SOFTWARE AND SYSTEMS ENGINEERING research group

Model-based testing under uncertainty

L1: Theoretical aspects & practical applications

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Formal Methods at Work @
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Outline

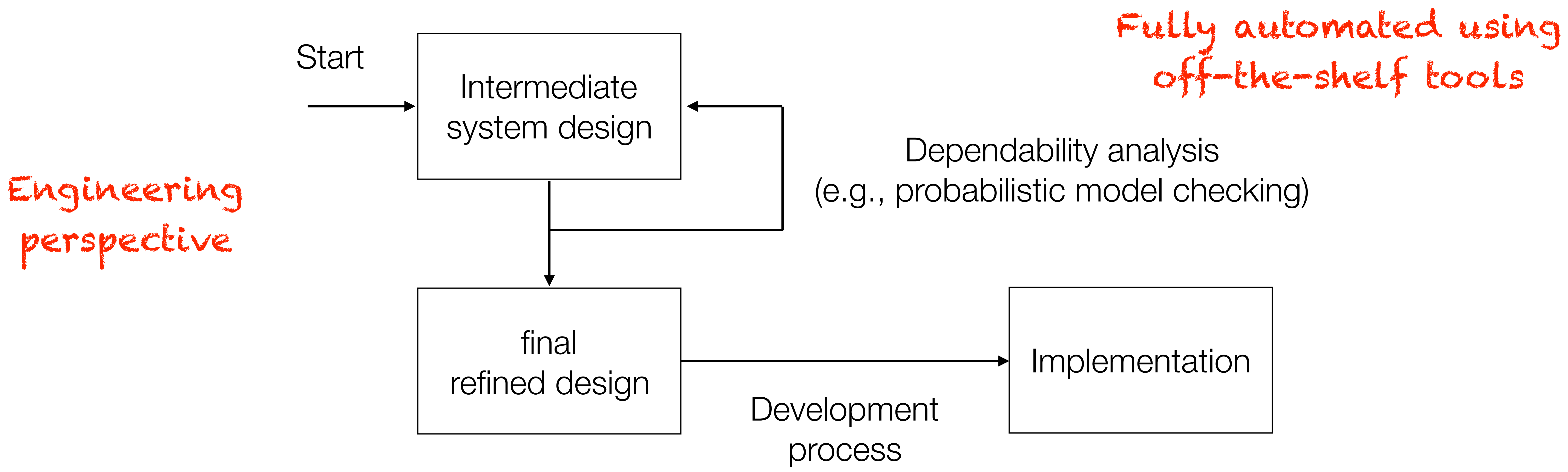
- Markov Decision Process
  - Structure
  - Rewards
  - Policy, best policy, value iteration
- Model-based testing (MBT)
  - Offline vs online approaches
  - Conformance relation
  - Probabilistic alternating simulation and refinement
- Online MBT under uncertainty
  - Problem statement
  - Uncertain model paramenters
  - Bayesian inference
  - Framework and test case generation strategies

Markov decision processes

- Structure
- Examples
- Rewards
- Policy, best policy
- Value iteration algorithms

Markov Models

- Basic notions
  - Formal framework for **performance** and **dependability** (reliability, availability, safety) analysis
  - **Dependability modeling** (upfront) at design time improves the quality of the system eventually produced
  - **Assumption** — the modeled system meets the **Markov property**[1,2] (memoryless)

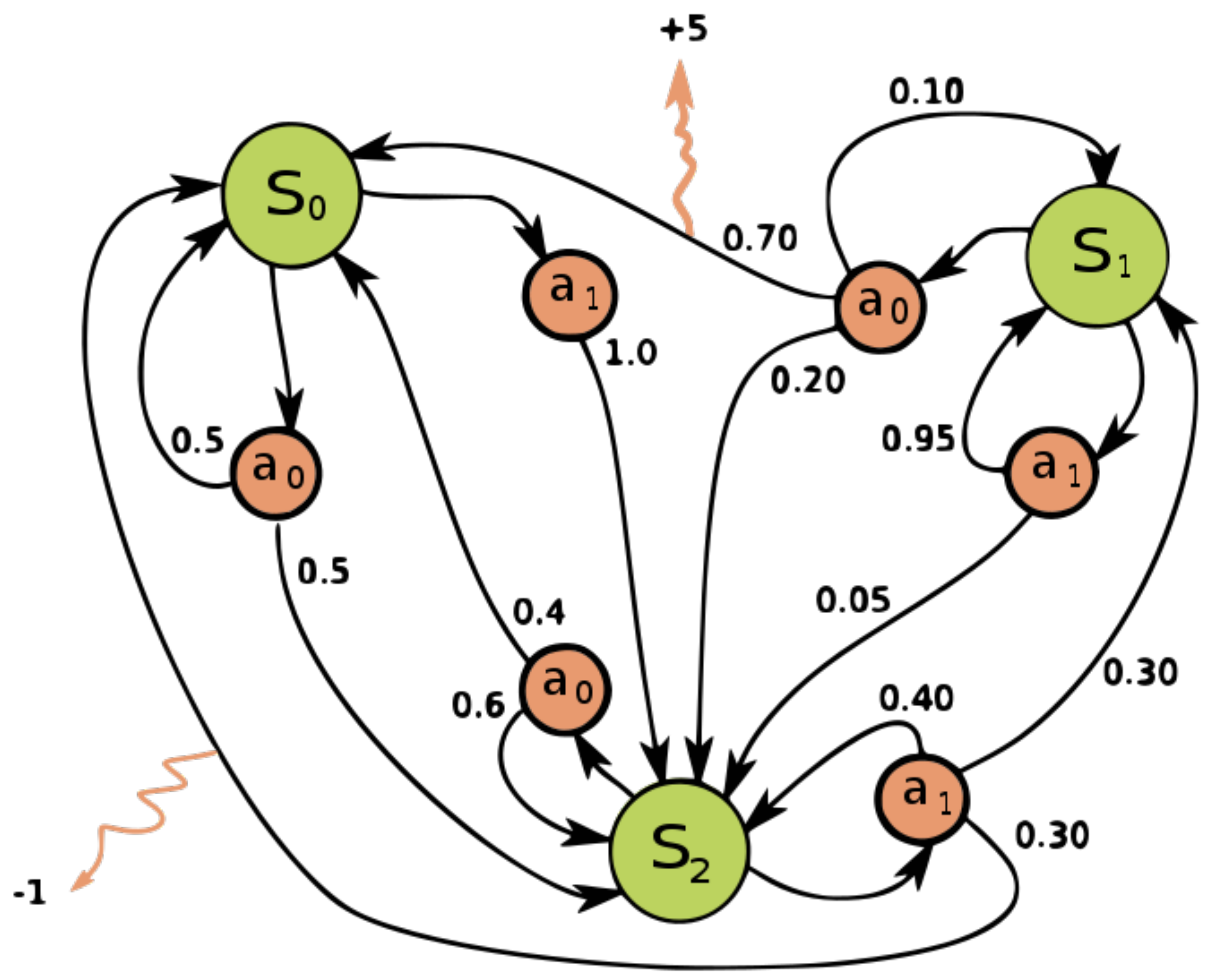


1. The the probability of moving to the next state only depends on the current state, not on the history that lead to that state.
2. R. C. Cheung, A user-oriented software reliability model, IEEE TSE, no. 2, pp. 118–125, 1980

Markov Decision Process

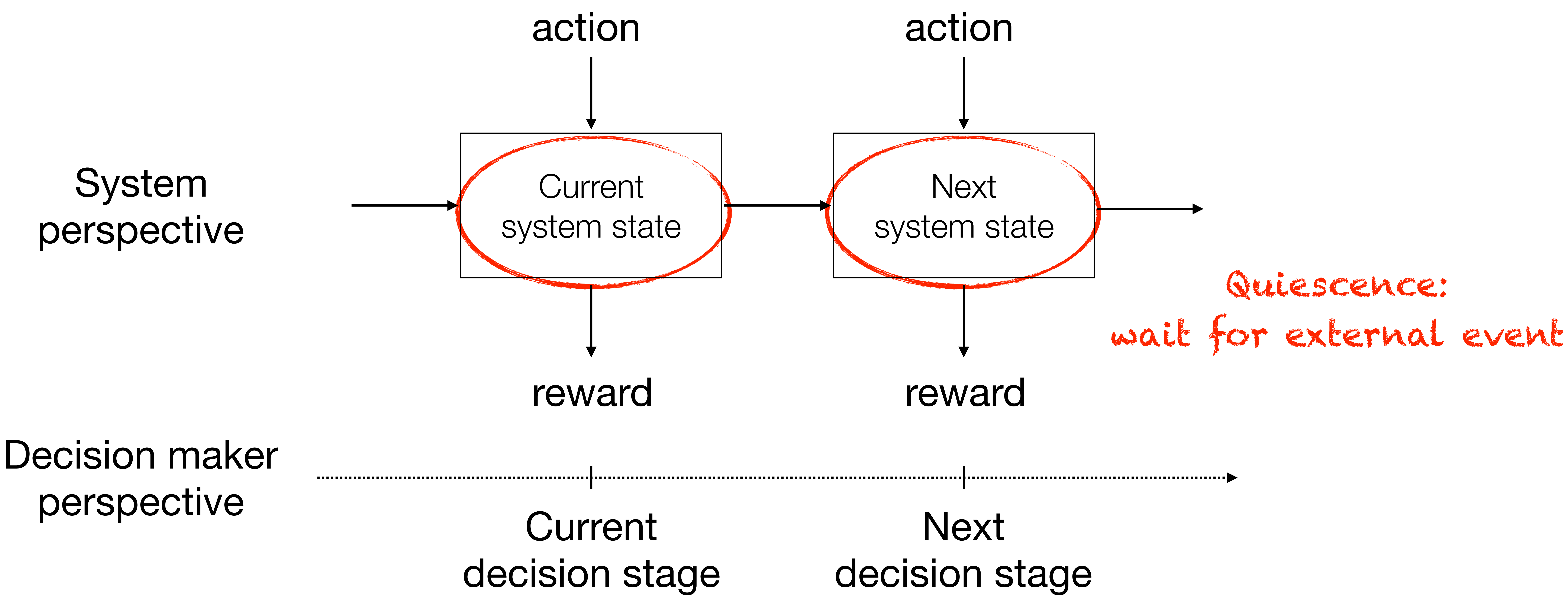
- Mathematical framework for modeling systems whose behavior is partially
  - Nondeterministic — actions (external stimuli) under the control of a decision maker
  - Stochastic — random outcome out of an executed action

- Formal structure
  - S: set of states
  - s_0: initial state
  - A: set of actions
  - P: S x A x S —> [0,1], $P(s, a, s') = P(s_{t+1} = s' \mid s_t = s, a_t = a)$
  - R: S x A x S —> $\mathbb{R}$, $R(s, a, s')$ reward for $(s_{t+1} = s', s_t = s, a_t = a)$



MDP behavior

- How does the model operates
  - The system must be in **one** of the **states** (finite countable set) at a time
  - The system makes a transition **s —> s'** when **one** of the available **actions** is selected



MDP behavior (2)

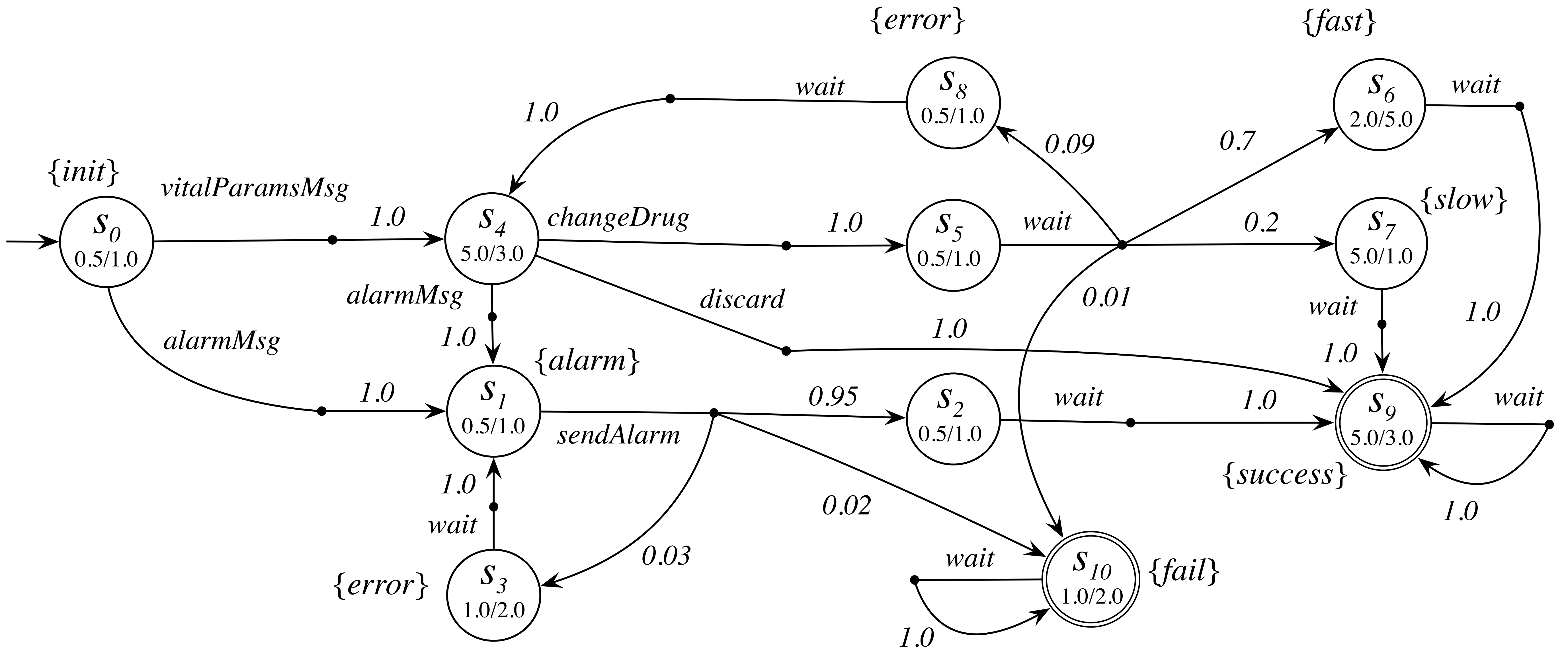
- States
  - System configurations or operational status of components
  - Instances of the system where
    - Components are operational or failed (e.g., enumeration of working/failed components)
    - Experienced specific sequences of events (e.g., events observed so far)
    - Operating in a fully-functioning mode, degraded mode, faulty, etc.
    - Undergoing recover/repair
- Actions
  - Possible inputs or external events
- Transitions
  - Define whether is possible to go from one state to another
  - Transition probability —> governs the likelihood of observing the transition

MDP examples

- Possible scenarios
  - Service-based systems
  - Web applications
  - Mobile applications
  - Cyber physical systems (CPSs)
  - etc.
- Selected examples
  - Tele assistant system (TAS) — example of service-based system
  - SafeHome — example of CPS

MDP examples — TAS

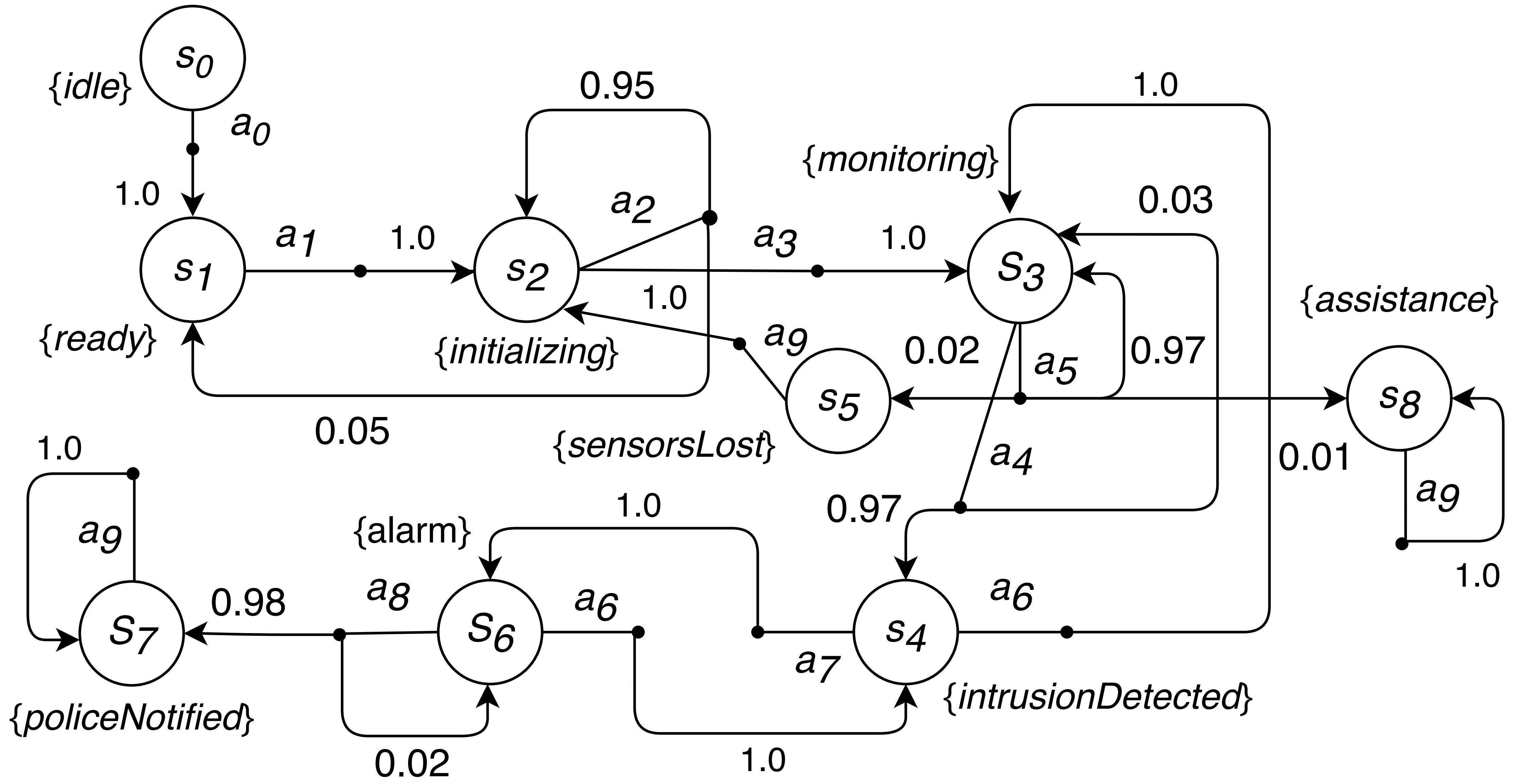
- TAS[1,2]
  - SBS providing health support to chronic condition patients at their homes
  - **wearable devices** (track vital parameters) + **remote services** (healthcare, pharmacy and emergency units)



1. D. Weyns et al., Tele Assistance: A Self-Adaptive Service-Based System Examplar, SEAMS 2015, IEEE
2. M. Camilli et al., Online Model-Based Testing under Uncertainty, ISSRE 2018, IEEE

MDP examples — SafeHome

- SafeHome[1,2]
  - Configure and control alarms along with related sensors that implement security and safety features
  - Here is the description of a part in charge of monitoring and intrusion detection
    - Modeled **phases**: sensors **initialization**, **monitoring**, **detection**, **notification**



Actions
a0: activate
a1: startInit
a2: initSensors
a3: startMonitoring
a4: intrusionOccurred
a5: sensorsCheck
a6: cancel
a7: turnAlarmOn
a8: notify
a9: wait

1. Roger S Pressman. 2005. Software engineering: a practitioner's approach. Palgrave Macmillan
2. Man Zhang, at al., Uncertainty-wise test case generation and minimization for Cyber-Physical Systems. 2019, JSS 153

MDP with rewards

- An MDP model can be augmented with multiple **reward structures**
- Reward structure
  - $R: S \times A \times S \longrightarrow \mathbb{R}$, $R(s, a, s')$ reward for ($s_{t+1} = s'$, $s_t = s$, $a_t = a$)
  - Describe **nonfunctional** aspects (e.g., energy consumption, computational cost, response time, …)

> **Mental note**
>
> This is the usual interpretation of a reward structure.
> We'll see how to leverage this notion in a "unconventional" way to drive testing.

MDP policy

- The notion of **policy π** refers to the way a **Decision Maker (DM) solves nondeterminism** of a MDP
- Deterministic policy[1,2]
  - π: S —> A, prescribes the action to take given a state
  - **DM objective**: choose π which **maximizes** the **expected cumulated reward** over an infinite horizon
    - This is called best policy π*
- Definition of the best deterministic policy
  - Given R(s, a), i.e., the one-step **expected reward**
  - We can compute the value function V(s) for each state

Bellman's equation

Best policy

$$R(s,a) = \sum_{s' \in S} p_{s,a,s'}\, r_{s,a,s'} \qquad V(s) = \max_{a \in A} \{R(s,a) + \gamma \sum_{s' \in S} p_{s,a,s'} V(s')\} \qquad \pi^*(s) = \arg\max_{a \in A} \{R(s,a) + \gamma \sum_{s' \in S} p_{s,a,s'} V(s')\}$$

1. Given a deterministic policy, the MDP reduces to a Discrete Time Markov Chain (DTMC).
2. Martin L. Puterman. 1994. Markov Decision Processes: Discrete Stochastic Dynamic Programming (1st. ed.). John Wiley & Sons, Inc., USA

MDP policy — Value iteration

```
Procedure ValueIteration(S,A,P,R,θ)
        Inputs
                S set of all states
                A set of all actions
                P transition function P(s,a,s')
                R reward function R(s,a,s')
                θ a threshold, θ>0
        Output
                π[S] optimal policy
                V[S] value function
        Local
                real array Vk[S] is a sequence of value functions
                action array π[S]
        assign V0[S] arbitrarily
        k ←0
        repeat
                k ←k+1
                for each state s do
                        Vk[s] = maxa ∑s' P(s,a,s') (R(s,a,s')+ γVk-1[s'])
                        π[S] = a
        until ∀s |Vk[s]−Vk-1[s]| < θ
        return π,Vk
```

$V_k[s] = \max_a \sum_{s'} P(s,a,s')\,(R(s,a,s') + \gamma V_{k-1}[s'])$

(current problem)

(subproblem)

Update V_k based on V_{k-1}

- The ValueIteration procedure uses dynamic programming
  - Memoization + recursion (or iteration)
- This procedure converges no matter what is the initial value function V_0

Model-based testing of probabilistic systems

- Offline vs online approaches
- Conformance relation
- Probabilistic alternating simulation and refinement

Model-based testing

- Basic idea
  - A formal model of the required behavior of the System Under Test (SUT) is used as baseline of
    - test case generation
    - Construction of the oracle
  - Test suites are automatically extracted from models and then executed
- Formal verification vs Model-based testing
  - Formal verification — prove that the model (i.e., formal specification) satisfies requirements
  - MBT — show that the SUT behaves as defined in the (verified) model
    - Limitation: testing is not complete (i.e., *“testing can only show the presence of errors, not their absence”*)

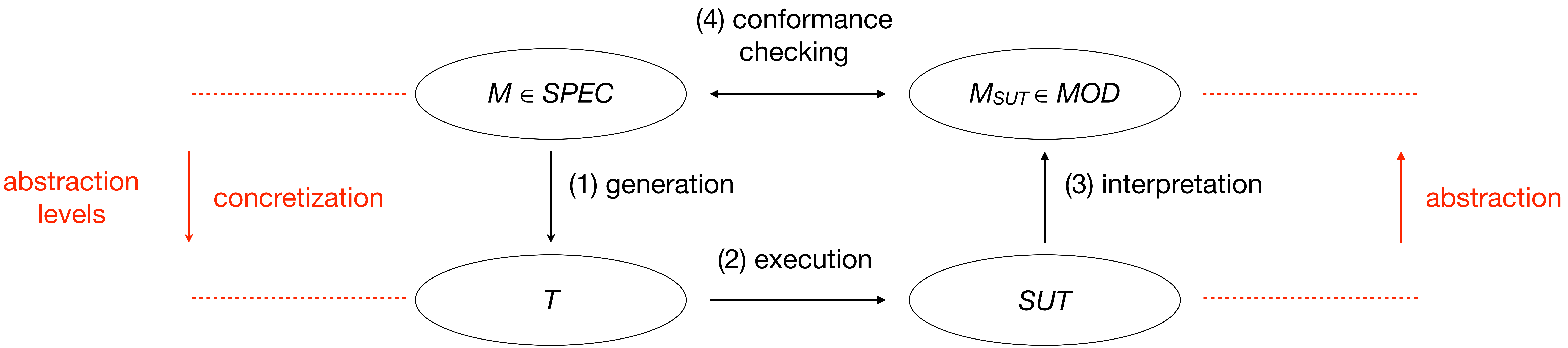
MBT — terminology

- Implementation or System Under Test
  - Piece of hardware/software, a software system, an embedded system, a CPS, etc.
  - The SUT is viewed as a black-box (secret internal structure)
  - The tester controls and observes the SUT via its interfaces (e.g., APIs)
- Specification
  - Describes what the SUT should do using a formal notation (or language)
  - *SPEC* — set of all valid models in a formal notation
    - A specification is $M \in SPEC$
- Conformance
  - Formalizes the notion of correct behavior of a SUT w.r.t. $M \in SPEC$
  - Problem: we'd like to define a relation between elements of different domains
    - *M* (formal entity) <—> SUT (not a formal entity)

MBT — terminology (2)

- Conformance Problem: *M* (formal domain) <—> SUT (not formal domain)
  - Trick — test assumption
    - The **SUT behavior** can be interpreted using the **same level of abstraction** of *M*
    - The SUT behavior is a model $M_{SUT} \in MOD \subseteq SPEC$
      - *MOD* — universe of implementation models
      - M_{SUT} not a-priori known
- Conformance (under the test assumption)
  - Can be expressed as a formal relation between *MOD* and *SPEC* elements
  - $conf \subseteq MOD \times SPEC$
  - M_{SUT} is correct w.r.t. *M* if M_{SUT} *conf S*
- Conformance checking
  - Assess by testing whether M_{SUT} *conf S*
  - *Create T (test suite) s.t.* M_{SUT} conf *M* => M_{SUT} *passes T* (sound but not complete)

MBT process

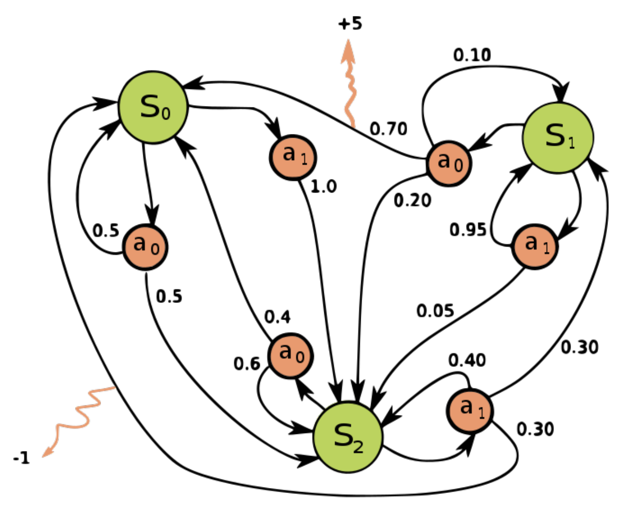


- Offline vs online [1]
  - Offline — steps 1-4 are **separated**
  - Online (or on-the-fly) — steps 1-4 are **merged** into a one-iteration step
    - Test cases are created dynamically and take advantage of the knowledge gained by exploring *M*
    - Iterative approach —> 2-players game: **controller + observer**

1. Utting, Mark, and Bruno Legeard. Practical model-based testing: a tools approach. Elsevier, 2010

Online MBT with MDPs

- Binding (concretization)
  - Defines a mapping between the MDP model spec and the SUT behavior



M

binding

APIs

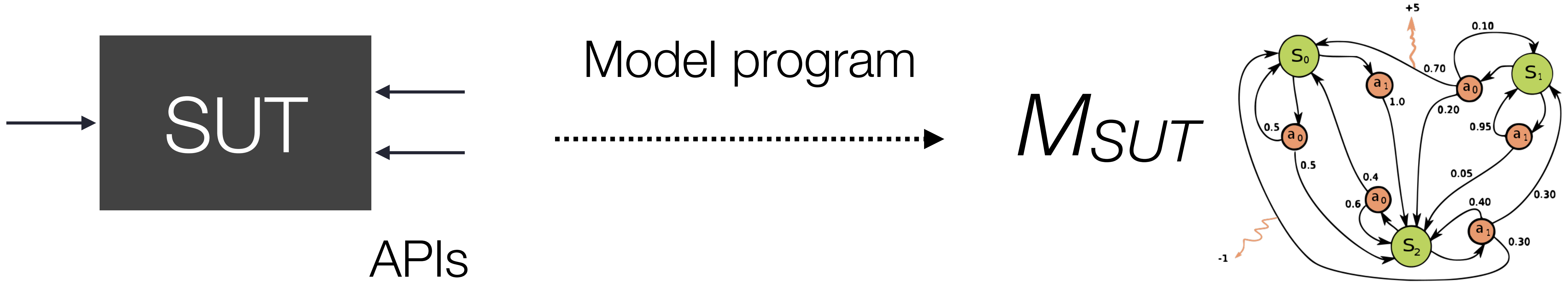
- Formal definition
  - Given a MDP M = (S, s_0, A, P) and a SUT, i.e., a set of exported services H (having signature and arguments), a binding is a tuple of partial functions (h, i, post) s.t.
    - h(s, a), a ∈ A(s) identifies a service ∈ H
    - i(s, a), a ∈ A(s) identifies a vector v_{in} for the service h(s, a)
    - post(s, a, s’), a ∈ A(s) maps to a post-condition that must hold for v_{out} resulting from the execution of the service h(s, a) on input v_{in}

Controllable SUT components

Observable SUT behavior

Online MBT with MDPs (2)

- Model program (abstraction)
  - Defines the abstract **interpretation** of the **SUT behavior** in terms of **MDP model**



- Formal definition
  - Given a MDP $M = (S, s_0, A, P)$ and a binding (h, i, post) the model program $M_{SUT} = (S', s_0', A', P')$ is a MDP model s.t.
    - $S' \subseteq S$ All observable SUT states exist in M
    - $A \subseteq A'$ All controllable actions in M are feasible in SUT
    - $s_0 = s_0'$
    - $P'(s, a, s') > 0$ iff there exists $v_{out} = h(s, a)(v_{in})$ s.t. post(s, a, s') holds for v_{out}

M_{SUT} transitions are defined in terms of SUT behavior

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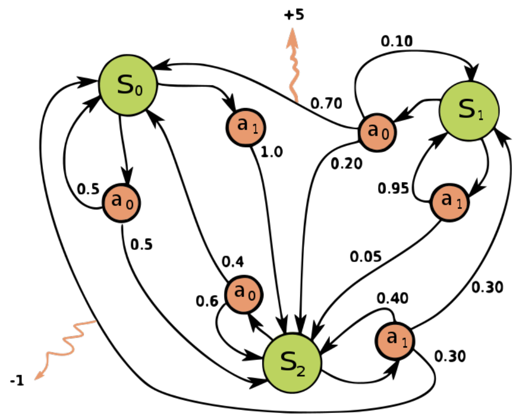
Online MBT with MDPs (3)

- Conformance checking
  - Needs the definition of conformance relation = probabilistic alternating simulation + refinement

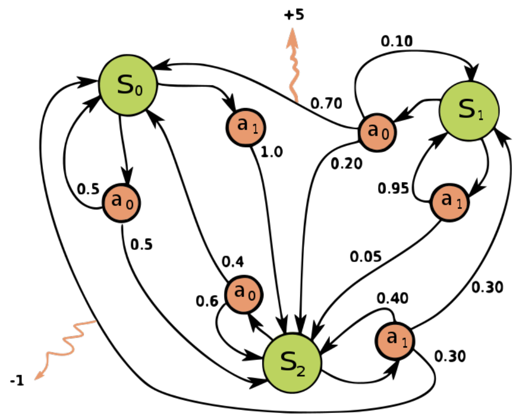
- Probabilistic alternating simulation
  - between M and M_{SUT} is a binary relation $\sigma \subseteq S \times S'$, s.t. for all $(s, s') \in \sigma$
    - $A(s) \subseteq A'(s')$
    - For each $t \in S : P(s, a, t) > 0$, there exists $t' \in S' : P'(s', a, t') > 0$ and $(t, t') \in \sigma$

M



$$\begin{array}{ccc} s & \overset{(s, s') \in \sigma}{\text{- - - - - - - -}} & s' \\ \downarrow a & & \downarrow a \\ t & \overset{(t, t') \in \sigma}{\text{- - - - - - - -}} & t' \\ \downarrow a' & & \downarrow a' \\ & \dots & \end{array}$$

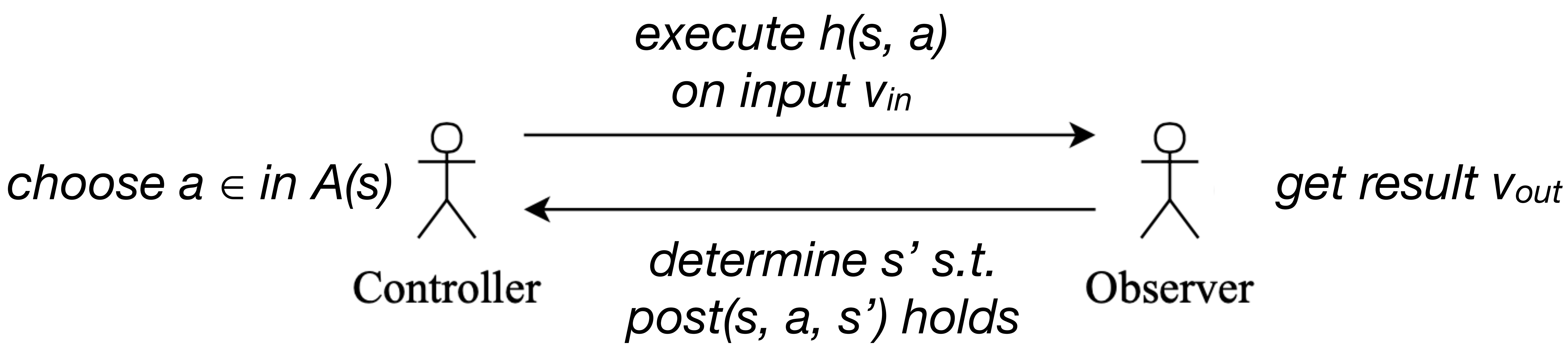
M_{SUT}



Online MBT with MDPs (4)

- Refinement
  - M_{SUT} refines M iff there exists a probabilistic alternating simulation σ s.t. $(s_0, s_0') \in \sigma$

- Conformance game
  - The notion of refinement is verified in practice by means of a conformance game between
    - Controller —> chooses actions based on a given test case generation strategy
    - Observer —> verifies the result out of a test execution



Online MBT under Uncertainty

- Problem statement
- Uncertain model paramenters
- Bayesian inference
- Framework and test case generation strategies

Problem statement

$$M \in SPEC \xrightarrow{\text{dev process}} SUT$$

Is it complete?
or is it based on partial knowledge
of the phenomenon of interest?

How to perform MBT if part of
M is uncertain?

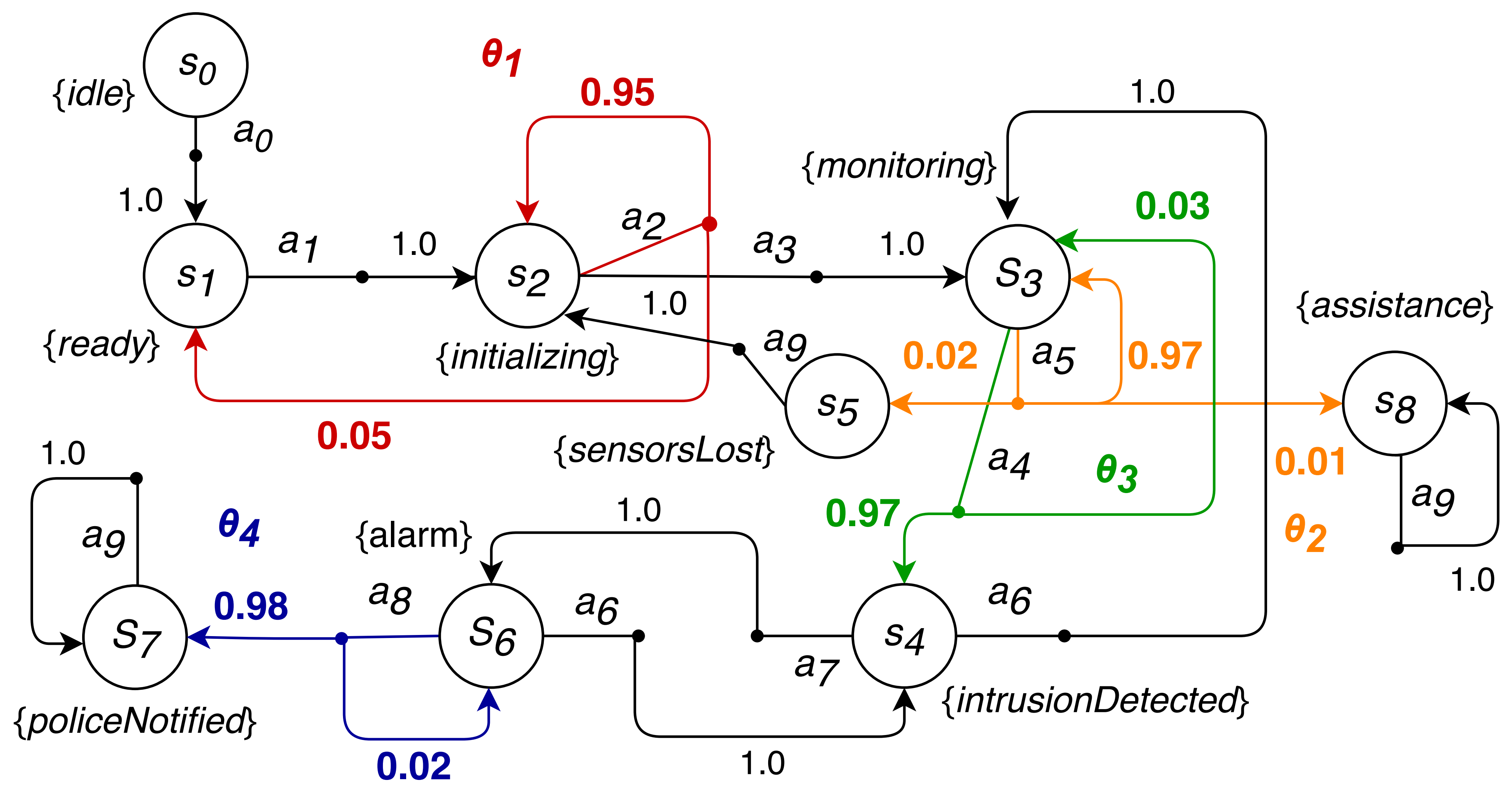
- Problem
  - Design-time models are imperfect and include **assumptions**
  - Assumptions are affected by sources of **uncertainty**
    - Uncertain **system** properties (e.g., algorithmic/structural uncertainty, performance)
    - Uncertain **environment** properties (e.g., usage profiles, failure rate of 3rd-party components, latency)

The very idea

- Objective
  - Reduce the discrepancy between design-time assumptions (uncertain mathematical models) and real-world entities (runtime evidence) by applying Inverse Uncertainty Quantification (IUQ)
- How
  - Assumption — sources of uncertainty affect model parameters (i.e., uncertain transition probability values in the MDP)
  - Calibration of uncertain model parameters during integration/system testing by combining
    - Online Model-based Testing
    - Bayesian inference

Uncertain model parameters — SafeHome

- Sources of uncertainty (in CPSs) [1]
  - Application level — uncertain events/data generated from software running upon physical units (e.g., θ_3)
  - Infrastructure level — uncertain reliability of networking and/or cloud infrastructure (e.g., θ_4)
  - Integration level — uncertain outcomes from interacting physical units (e.g., θ_1, θ_2)



Examples

θ_2: uncertain sensing capability from the monitoring state

θ_4: uncertain failure rate of police notification from the alarm state

1. Man Zhang, et al., Uncertainty-wise cyber-physical system test modeling. Software & Systems Modeling (2017), 1–40

Uncertain regions

- Uncertain regions
  - Uncertain transition probabilities θ_i grouped by <src-state, action>
  - Values in θ_i are uncertain parameters of a Categorical distribution
    - $\theta_i \sim Cat(p_1, \ldots, p_k)$

region	state-action	affected level	target states	probability values
θ_1	s_2-a_2	integration	s_2, s_1	0.95, 0.05
θ_2	s_3-a_4	integration	s_3, s_4	0.03, 0.97
θ_3	s_3-a_5	application	s_3, s_5, s_8	0.01, 0.97, 0.02
θ_4	s_6-a_8	infrastructure	s_6, s_7	0.02, 0.98

Region

Uncertain parameters of a Categorical distribution

—

hypothesis

- Intuition
  - Mitigate the uncertainty over θ regions by observing the SUT
  - Observation provides evidence to increase the confidence on transition probabilities

Bayesian inference

- Method used to **update the probability** for a **hypothesis** as more **evidence** becomes available
- Formulation [1]
  - To learn θ (phenomenon of interest) we collect a sample $y = (y_1, \ldots, y_n)$
  - *Posterior ∝ likelihood x Prior*
    - Prior $f(\theta)$ — *hypothesis on* θ
    - Likelihood $f(y \mid \theta)$ — *compatibility of the evidence with the given hypothesis*
    - Posterior $f(\theta \mid y)$ — ***best knowledge** on the hypothesis given the evidence*
- In our context
  - The natural conjugate **Prior** of the **Categorical** distribution is the **Dirichlet** distribution
  - $\text{Prior}_{\theta i} \sim Dir(\alpha_1, \ldots, \alpha_K)$
    - e.g., uninformative $\text{Prior}_{\theta 3} \sim Dir(0.5, 0.5, 0.5)$
    - e.g., informative $\text{Prior}_{\theta 3} \sim Dir(1.0, 97.0, 2.0)$
      100 observations = 1 s_3, 97 s_5, 2 s_8

region	state-action	affected level	target states	probability values
θ_1	s_2-a_2	integration	s_2, s_1	0.95, 0.05
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θ_4	s_6-a_8	infrastructure	s_6, s_7	0.02, 0.98

1. Robert, Christian. The Bayesian choice: from decision-theoretic foundations to computational implementation. Springer Science & Business Media, 2007

Bayesian inference (2)

- In our context
  - Updating rule Prior$_{\theta i}$ ~ $Dir(\alpha_1, ..., \alpha_K)$ —> Post$_{\theta i}$ ~ $Dir(\alpha_1 + n_1, ..., \alpha_K + n_k)$
    - e.g., $Dir(1.0, 97.0, 2.0)$ —> Post$_{\theta i}$ ~ $Dir(1.0 + 35, 97.0 + 955, 2.0 + 10)$
      1000 observations = 35 s_3, 955 s_5, 10 s_8
- Summarization
  - Prior/Posterior knowledge can be summarized by using
    - Mean — transition probability values $p_i = \alpha_i / \sum_{j=1}^{k} \alpha_j$
    - HPD region — degree of confidence $C = \{p : f(\cdot) \geq 0.95\}$

region	state-action	affected level	target states	probability values
θ_1	s_2-a_2	integration	s_2, s_1	0.95, 0.05
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θ_4	s_6-a_8	infrastructure	s_6, s_7	0.02, 0.98

Prior mean

inference

Posterior mean

0.033, 0.956, 0.011

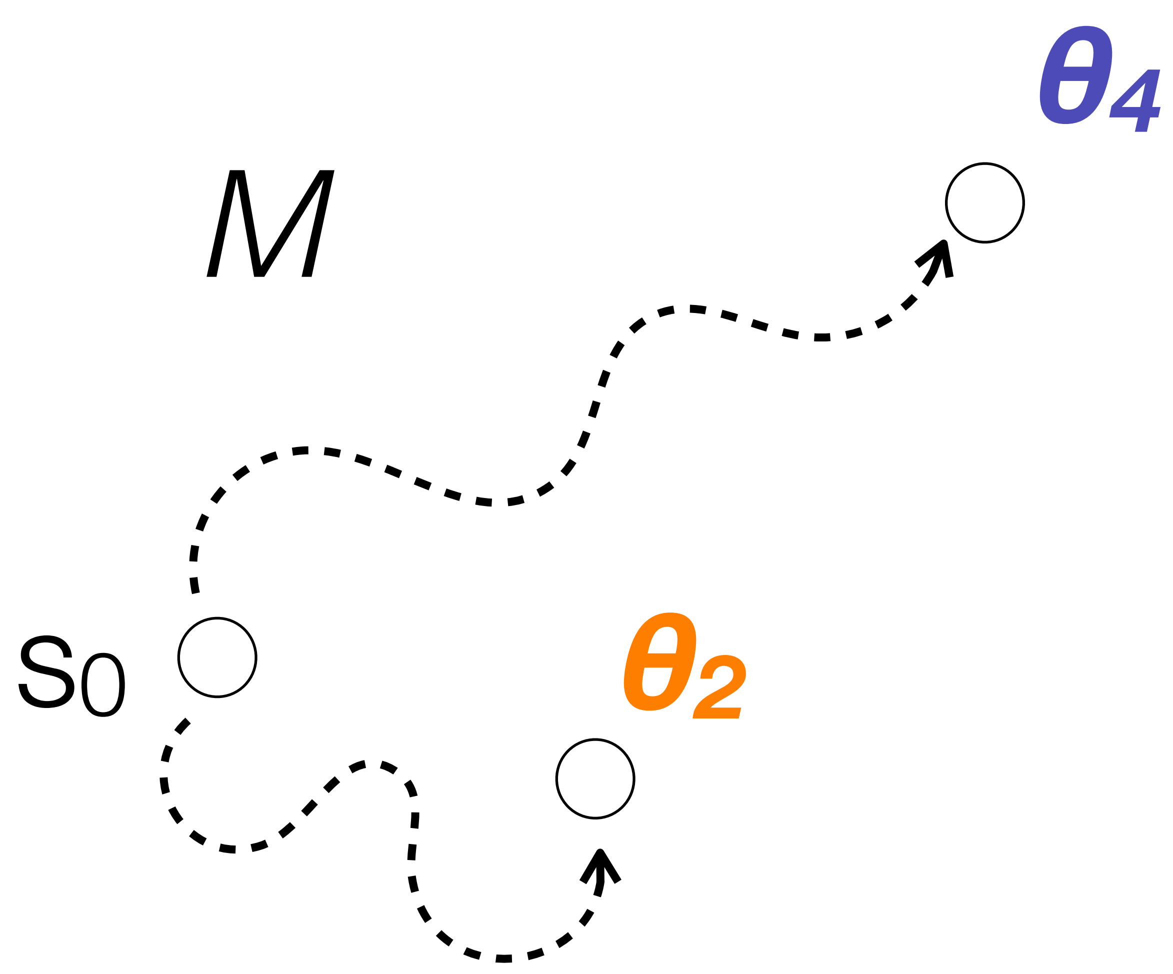
Prior HPD region = { [0.009, 0.019], [0.936, 0.996], [0.001, 0.047] }
Prior HPD width = 0.136

Posterior HPD region = { [0.009, 0.019], [0.936, 0.996], [0.001, 0.047] }
Posterior HPD width = 0.056

Lower value -> higher confidence

Online MBT + Bayesian inference

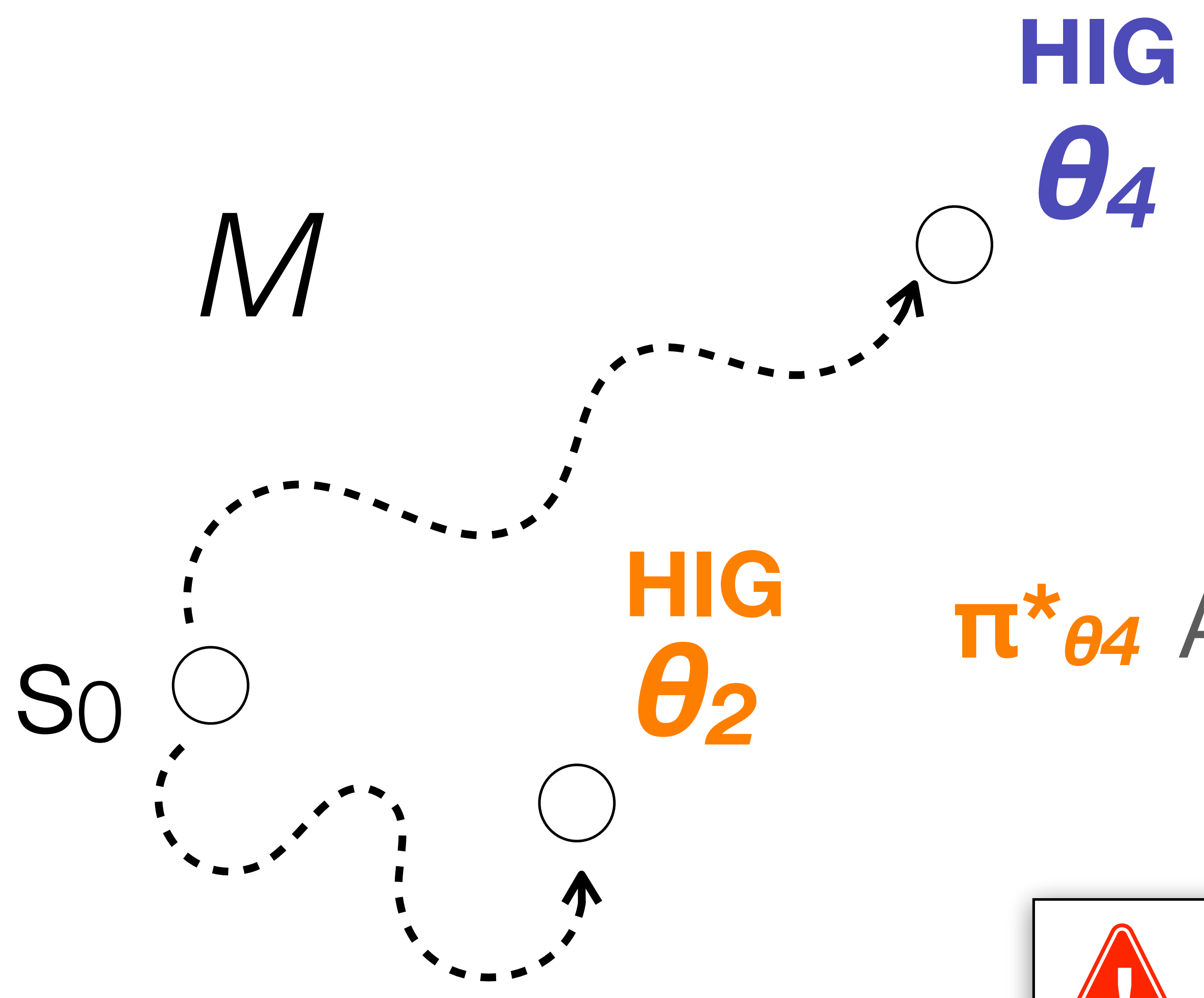
- Objective (reminder)
  - design-time assumptions <— gap reduction —> runtime evidence
- How
  - Perform a controlled exploration using online MBT to stress the uncertain components
  - Gather evidence and run bayesian inference to reduce the uncertainty



- Uncertainty-aware MBT strategy
  - Explore by maximize the probability of reaching θ regions
  - Reduces to an optimization problem:
    - Find out the actions a decision maker should take to maximize the exploration of θ regions

Uncertainty-aware strategy

- Computation of the best policies
  - For each θ_i
    - construct a **reward structure** that assigns **HIG** reward to θ_i transitions, **LOW** elsewhere
    - Compute the **best policy** $\pi^*_{\theta i}$ (value iteration)
      - For each state, it selects the action that maximizes the probability to reach θ_i



$\pi^*_{\theta 4}$ Actions maximizing the probability to reach θ_4

$\pi^*_{\theta 4}$ Actions maximizing the probability to reach θ_2

Back to our mental note

We'll see how to leverage rewards in a "unconventional" way to drive testing.

Uncertainty-aware strategy (2)

- How to combine the best policies $\pi^*_{\theta i}$?
  - Simple scenario —> there exists just a single θ region
  - Otherwise —> different exploration strategies may be constructed/adopted
    - Strategies represent **decision makers** (i.e., testers) that use a probabilistic function

$$\mathcal{P}(s,a) = \begin{cases} 0 & \omega(s,a) = 0 \\ \omega(s,a)/\sum_{a' \in A(s)} \omega(s,a') & otherwise \end{cases}$$

- The **ω weight** selectively increase/decrease the probability of choosing a specific action *a* from state *s*

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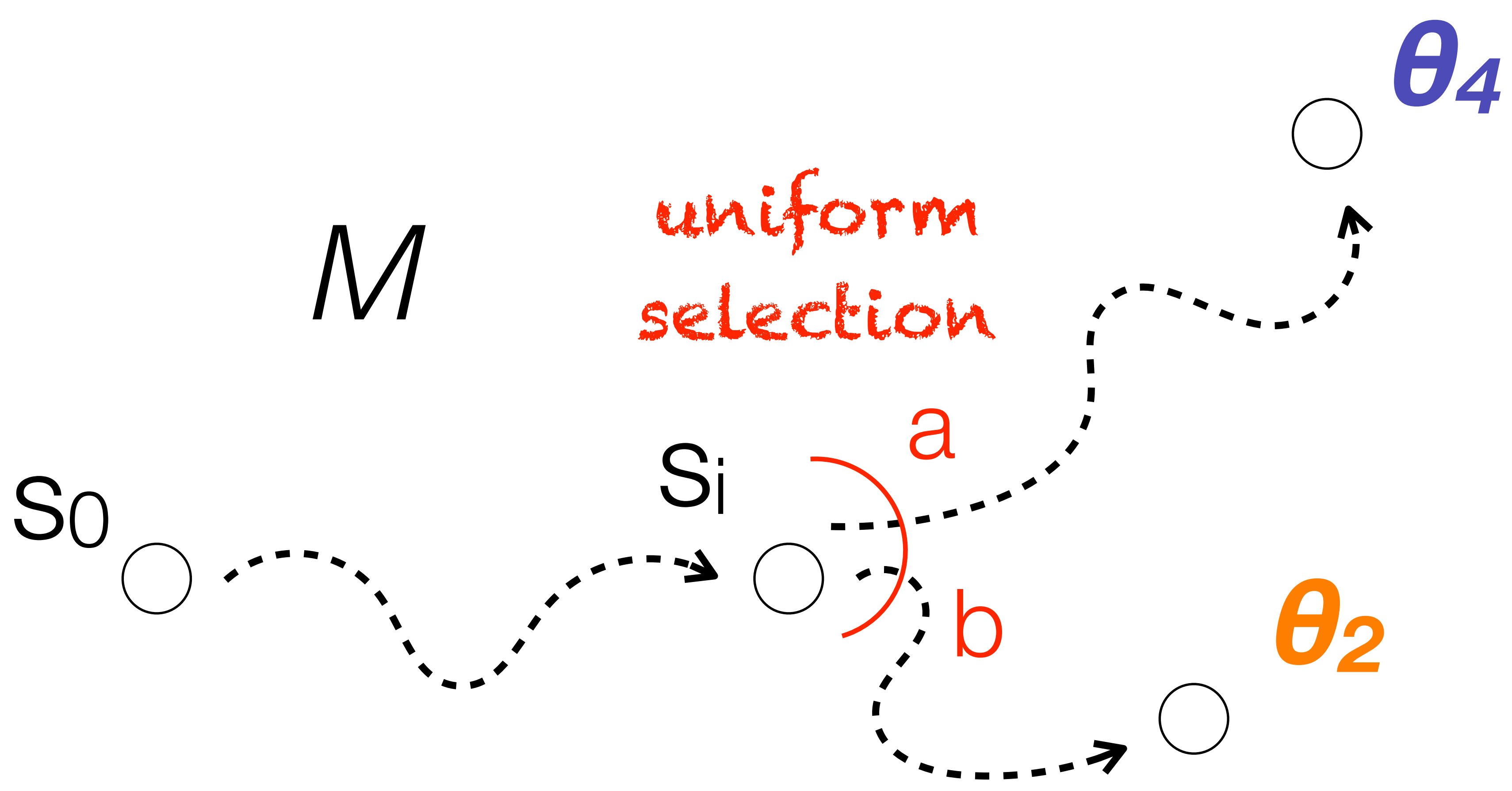
Uncertainty-aware strategy (3)

- Flat strategy
  - Actions selected by different policies $\pi^*_{\theta i}$ have equal probability
  - **Uniform random sampling** of the available policies

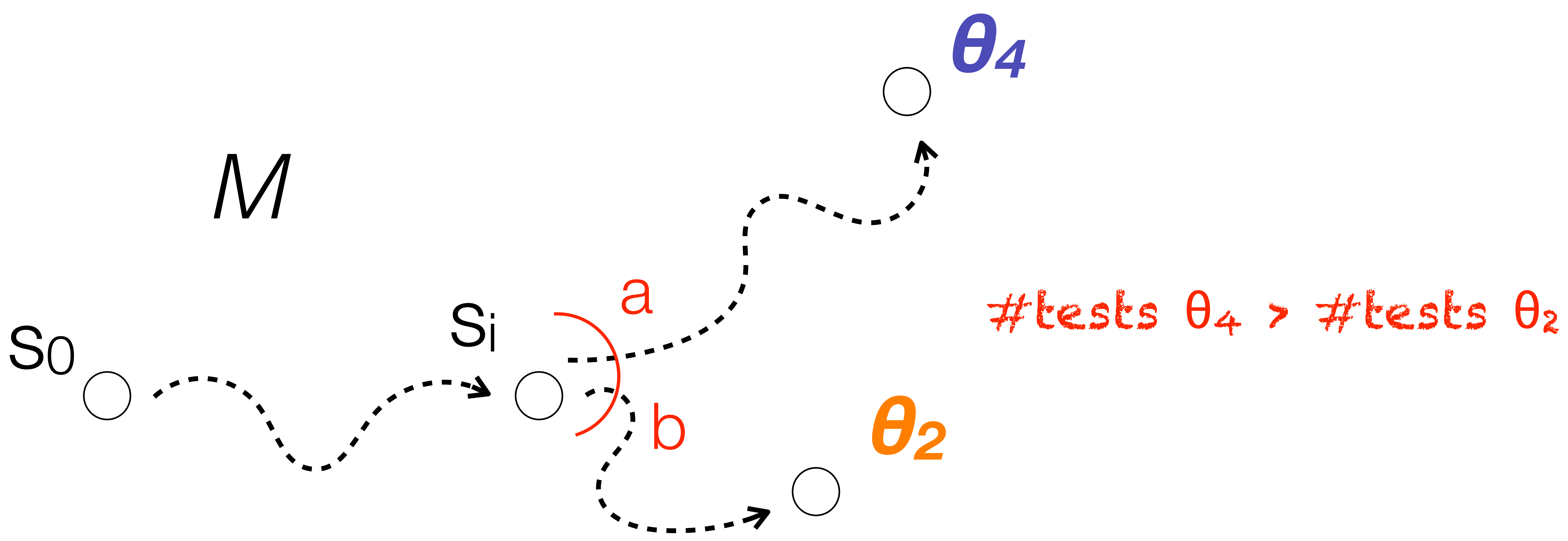
$$\omega^{RT}(s,a) = \begin{cases} 1 & \exists i : \pi_i^*(s) = a \\ 0 & otherwise \end{cases}$$



Uncertainty-aware strategy (4)

- History-based strategy
  - Tries to **keep balanced** the number of times θ regions are tested
  - We leverage decrementing weights **inversely proportional to #selections** of state-action pairs

$$\omega^{HT}(s,a) = \begin{cases} 1/\#(s,a) & \exists i : \pi_i^*(s) = a \\ 0 & otherwise \end{cases}$$

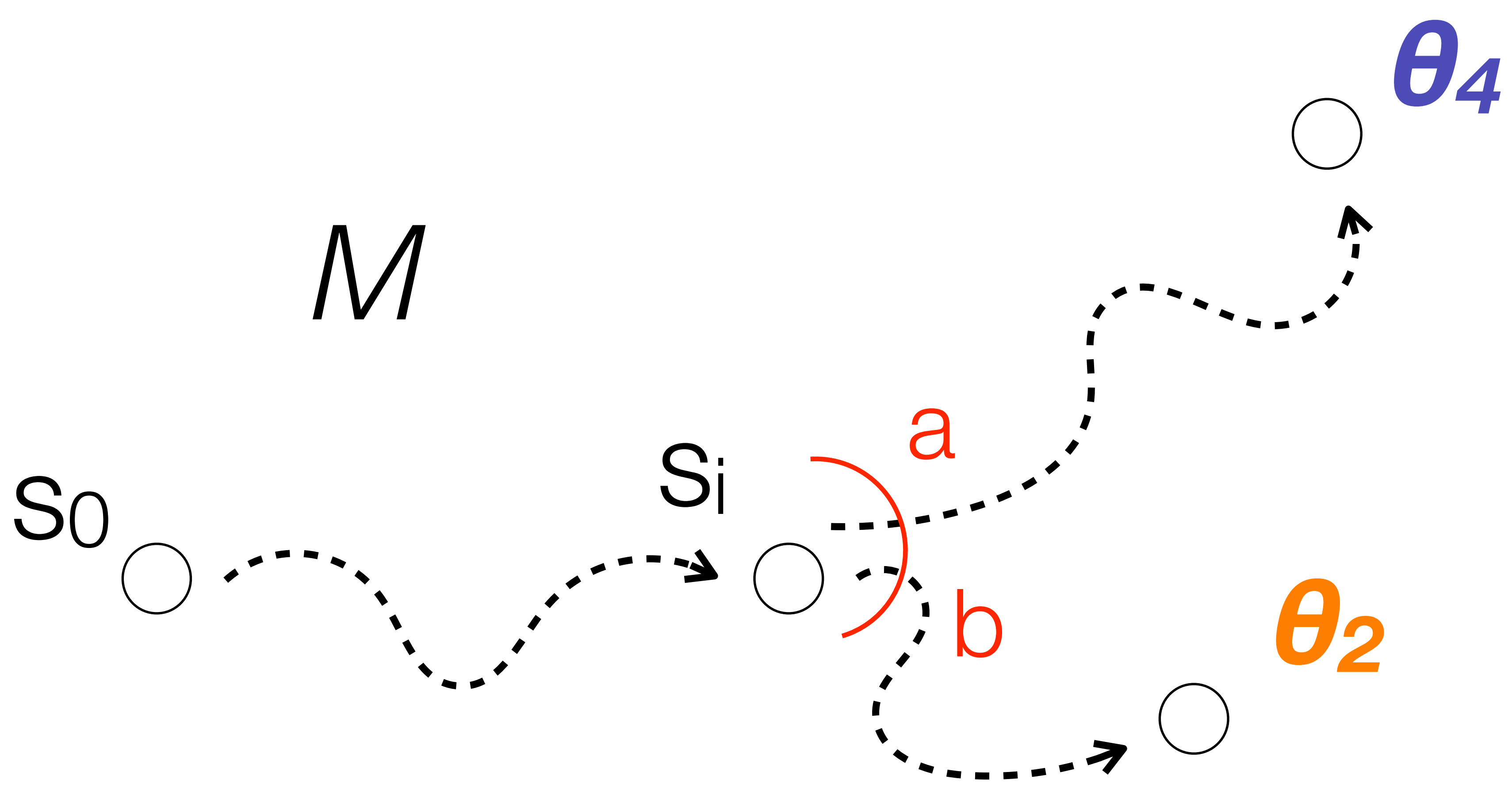


the higher the #selections of a,
the lower the likelihood of selecting it again

Uncertainty-aware strategy (5)

- Distance strategy
  - Tries to deliver **balanced degree of confidence** on θ regions
  - The weight is **proportional to the HPD width** of θ regions

$$\omega^{DT}(s,a) = \begin{cases} |\mathrm{HPD}_{\theta_i}| & \exists i : \pi_i^*(s) = a \\ 0 & otherwise \end{cases}$$



confidence θ_2 > confidence θ_4

the larger the HPD width of the target θ, the higher the likelihood of selecting it

Termination condition

- Limit on the effort
  - Traditional termination condition based on #tests limit
- Bayes factor
  - Tries to recognize when the inference process converges

$$\mathcal{F} = \frac{f(y|\theta)}{f(y|\theta')}$$

$$\mathcal{F} \in [10^0, 10^{1/2}]$$

Likelihood that data y are produced under different assumptions θ and θ′

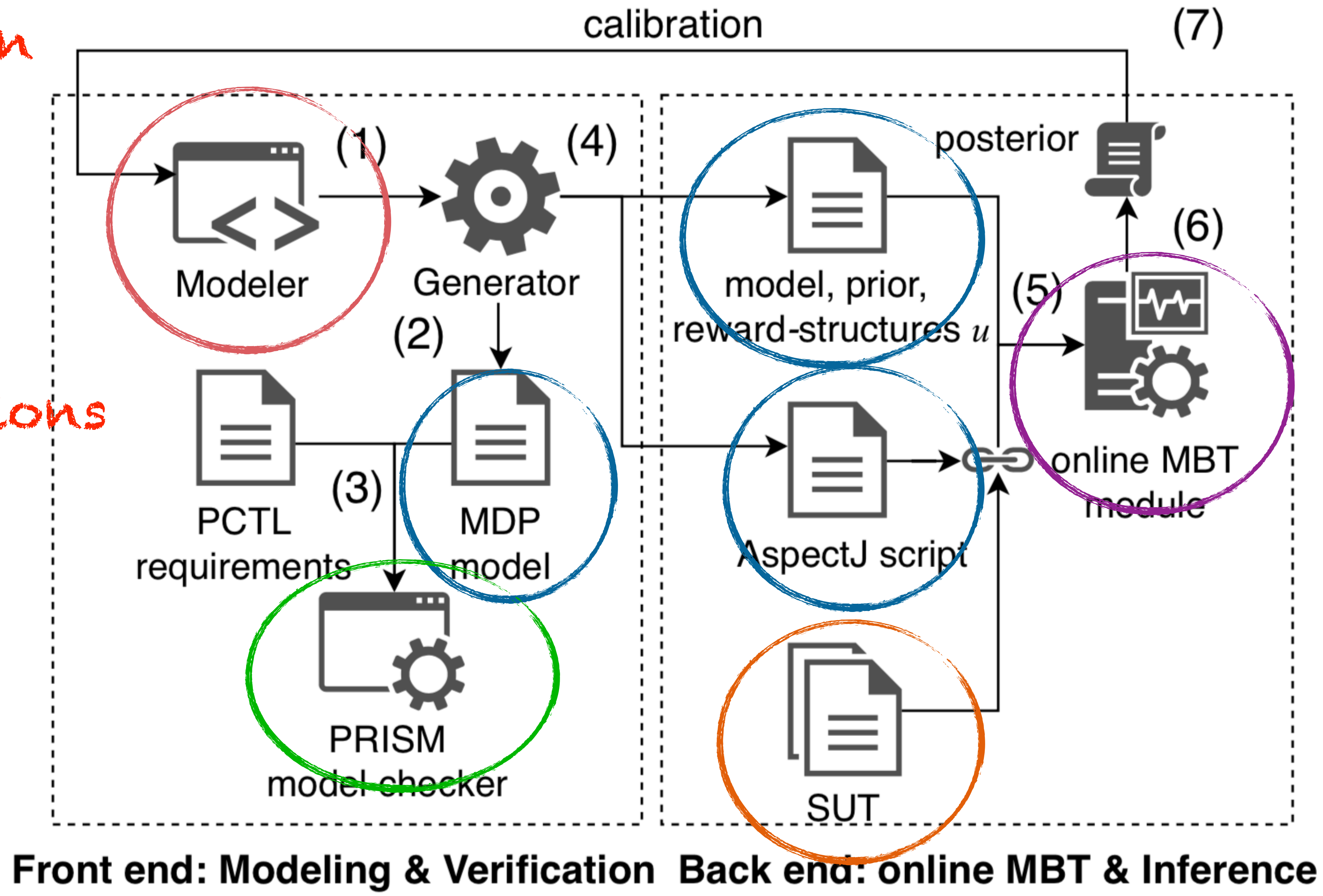
Difference between assumptions θ and θ′ is not substantial

Current toolchain implementation

MDP definition + uncertain transition probabilities θ

binding to the SUT:
actions -> inputs
acs -> routine postconditions

automatic generation



online MBT and inference/ calibration

model checking of PCTL requirements

Java program

Current toolchain implementation (2)

MBT module

https://github.com/SELab-unimi/mbt-module

Java 99.3%
Other 0.7%

- Current stage
  - Uncertainty-aware strategies have been implemented
  - Systematic evaluation of their cost-effectiveness —> currently inedited

Current toolchain implementation (3)

- Evaluation summary
  - We assessed statistical difference (Mann-Whitney U test [1])
  - We evaluated practical value (Vargha & Delaney's $\hat{A}_{12}$ measure [1])
    - In our context —> assuming same effort (i.e., #tests), the probability that target strategy yields smaller HPD width values than flat one (i.e., baseline)

Table 3: Vargha and Delaney's $\hat{A}_{12}$ measure

	%uncertainty			#actions		
balanced	20	50	80	5	10	20
hist	1.000	0.716	0.531	0.617	0.704	0.926
dist	1.000	0.790	0.679	0.741	0.802	0.951
unbalanced	20	50	80	5	10	20
hist	0.963	0.716	0.556	0.531	0.642	0.901
dist	0.988	0.951	0.691	0.741	0.741	0.975

1. Andrea Arcuri and Lionel Briand, A practical guide for using statistical tests to assess randomized algorithms in software engineering, ICSE'11, New York, NY, USA

Summary

- We discussed MBT for probabilistic systems and the problem of testing with uncertain model components
- Depending on the Prior knowledge (hypothesis) and information that can be gathered during testing, we derived different uncertainty-aware exploration strategies and evaluated their cost-effectiveness
  - Flat —> uniform selection
  - History —> balanced exploration
  - Distance —> balanced delivered confidence
- Next
  - Hands on session with the MBT module
  - Design/develop an additional exploration strategy

References

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