, best policy
- Value iteration algorithms

Markov Models

Basic notions

- The behavior of the target system (or phenomenon) of interest is partially/fully stochastic
- Formal framework for performance and dependability (reliability, availability, safety) analysis
- Dependability modeling (upfront) at design time improves the quality of the system eventually produced
- Assumption the modeled system meets the Markov property^{1,2} (memoryless)



^{1.} The the probability of moving to the next state only depends on the current state, not on the history that lead to that state.

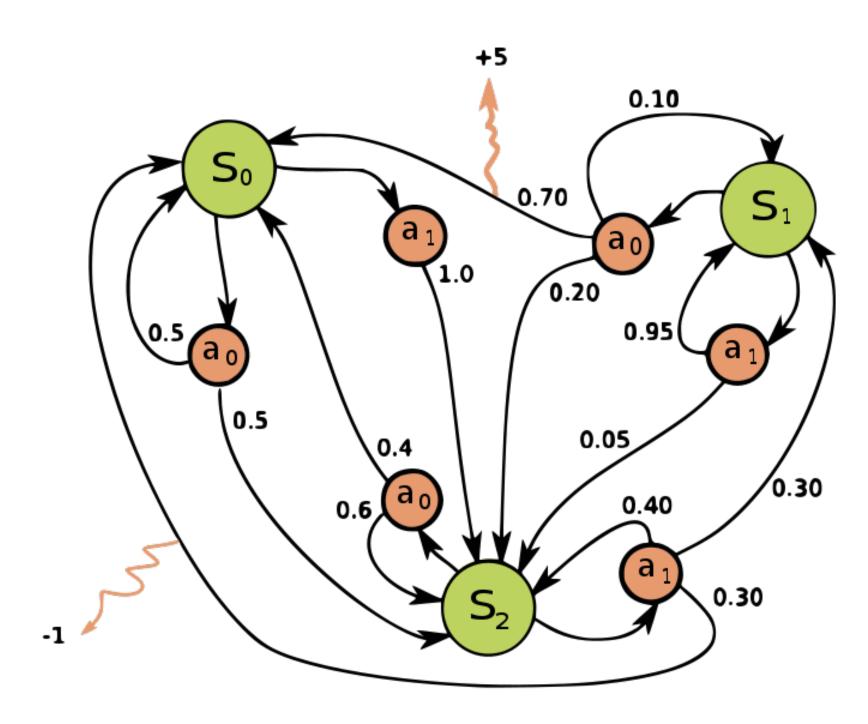
^{2.} R. C. Cheung, A user-oriented software reliability model, IEEE TSE, no. 2, pp. 118–125, 1980

Markov Decision Process

- Mathematical framework for modeling systems whose behavior is partially
 - Nondeterministic actions (external stimuli) under the control of a decision maker
 - Stochastic random outcome out of an executed action

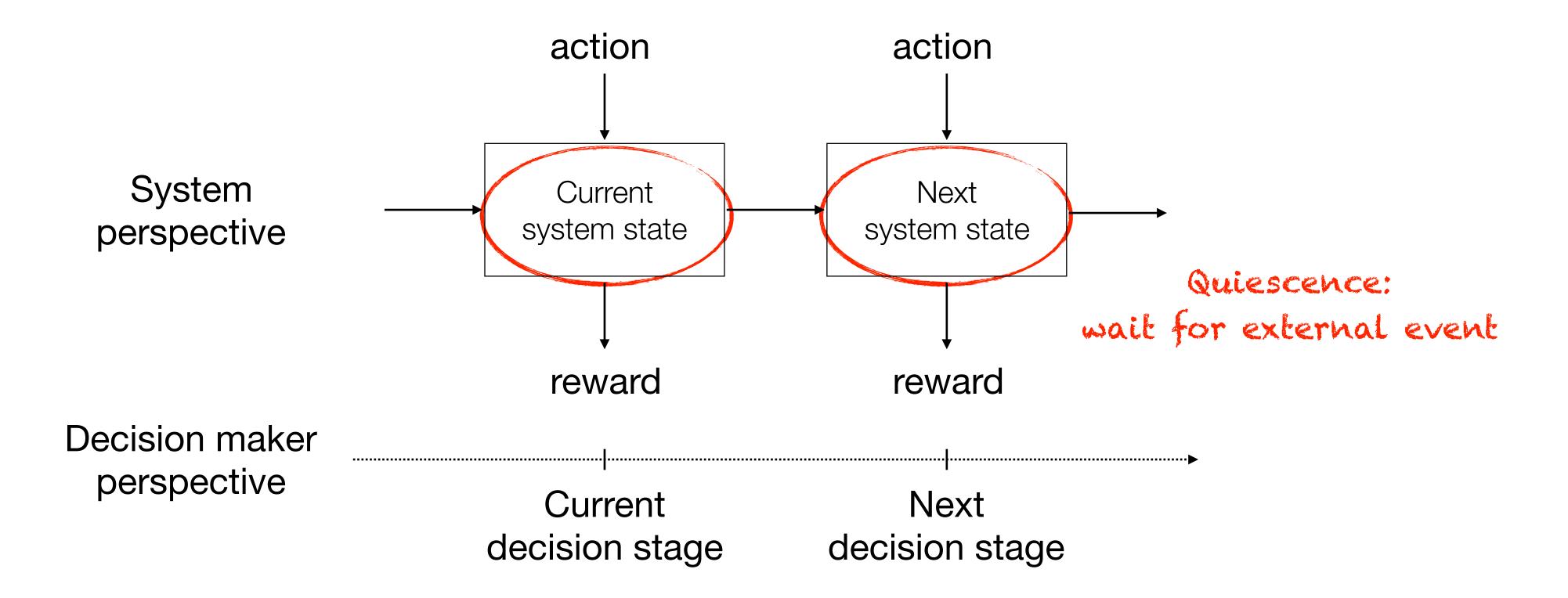
Formal structure

- S: set of states (finite/infinite)
- s₀: initial state
- A: set of actions (alphabet)
- P: S x A x S -> [0,1], P(s, a, s') = P(s_{t+1} = s' | s_t = s, a_t =a)
- R: S x A x S $\longrightarrow \mathbb{R}$, R(s, a, s') reward for (s_{t+1} = s', s_t = s, a_t = a)



MDP behavior

- How does the model operate
 - The system must be in one of the states (finite countable set) at a time
 - The system makes a transition s -> s' when one of the available actions is selected



MDP behavior (2)

States

- System configurations or operational status of components
- Instances of the system where
 - Components are operational or failed (e.g., enumeration of working/failed components)
 - Experienced specific sequences of events (e.g., events observed so far)
 - Operating in a fully-functioning mode, degraded mode, faulty, etc.
 - Undergoing recover/repair

Actions

Possible inputs or external events

Transitions

- Define whether is possible to go from one state to another
- Transition probability —> governs the likelihood of observing the transition

MDP examples

Possible scenarios

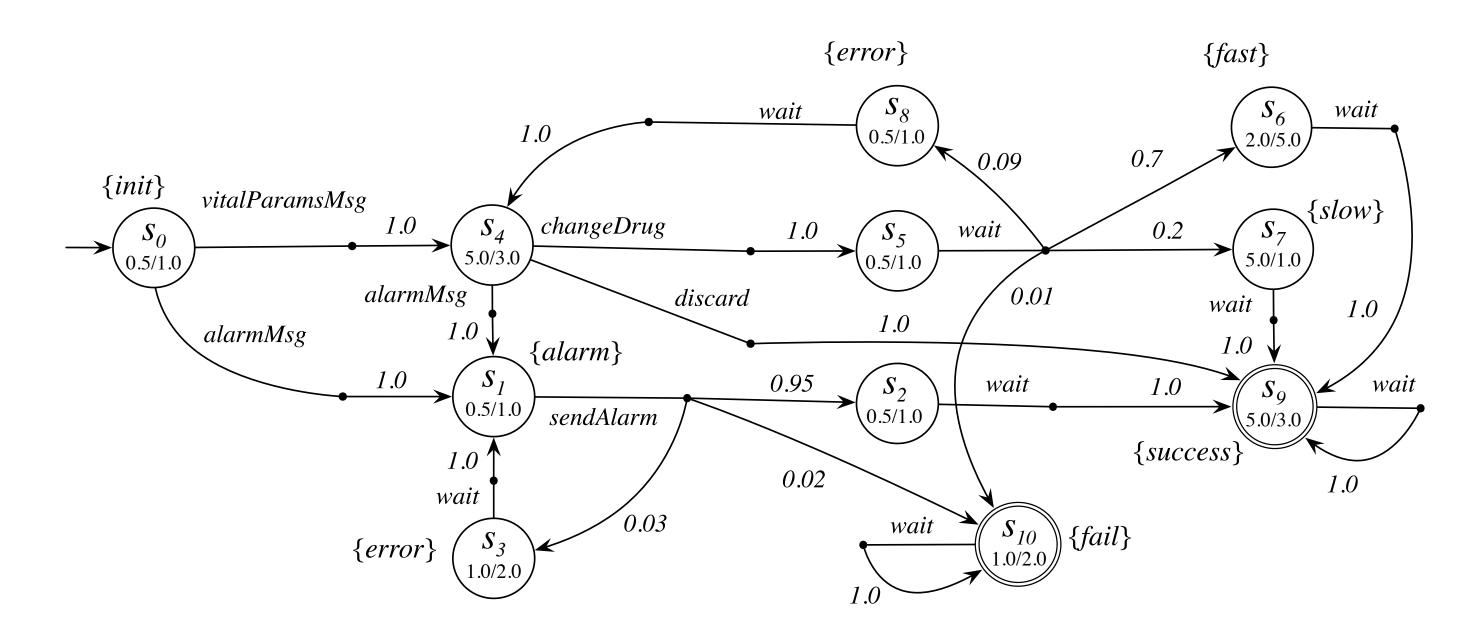
- Service-based systems
- Web applications
- Mobile applications
- Cyber physical systems (CPSs)
- Control policies in robotics
- Security protocols
- etc.

Selected examples

- Tele assistant system (TAS) example of service-based system
- SafeHome example of CPS

MDP examples — TAS

- TAS1,2
 - SBS providing health support to chronic condition patients at their homes
 - wearable devices (track vital parameters) + remote services (healthcare, pharmacy and emergency units)

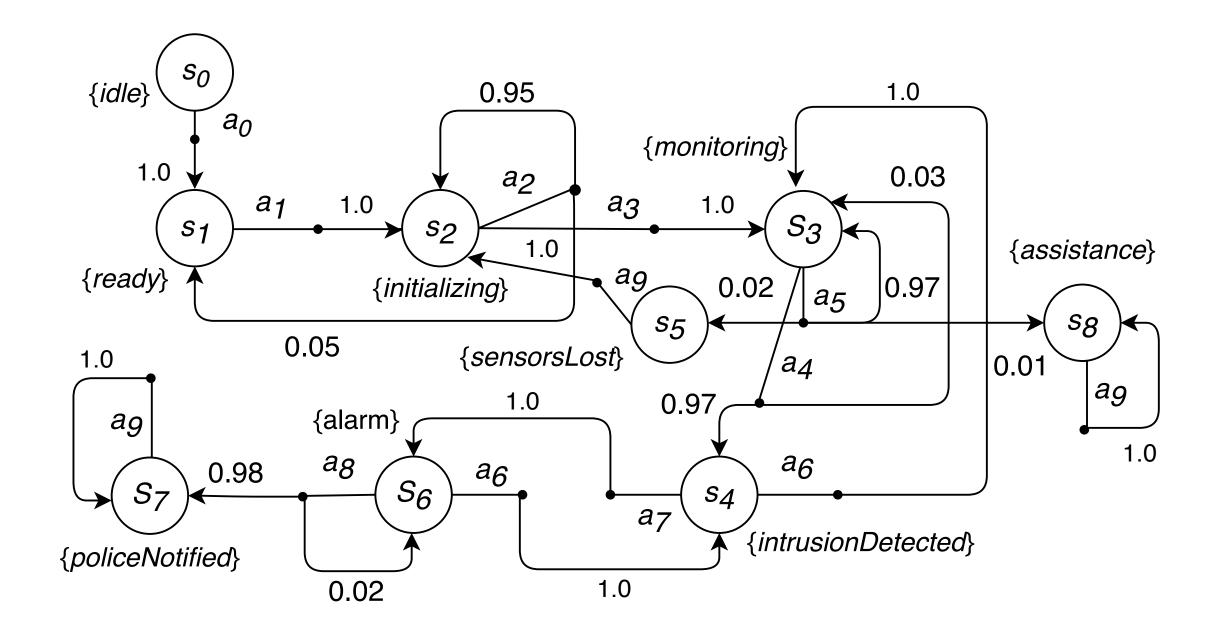


^{1.} D. Weyns et al., Tele Assistance: A Self-Adaptive Service-Based System Examplar, SEAMS 2015, IEEE

^{2.} M. Camilli et al., Online Model-Based Testing under Uncertainty, ISSRE 2018, IEEE

MDP examples — SafeHome

- SafeHome^{1,2}
 - Configure and control alarms along with related sensors that implement security and safety features
 - Here is the description of a part in charge of monitoring and intrusion detection
 - Modeled phases: sensors initialization, monitoring, detection, notification



Actions

- a0: activate
- a1: startInit
- a2: initSensors
- a3: startMonitoring
- a4: intrusionOccurred
- a5: sensorsCheck
- a6: cancel
- a7: turnAlarmOn
- a8: notify
- a9: wait

^{1.} Roger S Pressman. 2005. Software engineering: a practitioner's approach. Palgrave Macmillan

^{2.} Man Zhang, at al., Uncertainty-wise test case generation and minimization for Cyber-Physical Systems. 2019, JSS 153

MDP with rewards

- An MDP model can be augmented with multiple reward structures
- Reward structure
 - R: S x A x S $\longrightarrow \mathbb{R}$, R(s, a, s') reward for (s_{t+1} = s', s_t = s, a_t =a)
 - Describe nonfunctional aspects (e.g., energy consumption, computational cost, response time, ...)



Mental note

This is the usual interpretation of a reward structure. We'll see how to leverage this notion in a "unconventional" way to drive testing.

MDP policy

- The notion of policy π refers to the way a Decision Maker (DM) solves nondeterminism of a MDP
- Deterministic policy^{1,2}
 - π : S -> A, prescribes the action to take given a state
 - DM objective: choose π which maximizes the expected cumulated reward over an infinite horizon
 - This is called best policy π*
- Definition of the best deterministic policy
 - Given R(s, a), i.e., the one-step expected reward
 - We can compute the value function V(s) for each state

Bellman's equation

$$R(s,a) = \sum_{s' \in S} p_{s,a,s'} r_{s,a,s'} \qquad V(s)$$

$$R(s,a) = \sum_{s' \in S} p_{s,a,s'} \ r_{s,a,s'} \qquad V(s) = \max_{a \in A} \ \{ R(s,a) + \gamma \sum_{s' \in S} p_{s,a,s'} V(s') \}$$

Best policy

$$\pi^*(s) \neq \arg\max_{a \in A} \left\{ R(s, a) + \gamma \sum_{s' \in S} p_{s, a, s'} V(s') \right\}$$

^{1.} Given a deterministic policy, the MDP reduces to a Discrete Time Markov Chain (DTMC).

^{2.} Martin L. Puterman. 1994. Markov Decision Processes: Discrete Stochastic Dynamic Programming (1st. ed.). John Wiley & Sons, Inc., USA

MDP policy — Value iteration

```
1: Procedure ValueIteration(S,A,P,R,θ)

    The ValueIteration procedure uses dynamic programming

         Inputs
                S set of all states

    Memoization + recursion (or iteration)

                A set of all actions

    This procedure converges no matter what is the initial

                P transition function P(s,a,s')
                R reward function R(s,a,s')
                                                                  value function Vo
                \theta a threshold, \theta > 0
8:
         Output
                \pi[S] optimal policy
10:
                V[S] value function
11:
           Local
12:
                 real array V_k[S] is a sequence of value functions
13:
                 action array \pi[S]
14:
           assign V<sub>0</sub>[S] arbitrarily
                                                                                                                      (subproblem)
15:
           k ←0
                                                                                   (current problem)
16:
           repeat
17:
                 k \leftarrow k+1
                 for each state s do
18:
19:
                        V_k[s] = \max_a \sum_{s'} P(s,a,s') (R(s,a,s') + \gamma V_{k-1}[s'])
                                                                         Update Vk based on Vk-1
20:
                        \pi[S] = a
21:
           until \forall s |V_k[s] - V_{k-1}[s]| < \theta
22:
           return \pi, V_k
```

Model-based testing of probabilistic systems

- Offline vs online approaches
- Conformance relation
- Probabilistic alternating simulation and refinement

Model-based testing

Basic idea

- A formal model of the required behavior of the System Under Test (SUT) is used as baseline of
 - test case generation
 - Construction of the oracle
- Test suites are automatically extracted from models and then executed
- Formal verification vs Model-based testing
 - Formal verification prove that the model (i.e., formal specification) satisfies requirements
 - MBT show that the SUT behaves as defined in the (verified) model
 - Limitation: testing is not complete (i.e., "testing can only show the presence of errors, not their absence")

MBT — terminology

Implementation or System Under Test

- Piece of hardware/software, a software system, an embedded system, a CPS, etc.
- The SUT is viewed as a black-box (secret internal structure)
- The tester controls and observes the SUT via its interfaces (e.g., APIs)

Specification

- Describes what the SUT should do using a formal notation (or language)
- SPEC set of all valid models in a formal notation
 - A specification is $M \in SPEC$

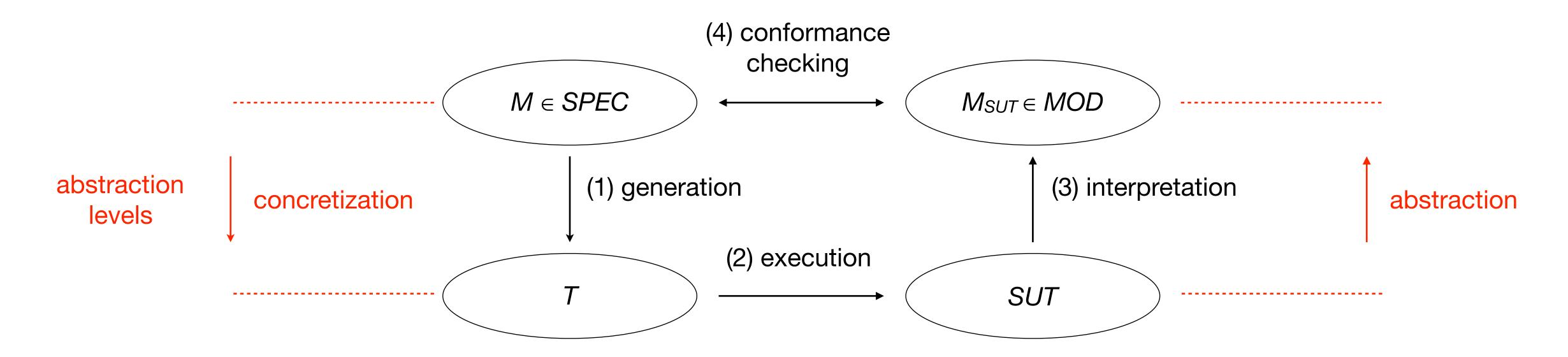
Conformance

- Formalizes the notion of correct behavior of a SUT w.r.t. $M \in SPEC$
- Problem: we'd like to define a relation between elements of different domains
 - M (formal entity) <-> SUT (not a formal entity)

MBT — terminology (2)

- Conformance Problem: M (formal domain) <-> SUT (not formal domain)
 - Trick test assumption
 - The SUT behavior can be interpreted using the same level of abstraction of M
 - The SUT behavior is a model *M*_{SUT} ∈ *MOD* ⊆ *SPEC*
 - MOD universe of implementation models
 - M_{SUT} not a-priori known
- Conformance (under the test assumption)
 - Can be expressed as a formal relation between MOD and SPEC elements
 - $conf \subseteq MOD \times SPEC$
 - M_{SUT} is correct w.r.t. M if M_{SUT} conf M
- Conformance checking
 - Assess by testing whether M_{SUT} conf M
 - Create T (test suite) s.t. M_{SUT} conf $M => M_{SUT}$ passes T (sound but not complete)

MBT process

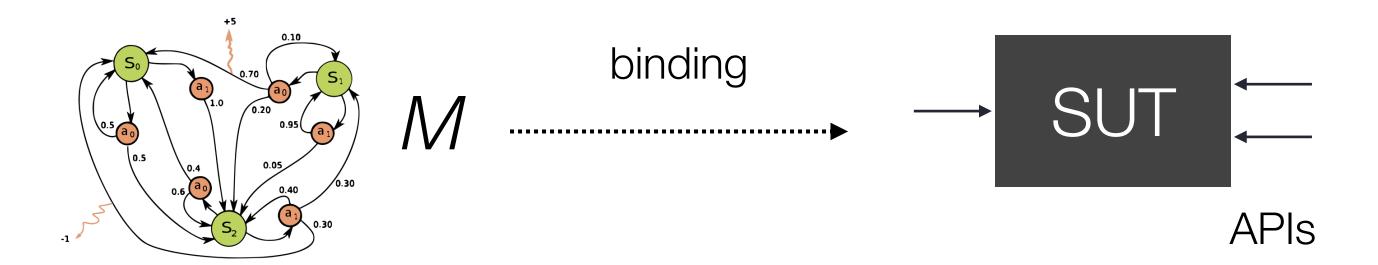


- Offline vs online 1
 - Offline steps 1-4 are separated
 - Online (or on-the-fly) steps 1-4 are merged into a one-iteration step
 - Test cases are created dynamically and take advantage of the knowledge gained by exploring M
 - Iterative approach —> 2-players game: controller + observer

^{1.} Utting, Mark, and Bruno Legeard. Practical model-based testing: a tools approach. Elsevier, 2010

Online MBT with MDPs

- Binding (concretization)
 - Defines a mapping between the MDP model spec and the SUT behavior



Formal definition

- Given a MDP $M = (S, s_0, A, P)$ and a SUT, i.e., a set of exported services H (having signature and arguments), a binding is a tuple of partial functions (h, i, post) s.t.
 - h(s, a), $a \in A(s)$ identifies a service $\in H$
 - i(s, a), $a \in A(s)$ identifies a vector v_{in} for the service h(s, a)

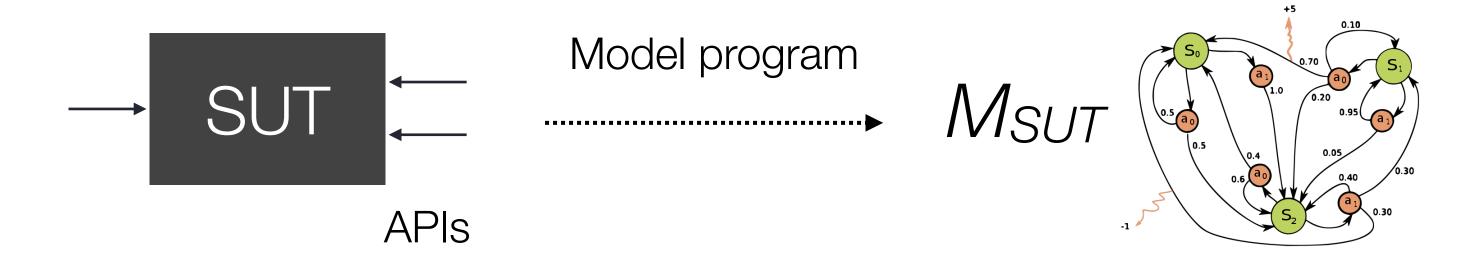
• post(s, a, s'), $a \in A(s)$ maps to a post-condition that must hold for v_{out} resulting from the execution of the service h(s, a) on input v_{in}

Controllable SUT components

> Observable SUT behavior

Online MBT with MDPs (2)

- Model program (abstraction)
 - Defines the abstract interpretation of the SUT behavior in terms of MDP model



Formal definition

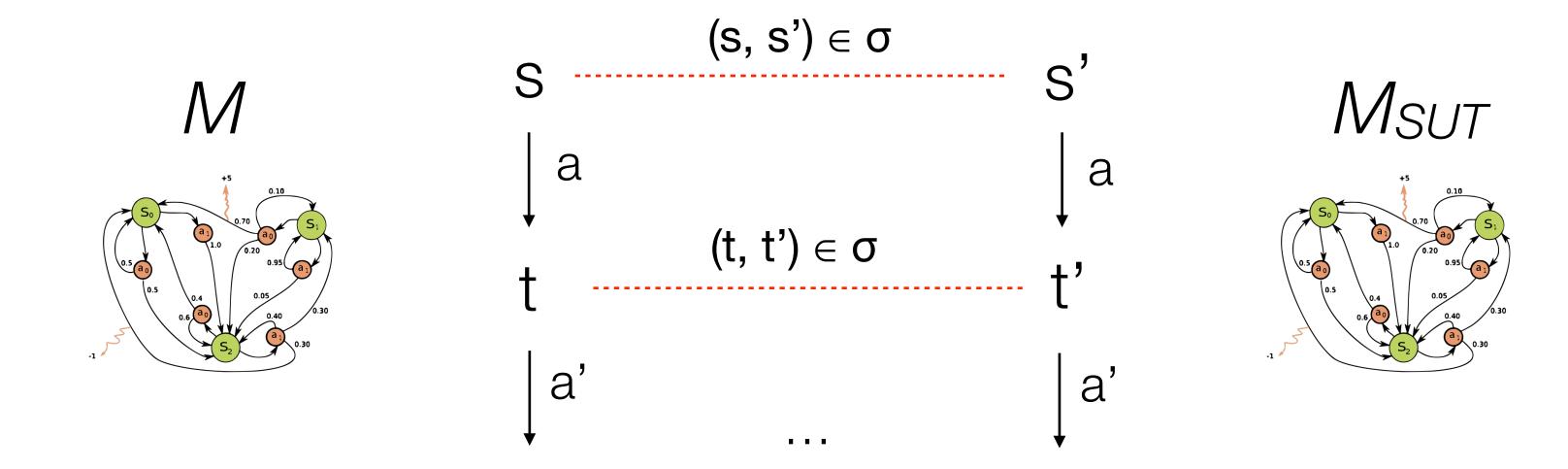
- Given a MDP $M = (S, s_0, A, P)$ and a binding (h, i, post) the model program $M_{SUT} = (S', s_0', A', P')$ is a MDP model s.t.
 - · S' ⊆ S ← All observable SUT states exist in M

 - $s_0 = s_0$
 - P'(s, a, s') > 0 iff there exists $v_{out} = h(s, a)(v_{in})$ s.t. post(s, a, s') holds for v_{out}

Mout transitions are defined in terms of SUT behavior

Online MBT with MDPs (3)

- Conformance checking
 - Needs the definition of conformance relation = probabilistic alternating simulation + refinement
- Probabilistic alternating simulation
 - between M and M_{SUT} is a binary relation $\sigma \subseteq S \times S'$, s.t. for all $(s, s') \in \sigma$
 - A(s) ⊆ A'(s')
 - For each $t \in S$: P(s, a, t) > 0, there exists $t' \in S'$: P'(s', a, t') > 0 and $(t, t') \in \sigma$



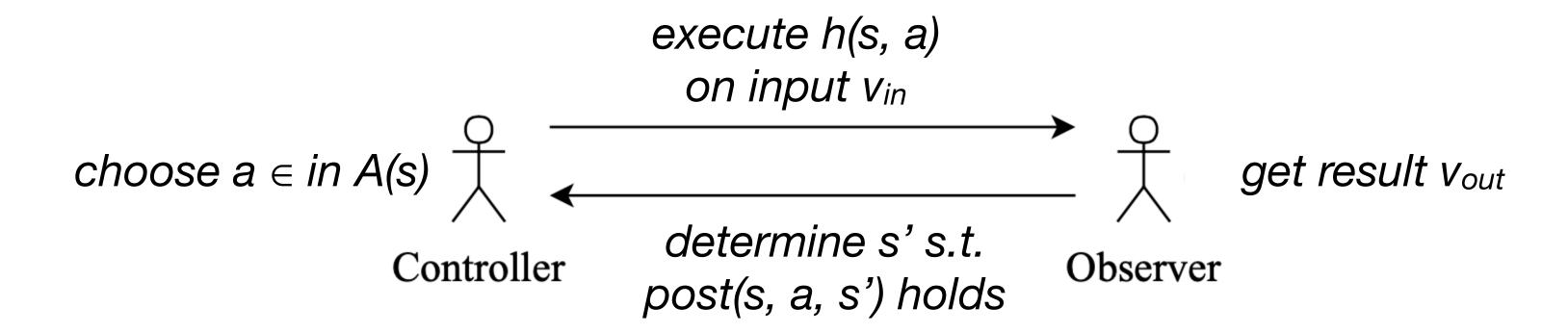
Online MBT with MDPs (4)

Refinement

• M_{SUT} refines M iff there exists a probabilistic alternating simulation σ s.t. (s₀, s₀') $\in \sigma$

Conformance game

- The notion of refinement is verified in practice by means of a conformance game between
 - Controller —> chooses actions based on a given test case generation strategy
 - Observer —> verifies the result out of a test execution



Online MBT under Uncertainty

- Problem statement
- Uncertain model paramenters
- Bayesian inference
- Framework and test case generation strategies

Problem statement



Problem

- Design-time models are imperfect and include assumptions
- Assumptions are affected by sources of uncertainty
 - Uncertain system properties (e.g., algorithmic/structural uncertainty, performance)
 - Uncertain environment properties (e.g., usage profiles, failure rate of 3rd-party components, latency)

The very idea

Objective

• Reduce the discrepancy between design-time assumptions (uncertain mathematical models) and real-world entities (runtime evidence) by applying Inverse Uncertainty Quantification (IUQ)

Assumption

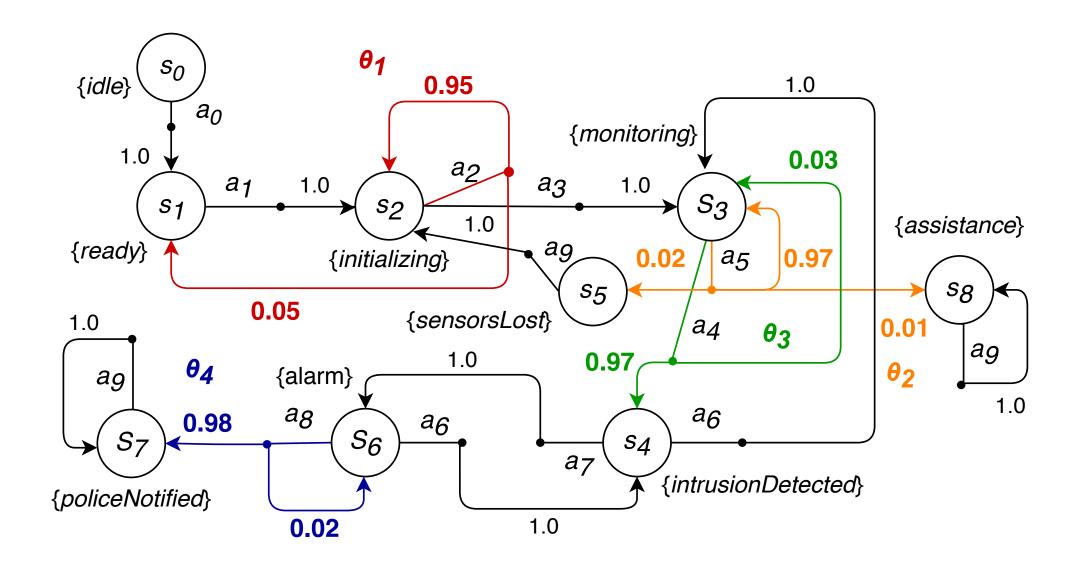
• sources of uncertainty affect model parameters (i.e., uncertain transition probability values in the MDP)

How

- Calibration of uncertain model parameters during integration/system testing by combining
 - Online Model-based Testing
 - Bayesian inference

Uncertain model parameters — SafeHome

- Sources of uncertainty (in CPSs) ¹
 - Application level uncertain events/data generated from software running upon physical units (e.g., θ_3)
 - Infrastructure level uncertain reliability of networking and/or cloud infrastructure (e.g., θ_4)
 - Integration level uncertain outcomes from interacting physical units (e.g., θ_1 , θ_2)



Examples

 θ_2 : uncertain sensing capability from the monitoring state

θ₄: uncertain failure rate of police notification from the alarm state

^{1.} Man Zhang, et al., Uncertainty-wise cyber-physical system test modeling. Software & Systems Modeling (2017), 1–40

Uncertain regions

Uncertain regions

- Uncertain transition probabilities θ_i grouped by <src-state, action>
- Values in θ_i are uncertain parameters of a Categorical distribution

•
$$\theta_i \sim Cat(p_1, ..., p_k)$$

	region	state-action	affected level	target states	probability values
	$ heta_1$	s_2 - a_2	integration	s_2, s_1	0.95, 0.05
	θ_2	<i>s</i> ₃ - <i>a</i> ₄	integration	s3, s4	0.03, 0.97
Region	θ_3	<i>s</i> ₃ - <i>a</i> ₅	application	s ₃ , s ₅ , s ₈	(0.01, 0.97, 0.02)
	$ heta_4$	<i>s</i> ₆ - <i>a</i> ₈	infrastructure	s ₆ , s ₇	0.02, 0.98

Uncertain parameters of a Categorical distribution

hypothesis

Intuition

- Mitigate the uncertainty over θ regions by observing (multiple times) the SUT
- Observation provides evidence to increase the confidence on transition probabilities

Bayesian inference

- Method used to update the probability for a hypothesis as more evidence becomes available
- Formulation ¹
 - To learn θ (phenomenon of interest) we collect a sample $y = (y_1, ..., y_n)$
 - - Prior $f(\theta)$ hypothesis on θ
 - Likelihood $f(y \mid \theta)$ compatibility of the evidence with the given hypothesis
 - Posterior $f(\theta \mid y)$ best knowledge on the hypothesis given the evidence
- In our context
 - The natural conjugate Prior of the Categorical distribution is the Dirichlet distribution
 - Prior $\theta_i \sim Dir(\alpha_1, ..., \alpha_K)$
 - e.g., uninformative Prior_{θ 3} ~ *Dir*(0.5, 0.5, 0.5)
 - e.g., informative Prior_{θ 3} ~ *Dir*(1.0, 97.0, 2.0) 100 observations = 1 s₃, 97 s₅, 2 s₈

region	state-action	affected level	target states	probability values
$egin{array}{c} heta_1 \ heta_2 \ heta_3 \ heta_4 \ \end{array}$	s ₂ -a ₂ s ₃ -a ₄ s ₃ -a ₅ s ₆ -a ₈	integration integration application infrastructure	s ₂ , s ₁ s ₃ , s ₄ s ₃ , s ₅ , s ₈ s ₆ , s ₇	0.95, 0.05 0.03, 0.97 0.01, 0.97, 0.02 0.02, 0.98

^{1.} Robert, Christian. The Bayesian choice: from decision-theoretic foundations to computational implementation. Springer Science & Business Media, 2007

Bayesian inference (2)

In our context

- Updating rule $Prior_{\theta i} \sim Dir(\alpha_1, ..., \alpha_K) \longrightarrow Post_{\theta i} \sim Dir(\alpha_1 + n_1, ..., \alpha_K + n_K)$
 - e.g., Dir(1.0, 97.0, 2.0) —> $Post_{\theta i} \sim Dir(1.0 + 35, 97.0 + 955, 2.0 + 10)$ $1000 \text{ observations} = 35 s_3, 955 s_5, 10 s_8$

Summarization

- Prior/Posterior knowledge can be summarized by using
 - Mean transition probability values $p_i = \alpha_i / \sum_{j=1}^k \alpha_j$
 - HPD region degree of confidence $C = \{p : f(\cdot) \ge 0.95\}$

region	state-action	affected level	target states	probability values			Posterior mean
$egin{array}{c} heta_1 \ heta_2 \end{array}$	s_2 - a_2 s_3 - a_4	integration integration	s ₂ , s ₁ s ₃ , s ₄	0.95, 0.05 0.03, 0.97	Prior mean	inference	1 OSCOTION MICAN
$egin{array}{c} heta_2 \ heta_3 \ heta_4 \end{array}$	$s_3 - a_5$ $s_6 - a_8$	application infrastructure	s ₃ , s ₄ s ₃ , s ₅ , s ₈ s ₆ , s ₇	0.01, 0.97, 0.02			0.033, 0.956, 0.011

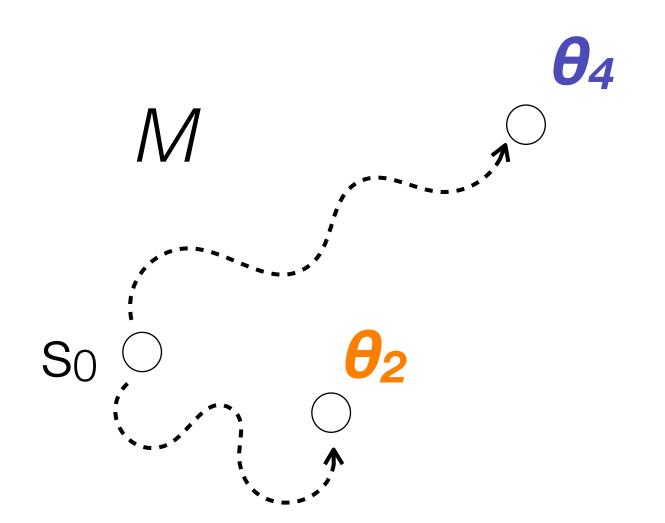
Prior HPD region = { [0.009, 0.019], [0.936, 0.996], [0.001, 0.047] } **Prior HPD width** = 0.136

Posterior HPD region = $\{ [0.009, 0.019], [0.936, 0.996], [0.001, 0.047] \}$ **Posterior HPD width** = 0.056

Lower value -> higher confidence

Online MBT + Bayesian inference

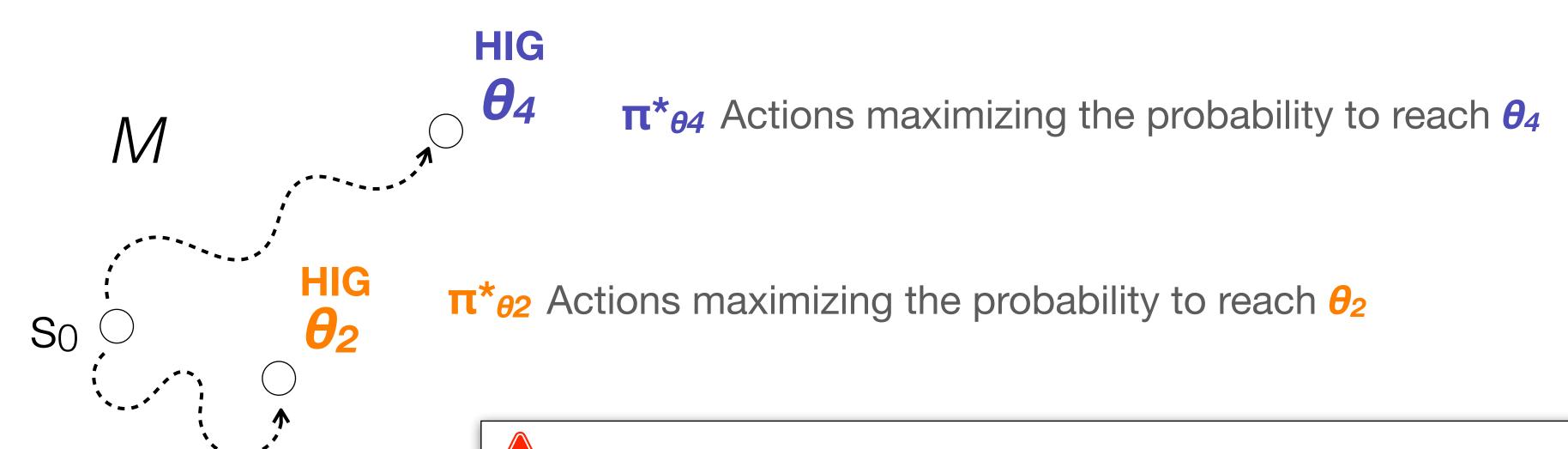
- Objective (reminder)
 - design-time assumptions < gap reduction —> runtime evidence
- How
 - Perform a controlled exploration using online MBT to stress the uncertain components
 - Gather evidence and run bayesian inference to reduce the uncertainty



- Uncertainty-aware MBT strategy
 - Explore by maximize the probability of reaching θ regions
 - Reduces to an optimization problem:
 - Find out the actions a decision maker should take to maximize the exploration of θ regions

Uncertainty-aware strategy

- Computation of the best policies
 - For each θ_i
 - construct a reward structure that assigns HIG reward to θ_i transitions, LOW elsewhere
 - Compute the best policy $\pi^*_{\theta i}$ (value iteration)
 - For each state, it selects the action that maximizes the probability to reach θ_i



Back to our mental note

We'll see how to leverage rewards in a "unconventional" way to drive testing.

Uncertainty-aware strategy (2)

- How to combine the best policies $\pi^*_{\theta i}$?
 - Simple scenario -> there exists just a single θ region
 - Otherwise —> different exploration strategies may be constructed/adopted
 - Strategies represent decision makers (i.e., testers) that use a probabilistic function

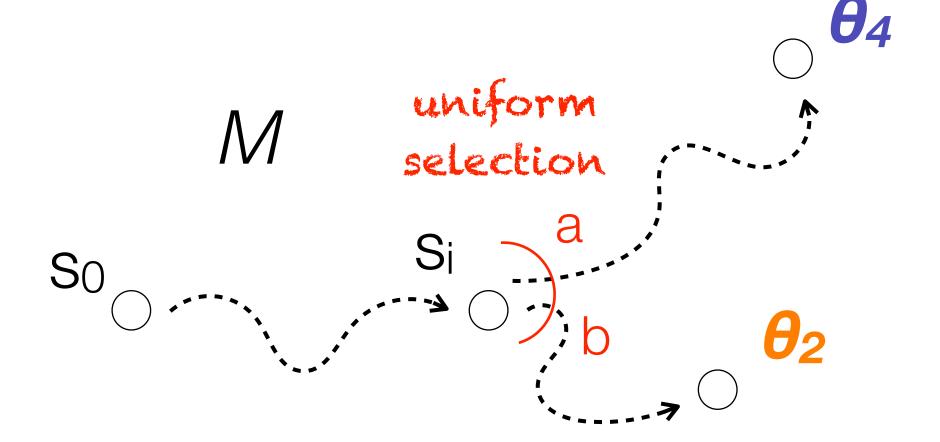
$$\mathcal{P}(s,a) = \begin{cases} 0 & \omega(s,a) = 0 \\ \omega(s,a)/\sum_{a'\in A(s)} \omega(s,a') & \text{otherwise} \end{cases}$$

• The ω weight selectively increase/decrease the probability of choosing a specific action a from state s

Uncertainty-aware strategy (3)

- Flat strategy
 - Actions selected by different policies $\pi^*_{\theta i}$ have equal probability
 - Uniform random sampling of the available policies

$$\omega^{RT}(s, a) = \begin{cases} 1 & \exists i : \pi_i^*(s) = a \\ 0 & otherwise \end{cases}$$



Uncertainty-aware strategy (4)

- History-based strategy
 - Tries to keep balanced the number of times θ regions are tested
 - We leverage decrementing weights inversely proportional to #selections of state-action pairs

$$\omega^{HT}(s,a) = \begin{cases} 1/\#(s,a) & \exists i : \pi_i^*(s) = a \\ 0 & otherwise \end{cases}$$

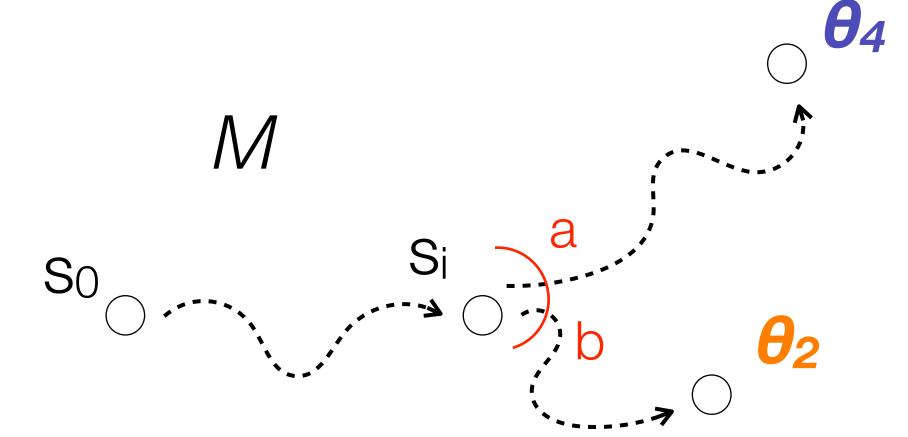
$$S_i \qquad \text{#tests } \theta_4 > \#\text{tests } \theta_2$$

the higher the #selections of a, the lower the likelihood of selecting it again

Uncertainty-aware strategy (5)

- Distance strategy
 - Tries to deliver balanced degree of confidence on θ regions
 - The weight is proportional to the HPD width of θ regions

$$\omega^{DT}(s, a) = \begin{cases} |\mathsf{HPD}_{\theta_i}| & \exists i : \pi_i^*(s) = a \\ 0 & otherwise \end{cases}$$



confidence θ_2 > confidence θ_4

the larger the HPD width of the target θ , the higher the likelihood of selecting it

Termination condition

- Limit on the effort
 - Traditional termination condition based on #tests limit
- Bayes factor
 - Tries to recognize when the inference process converges

$$\mathcal{F} = rac{f(y| heta)}{f(y| heta')}$$

 $\mathcal{F} \in [10^0, 10^{1/2}]$

likelihood that data y are produced under different assumptions θ and θ'

Difference between assumptions θ and θ' is not substantial

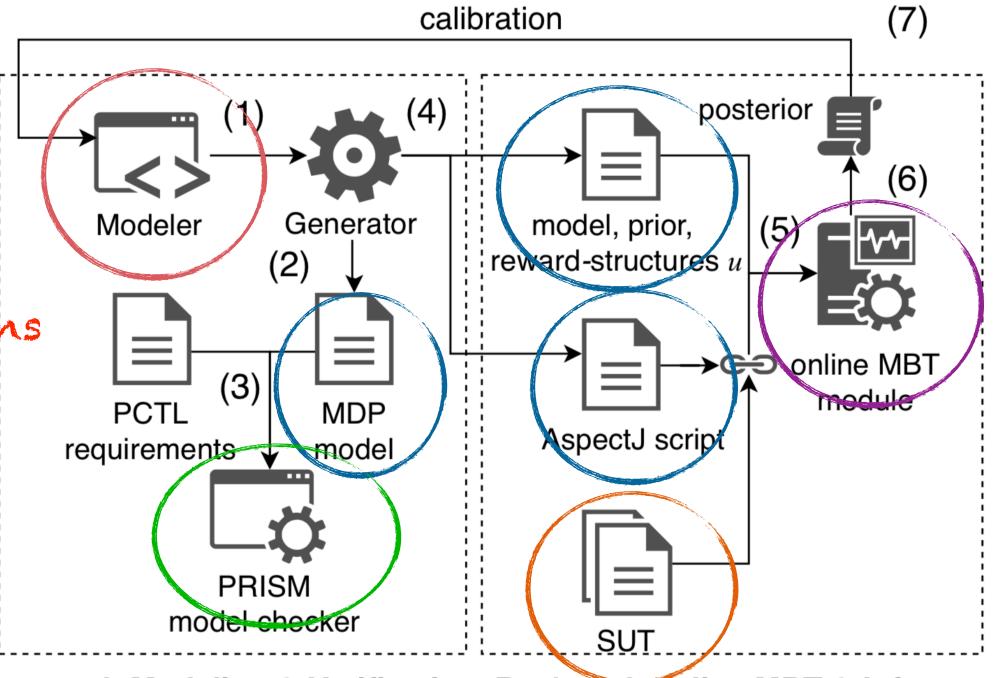
Current toolchain implementation

MDP definition + uncertain transition probabilities θ

binding to the SUT: actions -> inputs

acs -> routine postconditions

automatic generation



online MBT and inference/ calibration

Front end: Modeling & Verification Back end: online MBT & Inference

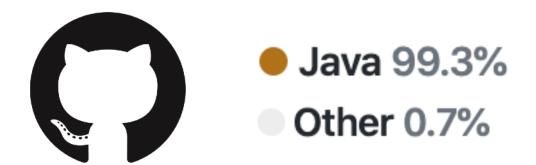
model checking of PCTL requirements

Java program

Current toolchain implementation (2)

MBT module

https://github.com/SELab-unimi/mbt-module



Current stage

- Uncertainty-aware strategies have been implemented
- Systematic evaluation of their cost-effectiveness —> currently inedited

Current toolchain implementation (3)

- Evaluation summary
 - We assessed statistical difference (Mann-Whitney U test 1)
 - We evaluated practical value (Vargha & Delaney's A₁₂ measure ¹)
 - In our context —> assuming same effort (i.e., #tests), the probability that target strategy yields smaller HPD width values than flat one (i.e., baseline)

Table 3: Vargha and Delaney's \hat{A}_{12} measure

	%u	ıncertair	ıty	#actions			
balanced	20	50	80	5	10	20	
hist	1.000	0.716	0.531	0.617	0.704	0.926	
dist	1.000	0.790	0.679	0.741	0.802	0.951	
unbalanced	20	50	80	5	10	20	
hist	0.963	0.716	0.556	0.531	0.642	0.901	
dist	0.988	0.951	0.691	0.741	0.741	0.975	

^{1.} Andrea Arcuri and Lionel Briand, A practical guide for using statistical tests to assess randomized algorithms in software engineering, ICSE'11, New York, NY, USA

Summary

- We discussed MBT for probabilistic systems and the problem of testing with uncertain model components
- Depending on the Prior knowledge (hypothesis) and information that can be gathered during testing, we
 derived different uncertainty-aware exploration strategies and evaluated their cost-effectiveness
 - Flat —> uniform selection
 - History —> balanced exploration
 - Distance —> balanced delivered confidence

Next

- Hands on session with the MBT module
- Design/develop an additional exploration strategy

One more thing: collaboration opportunities

- Relationship among uncertain regions
 - · Identify the uncertain regions depending on other uncertain regions along execution paths
 - Compute Posterior probabilities following the discovered relationships
- Refactoring of MDP models
 - Identify critical portion of the models w.r.t. requirements
 - Model-based refactoring and evaluation
- Automatic construction of MDP model from the implementation
 - Static analysis or symbolic execution
- Testing using Reinforcement Learning (based on MDP theory)
 - Discover the location of the uncertain regions

References

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