EECS 127/227AT Optimization Models in Engineering
Spring 2020

Discussion 8

1. **Complementary slackness**
   Consider the problem:

$$\begin{aligned} p^* = \min_{x \in \mathbb{R}} \ & x^2 \\ \text{s.t.} \ & x \geq 1, \ x \leq 2. \end{aligned}$$

   (a) Does Slater's condition hold? Is the problem convex? Does strong duality hold?

   (b) Find the Lagrangian $\mathcal{L}(x, \lambda_1, \lambda_2)$.

   (c) Solve for $x^*, \lambda_1^*, \lambda_2^*$ that satisfy KKT conditions.

   (d) Can you spot a connection between the values of λ_1^*, λ_2^* in relation to whether the corresponding inequality constraints are strict or not at the optimal x^*?

   (e) Find the dual function $g(\lambda_1, \lambda_2)$ so that the dual problem is given by,

$$d^* = \max_{\lambda_1, \lambda_2 \in \mathbb{R}^+} g(\lambda_1, \lambda_2). \tag{1}$$

   (f) Solve the dual problem in (1) for d^*.

2. **[Optional] Simple constrained optimization problem with duality**
   Consider the optimization problem

$$\begin{aligned} \min_{x_1, x_2 \in \mathbb{R}} \ & f(x_1, x_2) \\ \text{subject to} \ & 2x_1 + x_2 \geq 1 \\ & x_1 + 3x_2 \geq 1 \\ & x_1 \geq 0, \ x_2 \geq 0 \end{aligned}$$

   (a) Express the Lagragian of the problem $\mathcal{L}(x_1, x_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$

   Solve the following problems analytically and give the minimizing x_1^*, x_2^*: *Hint:* Use duality if the problem is hard to solve. Use the graphs in Figure 1 to "dualize" only some constraints:

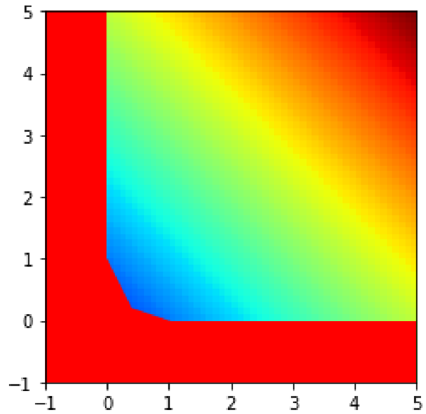
   (b) $f(x_1, x_2) = x_1 + x_2$

   (c) $f(x_1, x_2) = -x_1 - x_2$

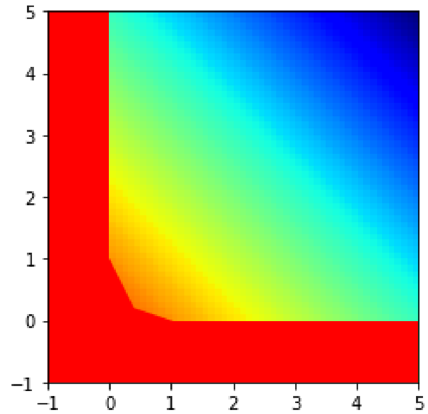
   (d) $f(x_1, x_2) = x_1$

   (e) $f(x_1, x_2) = \max\{x_1, x_2\}$

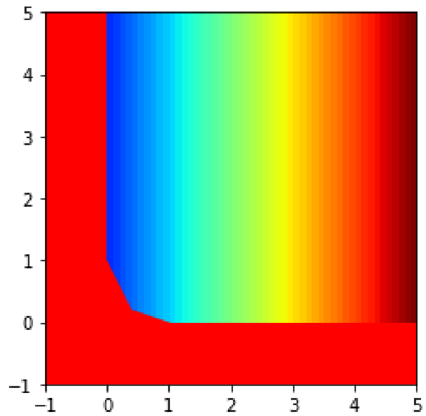
   (f) $f(x_1, x_2) = x_1^2 + 9x_2^2$



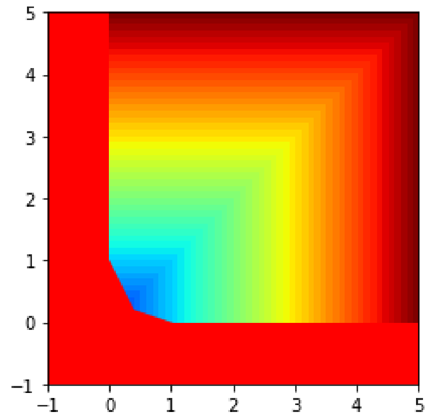
(b)



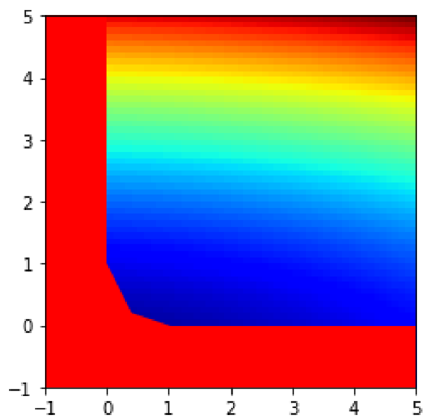
(c)



(d)



(e)



(f)

Figure 1: Heatmap of 2(b), 2(c), 2(d), 2(e) and 2(f): $\vec{x}^{\star} = (\frac{2}{5}, \frac{1}{5})$. In red is the unfeasible points, then the level sets are shown with colors; blue points are points (x_1, x_2) with the lowest value $f(x_1, x_2)$, red points are the ones with highest value.