EECS 127/227AT Optimization Models in Engineering
Spring 2020 **Discussion 6**

1. Simple constrained optimization problem with duality

Consider the optimization problem

$$\begin{aligned} \min_{x_1,x_2\in\mathbb{R}} & f(x_1,x_2) \\ \text{subject to } & 2x_1+x_2\geq 1 \\ & x_1+3x_2\geq 1 \\ & x_1\geq 0,\ x_2\geq 0 \end{aligned}$$

(a) Express the Lagragian of the problem $\mathcal{L}(x_1,x_2,\lambda_1,\lambda_2,\lambda_3,\lambda_4)$

(b) Show that $\mathcal{L}$ is concave in $(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$.

(c) Express the dual function of the problem, and show that it is concave.

(d) Assume f is convex. Show that $\mathcal{L}$ is convex in (x_1,x_2).

(e) Denoting $\mathcal{X}=\{(x_1,x_2)\mid 2x_1+x_2\geq 1,\ x_1+3x_2\geq 1,\ x_1\geq 0,\ x_2\geq 0\}$, show that

$$\max_{\lambda_1\geq 0,\lambda_2\geq 0,\lambda_3\geq 0,\lambda_4\geq 0}\mathcal{L}(x_1,x_2,\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\begin{cases} f(x_1,x_2) & \text{if } (x_1,x_2)\in\mathcal{X} \\ +\infty & \text{otherwise}\end{cases}$$

(f) Conclude that $\min_{(x_1,x_2)\in\mathcal{X}}\max_{\lambda_1\geq 0,\lambda_2\geq 0,\lambda_3\geq 0,\lambda_4\geq 0}\mathcal{L}(x_1,x_2,\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\min_{(x_1,x_2)\in\mathcal{X}} f(x_1,x_2)$

(g) Assuming f is convex, formulate the first order condition on $\mathcal{L}$ as a function of ∇f and $\lambda_1,\lambda_2,\lambda_3$ and λ_4 to solve:

$$\min_{x_1,x_2}\mathcal{L}(x_1,x_2,\lambda_1,\lambda_2,\lambda_3,\lambda_4)$$

2. Lagrangian Dual of a QP

Consider the general form of a convex quadratic program, with $Q\succ 0$:

$$\begin{aligned} \min_{\vec{x}} & \ \frac{1}{2}\vec{x}^\top Q\vec{x} \\ \text{s.t. } & A\vec{x}\leq\vec{b} \end{aligned}$$

(a) Write the Lagrangian function $\mathcal{L}(\vec{x},\vec{\lambda})$.

(b) Write the Lagrangian dual function, $g(\vec{\lambda})$.

(c) Show that the Lagrangian dual problem is convex by writing it in standard QP form. Is the Lagrangian dual problem convex in general?