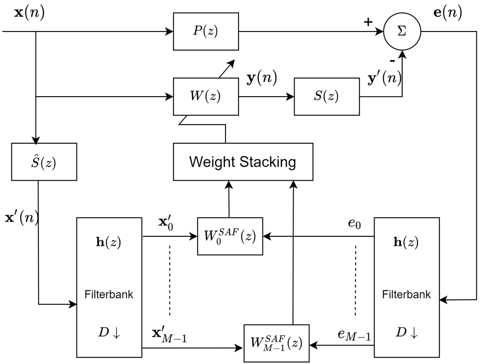
Delayless FX-LMS Active Noise Control

Delayless FX-LMS in subbands

Article [1] gives the following scheme for delayless active noise cancellation in subbands:



[1]: A. A. Milani, I. M. S. Panahi and P. C. Loizou, "A New Delayless Subband Adaptive Filtering Algorithm for Active Noise Control Systems," in IEEE Transactions on Audio, Speech, and Language Processing, vol. 17, no. 5, pp. 1038-1045, July 2009, doi: 10.1109/TASL.2009.2015691.

Implemented filter bank

Signal decomposition is implemented with an analysiss filter banck.
Article [1] uses uniform DFT filter banck (UDFT), with a decimation factor of $M/4$ (M: number of subbands).

Prototype filter: $H_0(z) = 1 + z^{-1} + \ldots + z^{M+1}$

Filter bank is given by the filters $H_k(z) = H_0(ze^{-j2\pi k/M})$

The analysis bank can be written as:

$$\boldsymbol{h}(z) = [H_0(z),\, H_1 U(z),\, \ldots,\, H_{M-1}(z)]^T$$

FX-LMS in subbands

Error: $e(n) = d(n) - \boldsymbol{s}(n) * [\boldsymbol{w}(n)^T \boldsymbol{x}(n)]$

Filters: $\boldsymbol{w}_k^{SAF}(n+D) = \boldsymbol{w}_k^{SAF}(n) + \mu \dfrac{\boldsymbol{x}_k'^{*}(n) e_k(n)}{\epsilon + ||\boldsymbol{x}_k'(n)||^2}$

with:

- μ step size
- ϵ positive small number
- $\boldsymbol{x}_k'(n) = [x_k'(n), x_k'(n-1), \ldots, x_k'(n - L_{SAF} + 1)]^T$
- $\boldsymbol{w}_k^{SAF}(n) = [w_{k,1}^{SAF}, w_{k,2}^{SAF}, \ldots, w_{k,L_{SAF}-1}^{SAF}]^T$
- L_{SAF} length of adaptive filters in subbands

Implemented filter bank

The filter bank is given by

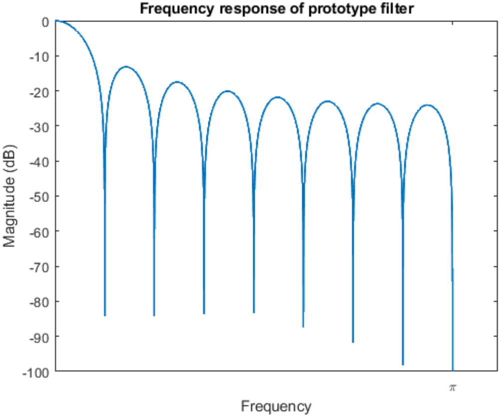
$$\boldsymbol{h}(z) = \frac{1}{M}\boldsymbol{F}^* \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-M+1} \end{bmatrix} =$$

$$\frac{1}{M}\begin{bmatrix} 1 & \dots & e^{-2\pi i(M-1)/M} \\ \vdots & & \vdots \\ e^{-2\pi i(M-1)/M} & \dots & e^{-2\pi i(M-1)(M-1)/M} \end{bmatrix}^* \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-M+1} \end{bmatrix} =$$

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} \in \mathbb{C}^{M\times M}$$

Frequency response of $H_0(e^{j\omega})$

$$H_0(e^{j\omega}) = \begin{cases} M & \omega = 0 \\ e^{-j\omega(M-1)/2} \dfrac{\sin(\omega M/2)}{\sin(\omega/2)} & \text{altrove} \end{cases}$$

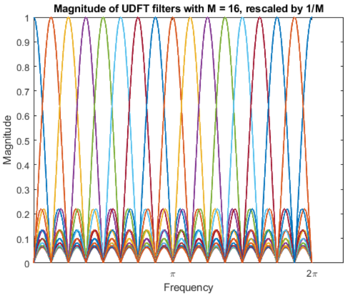


Frequency response of the prototype filter with $M = 16$. The frequency is normalized to 1

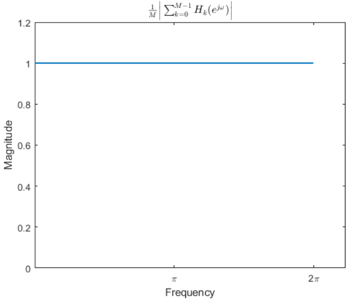
Flat frequency response

The condition of flat frequency response using the synthesis bank is respected:

$$\frac{1}{M}\left|\sum_{k=0}^{M-1} H_k(e^{j\omega})\right| = 1 \ \forall\omega$$



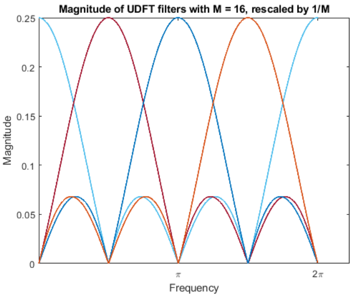
Frequency response of the filters $H_0, \ldots, H_{M-1}$, with $M = 16$



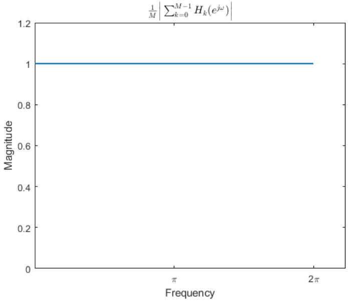
$\frac{1}{M}\left|\sum_{k=0}^{M} H_k(e^{j\omega})\right|$ for $M = 16$

Decimazione

Since the first 0 of the frequency response is $2\pi/M$, the usable band is $2\pi/M$. The decimation should not exceed $M/2$ and is chosen at $M/4$

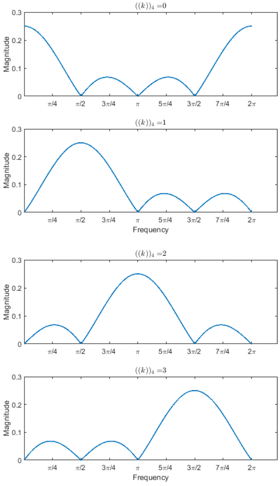


Frequency response of $H_0, \ldots, H_{M-1}$, con $M = 16$ decimated of $D = M/4$



$\frac{1}{D}\frac{1}{M}\left|\sum_{k=0}^{M}\sum_{d=0}^{D-1} H_k(e^{j(\omega-2\pi d)/D})\right|$ per $M = 16$ e $D = M/4$

Weight stacking



Filtri sottobanda per
$((k))_4 = 0, 1, 2, 3$

ω	$((k))_4$	Intervallo $W_k^{SAF}[\cdot]$
$[0, \pi/M)$	-	$W_0^{SAF}[0, \pi/M)$
$[(2k-1)\pi/M,$ $(2k+1)\pi/M)$	0	$W_k^{SAF}[7\pi/4, 2\pi)$ U $W_k^{SAF}[0, \pi/M)$
$[(2k-1)\pi/M,$ $(2k+1)\pi/M)$	1	$W_k^{SAF}[\pi/4, 3\pi/4$
$[(2k-1)\pi/M,$ $(2k+1)\pi/M)$	2	$W_k^{SAF}[3\pi/4, 5\pi/4)$
$[(2k-1)\pi/M,$ $(2k+1)\pi/M)$	3	$W_k^{SAF}[5\pi/4, 7\pi/4)$
$[2\pi - \pi/M, 2\pi)$	-	$W_{M-1}^{SAF}[7\pi/4, 2\pi)$

Mathematics of the weight stacking

From article [1]:

$$\begin{cases} l \in [0, N/2], & W[l] = W^{SAF}_{((\lfloor \frac{lM}{N} \rfloor))_M}\left[((l))_{\frac{4N}{M}}\right] \\ l \in (N/2, N), & W[l] = W[N-l] \end{cases} \tag{1}$$

Using the subband length $L_{SAF} = 4N/M$.

we have $W[N/2+1] = W[(N-2)/2]$ e $W[N-1] = W[1]$.

Tuttavia, **The algorithm diverges with this formula**, since the inverse transform $w(n) = \text{IFFT}[W(e^{j\omega})]$ is complex, since $M/2$ is complex.

[1]: A. A. Milani, I. M. S. Panahi and P. C. Loizou, "A New Delayless Subband Adaptive Filtering Algorithm for Active Noise Control Systems," in IEEE Transactions on Audio, Speech, and Language Processing, vol. 17, no. 5, pp. 1038-1045, July 2009, doi: 10.1109/TASL.2009.2015691.

Weight Stacking

Varying the weight stacking method:

$$\begin{cases} l \in [0, N/2), & W[l] = W^{SAF}_{((\lfloor \frac{lM}{N} \rfloor))_M}\left[((l))_{\frac{4N}{M}}\right] \\ l \in [N/2, N), & W[l] = W[N-l] \end{cases} \tag{2}$$

$W[N/2]$ does not update, so it is set to 0 (as in [2]).

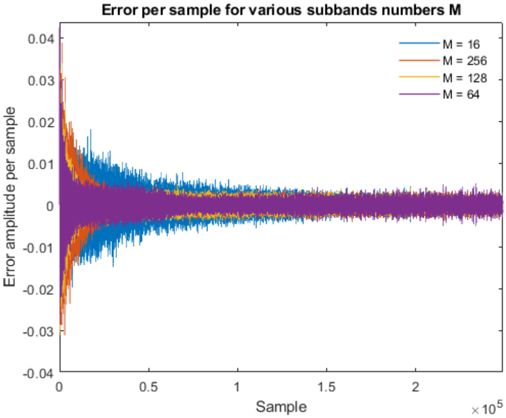
$W[N-1] = W[1]$. iFFT of W gives a real w filter. This allows the algorithm convergence.

[2]: Lee, Kong-Aik and Gan, Woon-Seng and Kuo, Sen M., Subband Adaptive Filtering: Theory and Implementation, 2009, Wiley Publishing

Results

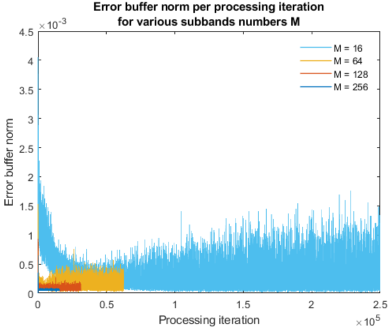
The tests have been executed with

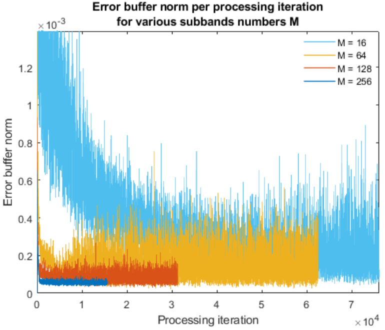
- $M = 16, 64, 128, 256$
- $\mu = 0.05$
- impulse responses $p(n)$ e $s(n)$ (of primary and secondary path) are of length 256.
- Audio signal is a white noise of $1 \cdot 10^6$ samples, $fs = 44.1$KHz



Error per sample, varying M

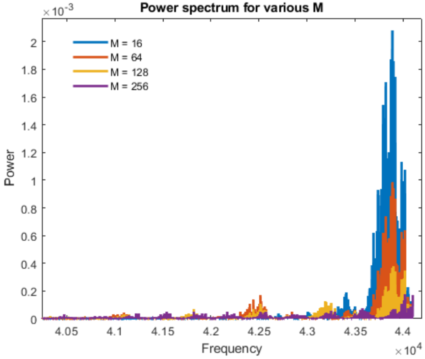
Risultati - Matlab





Results

Error power spectrum S_{ee} in the last 65,536 samples.

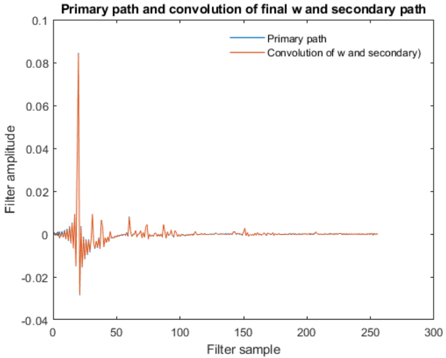


M	$\max(\lvert S_{ee}\rvert)$
16	$20.8 \cdot 10^{-4}$
64	$9.8 \cdot 10^{-4}$
128	$3.6 \cdot 10^{-4}$
256	$1.7 \cdot 10^{-4}$

Maximum power spectrum value varying M

Results

Using $M = 256$ it is obtained:



Superposition of $\boldsymbol{w}(n) * \boldsymbol{s}(n)$ and $\boldsymbol{p}(n)$

Computational complexity

Operation	Complexity	Type
$\boldsymbol{x}(n) \cdot \boldsymbol{w}(n)$	$\boldsymbol{O}(N)$	real, fp64
$\boldsymbol{x}(n) \cdot \hat{\boldsymbol{s}}(n)$	$\boldsymbol{O}(N)$	real, fp64

Operations implemented every sample.

Operation	Complexity	Type
$\boldsymbol{x}'(n) \cdot \boldsymbol{F}$	$\boldsymbol{O}(M \cdot (M/2+1))$	complex, fp64
$\boldsymbol{e}(n) \cdot \boldsymbol{F}$	$\boldsymbol{O}(M \cdot (M/2+1)$	complex, fp64
$\boldsymbol{w}_k^{SAF}$ update	$\boldsymbol{O}((M/2+1) \cdot 2 \cdot L_{SAF})$	complex, fp64
FFT di $\boldsymbol{w}_k^{SAF}$	$\boldsymbol{O}((M/2+1) \cdot L_{SAF} \log_2(L_{SAF}))$	complex, fp64
IFFT di W	$\boldsymbol{O}(N \log_2(N))$	complex, fp64

Operations implemented every $D = M/4$ sample.

La lunghezza dei filtri sottobanda $L_{SAF} = 4N/M$, varia al variare di M

Computational complexity

M	Complexity	Type
16	$\mathcal{O}(1736)$	complex, fp64
	$\mathcal{O}(512)$	real, fp64
64	$\mathcal{O}(590)$	complex, fp64
	$\mathcal{O}(512)$	real, fp64
128	$\mathcal{O}(665)$	complex, fp64
	$\mathcal{O}(512)$	real, fp64
256	$\mathcal{O}(1096)$	complex, fp64
	$\mathcal{O}(512)$	real, fp64

Computational complexity for various M, divided by D (considering $N = 256$).